A NEW MODEL FOR DESIGNING GAS DISTRIBUTION NETWORKS

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ABSTRACT

A new model for the design of the city gas distribution network has been presented. On the basis of mass flow rates and pressure balances a nonlinear system of equation is obtained. A method for solving the latter is outlined which gives accurate results. Also, this method, when compared to the other accessible ones, takes less computer time. Finally in this method the actual length of network and pipe diameter can be considered. This is in contrast to the other methods in which certain quantities called "equivalent length" for different pipe size should be used.

INTRODUCTION

Determination of pressure, pipe diameter and location of gas sources are the main problems in the design of gas distribution networks. There are different methods for calculating the above mentioned parameters. Almost all of them make use of the first and the second kirchoff's laws. Among these methods, Hardy Cross is the most famous one.

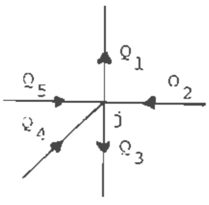
The intention of this report is first to review the above procedures and then present the mathematical model based on mass and pressure balance.

1-Numerical Methods for Design Calculation of Gas Distribution Network.

Methods which are used for the design calculation of gas distribution system are based on kirchoff's laws for steady state flow in a network. As a result of applying these laws, a system of equations is obtained. Kirchoff's laws for a network are:

1-1-Kirchoff's laws:

First law: The rate of flow into each junction or node must be equal to the rate of flow out of the junction. In other words, the algebraic sum of all flow rates must be equal to zero. The first law is applied to node j in Fig(1) as following.



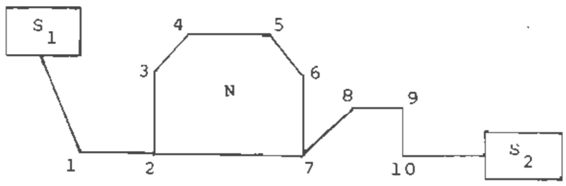
$$\sum_{i=1}^{5} Q_i = 0 \qquad 1$$

or:

$$-Q_1+Q_2-Q_3+Q_4+Q_5=0 \qquad 2$$

Fig(1)

Second law: The algebraic sum of the pressure differences across the pipe sections which form a closed loop must be equal to zero. This law when applied to Fig (2) will be,



Fig(2)

For loop N:

$$(P_3-P_2)+(P_4-P_3)+(P_5-P_4)+(P_6-P_5)+(P_7-P_6)+(P_2-P_7)=0 \qquad 3$$

For source connection S_1-S_2

$$(P_2-P_1)+(P_7-P_2)+(P_8-P_7)+(P_9-P_8)+(P_{10}-P_9)=(P_{10}-P_1) \qquad 4$$

In a gas piping system, the pressure quantity obtained through the use of the flow equation is expressed as $\Delta(P^2)$. Therefore, in Fig(2) kirchoff's second law is also expressed in terms of difference in the squares of the pressures across each pipe section. Thus equation 3 and 4 can be rewritten as:

$$(P_3^2-P_2^2)+(P_4^2-P_3^2)+(P_5^2-P_4^2)+(P_6^2-P_5^2)+(P_7^2-P_6^2)+(P_2^2-P_7^2)=0 \qquad 5$$

$$(P_2^2-P_1^2)+(P_7^2-P_2^2)+(P_8^2-P_7^2)+(P_9^2-P_8^2)+(P_{10}^2-P_9^2)=(P_{10}^2-P_1^2) \qquad 6$$

Therefore, the conditions imposed by kirchoff's laws results in a system of equ -

ations which can be solved by following procedures.

1-2-Simultanous solution of the system of Equations.

1-2-1-Trial-and-Error Solution:

For many years the only method used to solve network flow problems was the trial - and - error procedure.Although two investigators will never follow exactly the same path in the trial-and-error solution of a network problem,almost every one uses variations of the following steps:

a-A specific flow rate is assumed for each pipe section such that they satisfy kirchoff's first law at all nodes.

b-Pressure drops in the various pipe sections are calculated by use of a flow equation.

c-Pressure drops around closed loops and along continuous pipe sections between sources are summed.The values of these summations are then checked through kirchoff's second law.

d-The flow rates in step (a) are modified in response to the findings in step 'c'.

e-Repeat steps 'b','c' and 'd'.

f-Modification of the flow rates is continued until the calculated pressure drops satisfy kirchoff's second law within acceptable tolerances.Although the trial-and-error procedure is very tedious and errors are difficult to avoid,it is still widely used in the manual solution of network flow problems.

1-2-2-Systematic solution procedures

Several orderly numerical methods of solving gas distribution network flow problems have been developed.These methods solve the simultanous equations obtained by the application of kirchoff's laws to a specific gas distribution network problem.Two of the systematic solution procedures are as follows.

a-Hardy Cross Method(1,2)

An initial flow rate in each pipe section consistent with kirchoff's first law is assumed:then pressure losses are calculated for the assumed flow rate.The pressure losses must satisfy kirchoff's second law.Any unbalanced in the pressure drops is corrected by a modification in the flow rate at each loop.This correction is subtracted from the flow rate for each pipe section in the loop.With successive applications of this correction procedure to loops and source connections in a gas distribution network,a flow distribution that satisfies both of kirchoff's laws is gradually approached.

b-Pressure Balancing Method

In contrast to Hardy Cross method,in this method a pressure at each node in the gas distribution network is assumed. Then the flow rate for each pipe section is calculated.The flow rates must satisfy kirchoff's first law.Otherwise,they are corrected by successive calculation and application of pressure corrections until a set of pressure is obtained that satisfies both kirchoff's laws to the desired degree of accuracy.Since the pressure assumption required by this method are much easier to make than the flow rate assumptions of the Hardy Cross method,the design engineers favor the pressure-balancing method over the Hardy Cross.It is particularly preferred for the digital computer solution of very large gas distribution network problems.

2.Mathematical Model for the Design Calculation of Gas Distribution network

The mathematical model which is proposed here is based on steady state material balance in a gas distribution network. By this method a system of non-linear equations with respect to unknown pressures at each node is obtained in the following manner.

2-1-Generation the system of non-linear Equations

Consider a pipe section in a gas distribution network which connects node i to node j.The flow rate in the pipe section is related to pressure losses 3 according to the following relation:

$$Q_{ij}=C_{ij}(P_i^2-P_j^2)^M \qquad 7$$

Where M is a quantity less than one. In order to show the direction of gas flow rate equation 7 can be rearranged such that:

$$Q_{ij}=C_{ij}(P_i-P_j)(P_i+P_j)^M|P_i-P_j|^{M-1} \qquad 8$$

In the above equation,if $P_i > P_j$ then $Q_{ij} > 0$ and gas flows from node i to node j. If $P_i < P_j$ then $Q_{ij} < 0$ and gas flows from node j to node i. Material balance for each node j will be:

$$\sum_{i=1}^{n} Q_{ij}=\sum_{i=1}^{n} C_{ij}(P_i+P_j)^M(P_i-P_j)\,|P_i-P_j|^{M-1}=0 \qquad 9$$

In case, where sources are connected to node j, these sources are called z_j and then equation 9 will be changed to:

$$\sum_{i=1}^{n} Q_{ij}-\sum_{k=1}^{m}(z_j)_k=F_j(\vec{p})=0 \qquad 10$$

Where

$$\vec{P}=(P_1P_2P_3\ldots\ldots P_n)^t$$

Substituting equation 8 into 9 yields:

$$F_j(\vec{P})=\sum_{i=1}^{n} C_{ij}(P_i-P_j)(P_i+P_j)^M|P_i-P_j|^{M-1}-\sum_{k=1}^{m}(Z_j)_k=0 \qquad 11$$

Applying equation 11 to a gas distribution network with n nodes, a system of non-linear equations is obtained in which node pressures are the unknowns.

2-2-Solution to system of non-linear equations

let the general form of system of equation 11 be such that:

$$F_j(\vec{P})=0 \qquad j=1,2,3,\ldots\, n \qquad 12$$

Or

$$F_1(P_1P_2P_3\ldots P_n)=0$$
$$F_2(P_1P_2P_3\ldots P_n)=0 \qquad 13$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$F_n(P_1P_2P_3\ldots P_n)=0$$

If $\vec{p}=(p_1p_2\ldots p_n)^t$ is the solution to the equation 13 then $\vec{p}_n$ is the pressure at the proper node.

2-2-1-Procedure to calculate $\vec{p}$

A suitable rearrangement of the system of equation 13 can basically constitute the n functions $f_j(\vec{P})$ such that

$$\vec{P}_j=f_j(\vec{P}) \qquad 14$$

One can search for values $\vec{p}$ that will generate a solution that yields

$$\vec{P}_j=f_j(\vec{P}_j) \qquad 15$$

This is accomplished by assuming a pressure vector.

$\vec{P}_{j0}=[P_{10}P_{20}P_{30}\ldots P_{n0}]^t$ as a starting guess and substituting it into equation 14 which gives a new pressure vector. By successive iteration, finally a pressure vector $\vec{P}_{j1}=[P_{11}P_{21}P_{31}\ldots P_{n1}]^t$ will be reached that satisfies the following condition

$$\vec{P}_{i1}-\vec{P}_{i1-1}\leqslant h$$

Where h is a small value

Then pressure vector $\vec{P}_{i1}$ is the solution to the system of equation 13 or 14

Now consider equation 11 and rearrange it in the form of equation 14.

$$\sum_{i=1}^{n} C_{ij}(P_i-P_j)(P_i+P_j)^M|P_i-P_j|^{M-1}=\sum_{k=1}^{m}(Z_j)_k \qquad 16$$

or

$$\sum_{i=1}^{n} C_{ij}P_i(P_i+P_j)^M|P_i-P_j|^{M-1}-\sum_{i=1}^{n} C_{ij}P_j(P_i+P_j)^M|P_i-P_j|^{M-1}=\sum_{k=1}^{m}(Z_j)k \qquad 17$$

then

$$P_j=\frac{\sum_{i=1}^{n} C_{ij}P_i(P_i+P_j)^M|P_i-P_j|^{M-1}-\sum_{k=1}^{m}(Z_j)_k}{\sum_{i=1}^{n} C_{ij}(P_i+P_j)^M|P_i-P_j|^{M-1}}=f_j(\vec{P}) \qquad 18$$

When equation 18 is applied to gas distribution network with n nodes a system of equation in the form of equation 14 or 18 is obtained. Solution to the system of equation is outlined in 2-2-1 section and gives the pressure at each node of gas distribution network.

3- DISCUSSION

Several different gas distribution networks have been studied by the mathematical model. Since the results of a networks with 70 nodes were available, it is considered here for the sake of comparison.

The network with 70 nodes has 5 sources Fig(3). These sources are connected to nodes 2,8,38,54 and 61. Load in each node expressed in MCF/hr. In table(1) network specifications and in table (2) actual network load and calculated load are presented. Columns 2,3 and 4 in table (3) represent the calculated pressures through mathematical method using Spitzglass, Weymouth (4) and Monem Sofregas* flow equations respectively. Columns 5 and 6 represent pressure values obtained by means of Monem Sofregas and Hardy Cross methods both of which are being used by National Iranian Gas Company.

In Monem Sofregas method, system of non-

*Monem Sofregas is the name of computer program which is sold to Iranian National Gas Company.

linear equations are linearized while in the mathematical model such approximation is avoided.Thus it is expected that the results given in column 4 must be more accurate when compared with results in column 5 on table (3).

The use of actual pipe length in setting up the system of non-linear equation in the mathematical method in contrast to the use of equivalent length in Hardy Cross method is another advantage of the mathematical model.

To continue this work,one can consider other flow equations such as panhandle,modi - fied panhandle,etc,and compare the results with those given in table (3).A Newton-Raphson method can also used to solve the system of equations 13 .

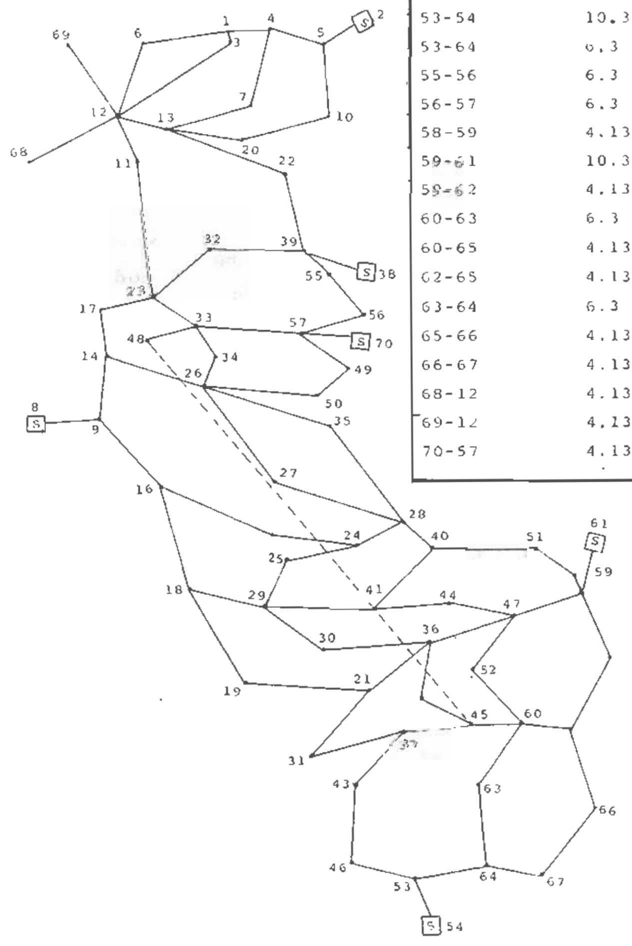
Table(1):Specification of 70 nodes net - work.

branch	Diameter(Inch)	Length(foot)
1-3	4.13	99
1-4	6.3	689
1-6	6.3	1542
2-5	15.3	990
3-12	4.13	3084
4-5	6.3	1214
4-7	4.13	1640
5-10	6.3	1640
6-11	6.3	1542
7-13	4.13	1640
8-9	10.3	492
9-14	6.3	1378
8-16	8.2	1312
10-20	6.3	1640
11-12	6.3	951
11-23	6.3	2854
12-13	6.3	623.4
13-20	6.3	1148.3
13-22	6.3	1706
14-17	6.3	804
14-16	6.3	1509
15-16	6.3	1312
15-24	6.3	1279.5
16-18	6.3	1312
17-23	6.3	804
18-19	6.3	1033.5
18-29	4.13	853
19-21	6.3	1033.5
21-31	6.3	1083
21-36	6.3	853
22-39	6.3	1706
23-32	6.3	1411
23-33	4.13	755
24-25	4.13	984
24-28	6.3	656
25-29	4.13	984
26-27	4.13	1640
26-34	4.13	722
26-35	4.13	1640
26-50	6.3	1509
27-28	4.13	1640
28-35	4.13	1640
28-40	6.3	525
29-30	4.13	984
29-41	4.13	853
30-36	4.13	1033.5

branch	Diameter(Inch)	Length(foot)
31-37	6.3	1083
32-39	6.3	1411
33-34	4.13	722
33-57	4.13	1509
36-42	4.13	1083
36-47	6.3	853
37-42	6.3	1033.5
37-43	4.13	853
38-39	10.3	492
39-55	4.13	918.6
40-41	6.3	820
40-51	4.13	1247

branch	Diameter(Inch)	length(foot)
41-44	6.3	982
42-45	4.13	1083
43-46	6.3	1033.5
44-47	6.3	984
45-48	6.3	1033.5
45-60	4.13	853
46-53	6.3	853
47-52	6.3	1083
47-59	8.23	853
43-33	6.3	1033.5
49-50	6.3	705.4
49-57	6.3	705.4
51-58	4.13	984
52-60	6.3	1083
53-54	10.3	328
53-64	6.3	853
55-56	6.3	1247
56-57	6.3	1247
58-59	4.13	984
59-61	10.3	328
59-62	4.13	1083
60-63	6.3	1033.5
60-65	4.13	853
62-65	4.13	1083
63-64	6.3	1033.5
65-66	4.13	853
66-67	4.13	1033.5
68-12	4.13	1837
69-12	4.13	1837
70-57	4.13	3281

Fig(3):70 nodes network map.



Table(2):Real and Calculated load for 70 nodes network.

NODE NO.	PRESSURE(PSI)	ACTUAL FLOW LOAD(MCF/HR)	CALCULATED FLOW LOAD(MCF/HR)
1	56.276	19.679	19.651
2	60.000	0.0	-423.995
3	55.846	122.470	122.465
4	57.104	25.210	25.206
5	59.600	0.0	-0.002
6	56.057	66.820	66.779
7	55.973	79.700	79.699
8	60.000	0.0	-700.645
9	59.500	0.0	-0.006
10	58.054	58.800	58.797
11	56.081	-0.0	-0.046
12	56.064	92.000	91.959
13	56.834	0.0	-0.010
14	56.205	0.0	-0.025
15	56.611	67.000	66.993
16	57.785	115.600	115.595
17	56.111	74.200	74.153
18	56.795	0.0	-0.008
19	56.382	90.550	90.515
20	57.339	0.0	-0.003
21	56.373	0.0	-0.031
22	57.719	66.570	66.568
23	56.131	0.0	-0.028
24	56.176	0.0	-0.019
25	56.020	32.170	32.165
26	53.632	0.0	-0.022
27	53.798	44.400	44.397
28	56.025	0.0	-0.029
29	56.119	0.0	-0.007
30	56.222	103.400	103.395
31	56.109	114.350	114.335
32	57.114	98.9	98.898
33	53.402	0.0	-0.103
34	52.767	98.900	98.896
35	52.760	105.900	105.899
36	56.525	0.0	-0.008
37	56.174	0.0	-0.012
38	60.000	0.0	-568.804
39	59.658	0.0	-0.003
40	56.090	74.340	74.319
41	56.244	21.200	21.188
42	53.240	132.570	132.567
43	56.697	109.300	109.296
44	56.663	78.220	78.215
45	54.068	0.0	-0.014
46	58.410	0.0	-0.001
47	57.794	0.0	-0.06
48	53.396	142.740	142.692
49	52.164	98.900	98.729
50	52.355	70.630	70.609
51	57.836	0.0	-0.001
52	56.833	117.530	117.521
53	59.794	70.630	70.628
54	60.000	0.0	-538.575
55	53.274	0.0	-0.014
56	52.297	98.900	98.859
57	52.177	35.300	35.221
58	59.185	62.790	62.789
59	59.603	0.0	-0.002
60	56.745	0.0	-0.010
61	60.000	0.0	-771.517
62	56.443	97.540	97.537
63	56.985	114.000	113.997
64	58.234	0.0	-0.002
65	56.471	0.0	-0.003
66	56.276	71.120	71.119
67	57.355	0.0	-0.000
68	54.982	37.000	37.000
69	53.424	60.000	60.000
70	50.286	35.300	35.299

Negative sign shows the feed sources that their pressures are 60 psig.

Table(3):Comparing the results of various methods with the mathematical method.

Node	new method			models that use in Iranian Gas Co	
	with spitzglass equation	with weymouth equation	with monem-sofre gas equation	monem sofre-gas model	hardy cross model
1	56.275	52.614	53.407	55.238	52.5
2	60.	60.	60.	60.	60.
3	55.848	51.856	52.773	54.71	51.7
4	57.104	54.214	54.865	56,215	54.1
5	59.6	59.206	59.006	59.427	59.1
6	56.07	52.265	53.038	55.139	52.1
7	55.873	52.354	53.042	54.654	52.3
8	60.	60.	60.	60.	60.
9	59.5	58.941	59.027	59.277	58.8
10	58.054	56.169	56.54	57.423	56.1
11	56.081	52.298	59.093	55.327	52.2
12	56.064	52.275	53.056	55,322	52.1
13	56.834	53.867	54.414	56.087	53.7
14	56.205	52.5	53.403	55.499	52.3
15	56.611	53.003	53.828	56.743	52.8
16	57.785	55.243	55.823	54.663	55
17	56.111	52.344	53.188	55.318	52.2
18	56.795	53.328	54.103	54.443	53.1
19	56.382	52.587	53.403	54.374	52.4
20	57.339	54.824	55.297	56.639	54.7
21	56.373	52.577	53.393	54.665	52.3
22	57.716	55.495	55.954	57.087	55.4
23	56.13	52.369	53.204	55.343	52.2
24	56.176	52.252	53.107	54.977	52
25	56.02	52.053	52.9	54.254	51.8
26	53.633	52.57	49.185	52.931	47.4
27	53.798	47.88	49.462	52.898	47.7
28	56.025	51,996	52.854	54.41	51.8
29	56.119	52.192	53.064	54.246	52
30	55.222	50.702	51.743	53.088	50.5
31	56.109	52.153	52.461	54.648	51.9
32	57.114	54.185	54.918	56.253	54.1
33	53.403	47.481	48.901	53.225	47
34	52.767	46.268	47.913	52.359	45.9
35	52.76	46.202	47.413	51.002	46.1
36	56.525	52.812	53.657	55.00	52.6
37	56.174	52.266	53.101	55.387	52
38	60.	60.	60.	60.	60.
39	59.658	59.287	59.307	59.366	59.2
40	59.09	52.116	52.988	54.4	51.9
41	56.244	52.357	53.242	54.549	52.1
42	53.24	47.157	48.845	52.748	46.9
43	56.697	53.248	54.069	55.775	53.1
44	56.663	53.079	53.945	55.13	52.9
45	54.068	48.682	50.111	55 797	48.3
46	58.41	56.779	57.142	57.776	56.7
47	57.794	55,259	55.881	56.854	55
48	53.396	47.481	48.886	56.346	47
49	52.164	44.83	46.809	52.362	44.6
50	52.355	45,14	47.091	52.362	44.9
51	57.836	55.636	46.147	55.016	55.5
52	56.833	53.549	54.316	55.871	53.4
53	59.794	59.567	59.584	59.388	59.5
54	60.	60.	60.	60.	60.
55	53.273	47.013	49.137	57.089	46.9
56	52.297	45.055	47.159	53.878	44.9
57	52.176	44.86	46.061	52.824	44.7
58	59.185	58.294	58.543	55.498	58.2
59	59.603	59.132	59.203	59.423	59
60	56.745	53.452	54.206	55.87	53.3
61	60.	60.	60.	60.	60.
62	56.443	53,192	54.-21	55.302	53.1
63	56.985	53.863	54.631	55.995	53.7
64	58.234	56.347	56.832	57.542	56.2
65	56.471	53.2	54,021	55.369	53.1
66	56.276	52.983	53,79	55.027	52.9
67	57.355	54.846	55.473	56.366	54.7
68	54.982	50.606	51.463	53.838	50.5
69	53.423	47.493	49.145	52.243	47.8
70	50.286	41.769	43.952	56.366	41.6

The relative relations are in appendix.

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