Pseudo Steady State Gas Flow in Tight Reservoir under Dual Mechanism Flow

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ABSTRACT: *Gas reservoirs with low permeability (k<0.1 mD) are among the unconventional reservoirs and are commonly termed as "Tight Gas Reservoirs". In conventional gas reservoirs that have high permeability, the flow of gas is basically controlled by the reservoir permeability and it is calculated using the Darcy equation. In these reservoirs, gas flow due to gas diffusion is ignored compared to Darcy flow. However, diffusion phenomenon has a significant impact on the gas flow in tight gas reservoirs and the mechanism of gas diffusion can no longer be ignored in comparison to Darcy flow. In this study, a dual mechanism based on Darcy flow as well as diffusion is used for the gas flow modeling in tight gas reservoirs. The diffusivity equation is obtained using this method that it indicates the gas flow in a porous media. The conventional dry gas pseudo pressure function is not able to linearize the diffusivity equation including diffusion effect. Subsequently, a new real gas pseudo pressure function is used and a novel real gas pseudo time function is introduced. These pseudo functions consider changes in gas properties with pressure and linearize the diffusivity equation. The linear diffusivity equation is solved analytically for constant gas flow boundary condition under Pseudo Steady State (PSS) situation. Then, pseudo steady state analytical solution, based on new functions of pseudo pressure and pseudo time, is obtained. The calculation of reservoir parameters such as permeability, effective diffusion coefficient and original gas in place (OGIP) using reservoir data is the first application of analytical solution. Reservoir data is required to analysis the results of application of introduced model in low permeability gas reservoir.*

KEY WORDS: *Tight gas reservoir, Real gas pseudo pressure function, Pseudo time function, Effective diffusion coefficient, OGIP.*

INTRODUCTION

Gas reservoirs with permeability less than 0.1 mD (9.869E-17 m^2) are termed as "Tight Gas" reservoirs. These reservoirs worldwide present a considerable potential for hydrocarbon production [1-3].

Al-Hussainy et al. [4] introduced a pseudo pressure function for real gas as,

$$m(p) = 2\int_{p^o}^{p} \frac{p}{\mu z} dp \qquad (1)$$

Where gas properties are considered functions of pressure. Then, he used the aforementioned relation to linearize the diffusivity equation for real gases. Also,

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this function is known as conventional dry gas pseudo pressure function. *Kikani & Pedrosa* [5] modified real gas pseudo pressure function for situations where the reservoir permeability (k) is a function of pressure as,

$$m'(p) = 2\int_{p^o}^{p} \frac{p\,k(p)}{\mu z} dp \tag{2}$$

Agarwal [6] and *Lee & Holditch* [7] employed pseudo time concept to linearize the transient gas flow equation for gas reservoirs having large hydraulic fractures as shown below,

$$t_a = \int_0^t \frac{1}{\mu(p_w)\,c_t(p_w)} dt \tag{3}$$

Variations of viscosity as well as compressibility were considered as a function of wellbore pressure (p_w). *Fraim & Wattenbarger* [8] proposed a normalized pseudo time function by considering the variations of gas properties as follow,

$$t_n = \int_0^t \frac{(\mu\, c_t)_i}{\mu(\overline{p})\; c_t(\overline{p})} dt \tag{4}$$

Later, *Ibrahim et al.* [9] introduced a new normalized pseudo time function by considering reservoir properties to be functions of average pressure as follow,

$$t_n = (\varphi\mu\, c_t)_i \int_0^t \frac{1}{\varphi(\overline{p})\mu(\overline{p})\; c_t(\overline{p})} dt \tag{5}$$

Ertekin et al. [10] suggested that flow mechanisms of gas in tight reservoirs are influenced by Darcy flow as well as gas molecular concentration gradient. In fact, in tight porous systems gas flow due to diffusion effects might be the dominant mechanism as discussed by *Ayala et al.* [11, 12]. Consequently, gas flow velocity in tight reservoirs may be formulated as below:

a. Darcy flow velocity due to pressure gradient [13],

$$v^D = -\frac{k}{\mu}\nabla p \tag{6}$$

b. Diffusion flow velocity due to gas molecular concentration gradient,

$$v^F = -\varphi S_g \frac{D_e}{\overline{\rho}_g}\nabla\overline{\rho}_g = -\varphi S_g D_e c_g \nabla p \tag{7}$$

Therefore, the total gas flow velocity will be,

$$v^T = v^D + v^F \tag{8}$$

Applying some simple manipulations,

$$v^T = -(\frac{k}{\mu} + \varphi S_g D_e\, c_g)\nabla p \tag{9}$$

Note that, D_e represents the combined diffusion mechanisms that might happen in the porous media. In gas reservoirs with reasonable permeability, the contribution of diffusion effects to flow could be ignored and Eq. (9) reduces to Darcy flow, Eq. (6).

In the previous study [14], a dual mechanism based on Darcy flow as well as diffusion was presented for the gas flow modeling in homogeneous porous media. Then, a new pseudo pressure function was introduced and used to linearize the gas flow governing differential equation with the diffusion effect. Finally a set of analytical solutions was presented for analyzing steady-state and transient gas flow through porous media including effective diffusion.

THEORITICAL SECTION

Mathematical model

Assuming a homogeneous tight gas reservoir, the flow of gas is obtained from the continuity equation as below [14],

$$\nabla\cdot\left(\rho\; v^T\right) = -\frac{\partial}{\partial t}(\rho\varphi) \tag{10}$$

Substitution of total gas flow velocity, v^T, from Eq. (9) and real gas equation of state as $\rho = pM/zRT$ in Eq. (10) and employing some manipulation yields [14],

$$\nabla.\left(\frac{p}{z}\left(\frac{k}{\mu} + \varphi S_g D_e c_g\right)(\nabla p)\right) = \varphi\frac{p}{z}c_t\frac{\partial p}{\partial t} \tag{11}$$

The above nonlinear partial differential equation represents the flow of gas in tight reservoirs including diffusion effect. The conventional dry gas pseudo pressure function, Eq. (1) is not able to linearize Eq. (11). Therefore, the followings pseudo functions are introduced,

Pseudo pressure function

In order to linearize the Eq. (11), a new pseudo pressure function was defined as follows [14],

$$m'(p) = \int_{p^o}^{p} \frac{2p}{z}\left(\frac{k}{\mu} + \varphi S_g D_e c_g\right) dp \qquad (12)$$

Using Eq. (12) in Eq. (11) and employing some simple mathematical manipulation yields [14],

$$\nabla.(\nabla m'(p)) = \frac{\varphi c_t}{(\frac{k}{\mu} + \varphi S_g D_e c_g)} \frac{\partial m'(p)}{\partial t} \qquad (13)$$

Now, the Left Hand Side (LHS) of Eq. (13) is linear. However, the Right Hand Side (RHS) of Eq. (13) contains strongly pressure dependent parameters and it is nonlinear; especially during pseudo steady state flow situation and buildup tests while the average reservoir pressure changes sharply.

Pseudo time function

In order to linearize the RHS of Eq. (13), a novel pseudo time function is introduced as below,

$$t_a'(t) = \int_0^t \frac{(\frac{k}{\mu(\overline{p})} + \varphi(\overline{p}) S_g(\overline{p}) D_e c_g(\overline{p}))}{\varphi(\overline{p}) c_t(\overline{p})} dt \qquad (14)$$

Derivative of pseudo time function with respect to time is shown below,

$$\frac{\partial t_a'}{\partial t} = \frac{(\frac{k}{\mu(\overline{p})} + \varphi(\overline{p}) S_g(\overline{p}) D_e c_g(\overline{p}))}{\varphi(\overline{p}) c_t(\overline{p})} \qquad (15)$$

Gas flow equation with diffusion effect

Substitution of Eq. (15) in Eq. (13) becomes as,

$$\nabla.(\nabla m'(p)) = \frac{\partial m'(p)}{\partial t_a'} \qquad (16)$$

In cylindrical coordinate system Eq. (16) becomes as,

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial m'(p)}{\partial r}\right] = \frac{\partial m'(p)}{\partial t_a'} \qquad (17)$$

Analytical solutions

The problem under consideration is that of radial flow from a finite cylindrical gas reservoir with sealed upper and lower surfaces at a constant production rate. The inner boundary is the well. The reservoir is volumetric, that is, the outer boundary is sealed. Under constant production rate boundary condition, after pressure variation reaches the boundaries of reservoir, System enters into pseudo steady state. The initial and boundary conditions for pseudo steady state flow are as follows,

a. Constant gas flow rate at well,

$$\left(r \frac{\partial m'(p)}{\partial r}\right)_{r=r_w} = \frac{p_s q_s T}{\pi h T_s} \qquad \text{for all } t_a' \rangle 0 \qquad (18)$$

Where q_s is a constant gas flow rate.

b. Initially, uniform pressure prevails throughout the system,

$$m'(p) = m'(p_i) \qquad \text{at} \quad t_a' = 0, \ \text{for all r} \qquad (19)$$

c. No flow exists at the boundaries,

$$\left.\frac{\partial m'(p)}{\partial r}\right|_{r=r_e} = 0 \qquad , \text{ for all } t_a' \qquad (20)$$

In this situation using the Laplace Transformation, *Van Everdingen & Hurst* [15, 16] introduced the following analytical solution for gas flow equation (Eq. (17)) [17],

$$m'(p_i) - m'(p_w) = \qquad (21)$$
$$\frac{2 p_s q_s T_i}{T_s V_b} t_a'(t) + \frac{p_s q_s T_i}{\pi h T_s}\left[\ln(\frac{r_e}{r_w}) - \frac{3}{4}\right]$$

Noting that; original gas in place, $G = V_{pi} S_{gi} / B_{gi}$ pore volume, $V_{Pi} = \varphi_i V_b$ the bulk volume, $V_b = \pi(r_e^2 - r_w^2)h$ and $B_{gi} = T_i p_s z_i / T_s p_i$, Then Eq. (21) may be rewritten as,

$$m'(p_i) - m'(p_w) = \qquad (22)$$
$$= \frac{2 \varphi_i S_{gi} p_i q_s}{z_i G} t_a'(t) + \frac{p_s q_s T_i}{\pi h T_s}\left[\ln(\frac{r_e}{r_w}) - \frac{3}{4}\right]$$

Eq. (22) suggests that a graph of $[m'(p_i) - m'(p_w)]$ versus $t_a'(t)$ will be a straight line of slope;

$$SL = \frac{2 \varphi_i S_{gi} p_i q_s}{z_i G} \qquad (23)$$

And intercept,

$$I = \frac{p_s q_s T_i}{\pi h T_s}\left[\ln(\frac{r_e}{r_w}) - \frac{3}{4}\right] \qquad (24)$$

RESULTS AND DISCUSSION

It should be noted that, in the course of oncoming calculations one requires knowledge of the average reservoir pressure ($\overline{p}$), as a function of time. Since $\overline{p}$ is not known, calculations should be iterative. In the followings, the procedure to calculate $\overline{p}$ and G is explained:

Assuming k and D_e to be constant, the pseudo pressure function Eq. (12), can be written as bellow:

$$m'(p) = k\ m_1'(p) + D_e\ m_2'(p) \tag{25}$$

Where,

$$m_1'(p) = \int_{p^o}^{p} \frac{2p}{\mu z} dp \tag{26}$$

And,

$$m_2'(p) = \int_{p^o}^{p} \frac{2p\varphi S_g c_g}{z} dp \tag{27}$$

Similarly, Eq. (14), representing pseudo time function can be written as bellow:

$$t_a'(t) = k\ t_{a1}(t) + D_e\ t_{a2}(t) \tag{28}$$

Where,

$$t_{a1}(t) = \int_0^t \frac{dt}{\mu(\overline{p})\varphi(\overline{p})c_t(\overline{p})} \tag{29}$$

And,

$$t_{a2}(t) = \int_0^t \frac{S_g(\overline{p})c_g(\overline{p})\,dt}{c_t(\overline{p})} \tag{30}$$

As mentioned earlier, we need to know $\overline{p}$ in order to calculate the pseudo time functions presented in the Eqs. (29) and (30). Since $\overline{p}$ is not known, it is calculated from the gas material balance equation, which is given as [18],

$$\left(\frac{\overline{p}}{\overline{z}}\right) = \left(\frac{p_i}{z_i}\right)\left(1 - \frac{G_p}{G}\right) \tag{31}$$

A logical iteration technique consists of, evaluating $\overline{p}$ from an initial guess of the original gas in place (G). This initial estimate could be a value a little bit greater than the final cumulative gas production (G_p). The average pressure, $\overline{p}$ obtained in this way is used to calculate the pseudo time functions and then $[m'(p_i) - m'(p_w)]$ versus $t_a'(t)$ plot could be constructed. The slope of this plot is used to evaluate G, as suggested by Eq. (23). Subsequently, new value of G is used for the next iteration and this process continues until convergence tolerance for G is met. An algorithm for these calculations is presented in follow.

Determination of reservoir permeability, effective diffusion coefficient and OGIP

In a real situation, the production data, i.e., wellbore pressure versus time and flow rate versus time are known. These data is employed in the analytical solution in conjunction with an iterative technique to obtain reservoir permeability (k), reservoir effective diffusion coefficient (D_e) and original gas in place (G);

Noting that Eq. (22) represents pseudo steady state gas flow in the reservoir and assuming permeability (k) and effective diffusion coefficient (D_e) to be constant, the left hand side of the Eq. (22) can be expanded as below:

$$m'(p_i) - m'(p_w) = k\,(m_1'(p_i) - m_1'(p_w)) + D_e\,(m_2'(p_i) - m_2'(p_w)) \tag{32}$$

Where

$$m_1'p_w = (m_1'(p_i) - m_1'(p_w)) \tag{33}$$

And

$$m_2'p_w = (m_2'(p_i) - m_2'(p_w)) \tag{34}$$

Then

$$m'(p_i) - m'(p_w) = k\ m_1'p_w + D_e\ m_2'p_w \tag{35}$$

Therefore

$$k\,m_1'p_w + D_e\,m_2'p_w = \frac{2\varphi_i S_{gi} p_i q_s}{z_i G}(k\,t_{a1}(t) + D_e t_{a2}(t)) + \frac{p_s q_s T_i}{\pi h T_s}\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4}\right] \tag{36}$$

Employing well bore pressure data versus time, the values of $m_1'p_w$ and $m_2'p_w$ can be obtained. To determine the values of $t_{a1}(t)$ and $t_{a2}(t)$ (Eq. (29) and Eq. (30)) the values of average reservoir pressure ($\overline{p}$) is needed which can be obtained from Eq. (31). Using above defined parameters, Eq. (36) is simplified as,

$$k\ m_1'p_w + D_e\ m_2'p_w = SL\ (k\ t_{a1}(t) + D_e\ t_{a2}(t)) + I \tag{37}$$

Eq. (37) is rearranged as Eq. (38),

$$k(m_1'p_w - SL\ t_{a1}(t)) + D_e(m_2'p_w - SL\, t_{a2}(t)) = I \qquad (38)$$

External reservoir radius is determined as,

$$r_e = \sqrt{\left(\frac{G\, p_s\, z_i T_i}{\pi h \varphi_i\, S_{gi}\, p_i\, T_s} + r_w^2\right)} \qquad (39)$$

Cumulative gas production can be determined as below,

$$G_p = \int_0^t q dt = t\, q_s \qquad (40)$$

Iterative procedure to determine the average reservoir pressure $\bar{p}$, k, D_e and OGIP from PSS gas flow data

The average reservoir pressure ($\bar{p}$) vs. time, reservoir permeability (k), reservoir effective diffusion coefficient (D_e) and reservoir original gas in place (G) could be determined employing an iterative technique as below,

1- Cumulative gas production (G_p) with respect to time is calculated using Eq. (40).

2- Make an initial guess for G which should be grater than final G_p.

3- Using G and Eq. (31) determine $\bar{p}$ versus time.

4- Using $\bar{p}$ and Eqs. (29) and (30) determine $t_{a1}(t)$ and $t_{a2}(t)$.

5- Using Eqs. (23), (39) and (24) determine values of SL and I.

6- Using wellbore pressure data and Eqs. (33) and (34) calculate the values of $m_1'p_w$ and $m_2'p_w$.

7- Using Eq. (38), and values of SL and I determine k and D_e employing least square technique.

8- From Eq. (37), determine G from the slope of $(k\ m_1'p_w + D_e\ m_2'p_w)$ curve vs. $(k\ t_{a1}(t) + D_e\ t_{a2}(t))$.

9- Go to step 1 and iterate until the value of G converges to a constant value.

CONCLUSIONS

Flow of gas in tight gas reservoirs was modeled using dual flow mechanism. Linearization of the equation was made by introducing new pseudo pressure and pseudo time functions. The equation was then solved analytically. An algorithm to evaluate reservoir parameters such as permeability (k), effective diffusion coefficient (D_e) and OGIP (G) was introduced.

Nomenclature

c	Compressibility, 1/Pa
D	Diffusion coefficient, m^2/s
h	Formation thickness, m
k	Permeability, m^2
M	Molecular weight of gas, kg/kmol
$m'(p)$	New real gas pseudo pressure, Pa.m^2/s
$m'(p_i)$	m'(p) at initial reservoir pressure
$m'(p_w)$	m'(p) at wellbore pressure
p	Pore pressure, Pa
q	Gas flow rate, m^3/s
r	Radial distance from center of Wellbore, m
R	Universal gas constant
S	Fluid saturation, fraction
T	Temperature, K
t	Time, s
$t_a'(t)$	Pseudo time function, m^2
v	Velocity, m/s
z	Gas deviation factor

Greek Letters

μ	Gas viscosity, Pa.s
ϕ	Porosity, fraction
$\bar{\rho}$	Molar density, kmol/m^3
ρ	Fluid density, kg/m^3

Subscripts

e	Effective
g	Gas
i	Initial
s	Standard conditions
w	Wellbore

Superscripts

D	Darcian component
F	Fickian component
T	Total

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