

APENDIX

The governing partial differential equations of our problem are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_c B_o^2}{\rho_f} u - \frac{\nu}{k_o} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left\{ D_b \frac{\partial c}{\partial y} \frac{\partial T}{\partial y} + D_t \left(\frac{\partial T}{\partial y} \right)^2 \right\} \quad (3)$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D_b \frac{\partial^2 c}{\partial y^2} + \frac{D_t}{T_f} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

$$u \frac{\partial n}{\partial x} + v \frac{\partial n}{\partial y} + \left(\frac{b_{ch} W_{cs}}{c_w - c_f} \right) \left\{ \frac{\partial}{\partial y} \left(n \frac{\partial c}{\partial y} \right) \right\} = D_m \frac{\partial^2 n}{\partial y^2} \quad (5)$$

To transform the above system of equations into Ordinary ones, we will utilize appropriate transformations given above in (8).

Since in terms of stream function, the velocity components have form:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}.$$

$$u = \frac{\partial \psi}{\partial y} = \left(\frac{a_o}{v} \right)^{\frac{1}{2}} x f' (a_o v)^{\frac{1}{2}} = a_o x f', \quad v = -\frac{\partial \psi}{\partial x} = -\left(a_o v \right)^{\frac{1}{2}} f$$

By taking derivatives, we obtain:

$$\frac{\partial u}{\partial x} = a_o f' \Rightarrow \frac{\partial^2 u}{\partial x^2} = 0, \quad \frac{\partial u}{\partial y} = \left(\frac{a_o}{v} \right)^{\frac{1}{2}} a_o x f'' \Rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{a_o^2}{v} x f'''$$

And

$$\frac{\partial v}{\partial y} = -a_o f'$$

Now, by putting values of derivatives in Eq. (1), it is identically satisfied.

In the similar manner, Substitution of values of derivatives in Eq. (2) yields the form:

$$a_o x f' (a_o f') + \left(-\left(a_o v \right)^{\frac{1}{2}} f \right) \frac{a_o^{\frac{3}{2}}}{v^{\frac{1}{2}}} x f'' =$$

$$\nu \left(\frac{a_o^2}{v} x f''' \right) - \frac{\sigma_c B_o^2}{\rho_{fl}} a_o x f' - \frac{\nu}{k_o} a_o x f'$$

By simplifying above equation, we have obtained Eq. (9):

$$f''' + f f'' - f'^2 - M(f') - \gamma(f') = 0$$

Similarly, by attaining derivatives and substituting their values, we have converted Eqs. (3)-(5) into Eqs. (10)- (12) respectively.