Investigation of Natural Convection in a Vertical Cavity Filled with a Anisotropic Porous Media

Muasavi, Sayed Mojtaba*+; Shahnazari, Mohammad Reza

Faculty of Mechanical Engineering, K.N. Toosi University of Technology,
P.O. Box 19395-1999 Tehran, I.R. IRAN

ABSTRACT: *In present paper, a numerical analysis for a rectangular cavity filled with a anisotropic porous media has been studied. It is assumed that the horizontal walls are adiabatic and impermeable, while the side walls of the cavity are maintained at constant temperatures and concentrations. The buoyancy force that induced the fluid motion are assumed to be cooperative. In the two extreme cases of heat-driven ($N \leq 1$) and solute-driven ($N \geq 1$) natural convection, scale analysis is applied to predict the order of magnitudes involved in the boundary layer regime. Especially, the effects of anisotropic properties on heat and mass transfer have been considered. The variation of Nusselt and Sherwood numbers for values of permeability ratio for a wide range of thermal Rayleigh number, buoyancy ratio, and Lewis number are presented. It is demonstrated that the anisotropic properties of the porous medium considerably modify the heat and mass transfer rates from that expected under isotropic conditions.*

KEY WORDS: *Natural convection, Vertical cavity, Non-isotrop, Porous media.*

INTRODUCTION

Natural convection through anisotropic porous media is of considerable interest in many industrial applications of geophysics, hydrology, oil extraction and other processes. Anisotropy usually arises from a symmetrical geometry of grains or fiber or scientific conditions. Precise studies for natural heat transfer in anisotropic porous media, despite its wide range of application, are few in number. Previous studies were about natural heat transfer in concentrated anisotropic porous media upon anisotropic environment for developing the flow for heat transfer in horizontal layers under lower heating . Earlier studies on natural convection in saturated anisotropic porous media have generally been confined to the effects of anisotropy on the development of the convective flow and heat transfer in a porous layer of infinite horizontal extent heated from below. A literature review may be found in the article of *Nilsen* and *Storesletten* [1] as well as in the book by *Nield* and *Bejan* [2] and *Ingham* and *Pop* [3]. Several studies have also been reported concerning natural convection in vertical anisotropic porous layers heated from the side. About concerned cause that is a square cavity filled with anisotropic porous media heated by two sides, *Ni* and *Beckerman* [4] presented a numerical solution. All studies performed [5,6] includes

* To whom correspondence should be addressed.
\+ E-mail: aerospace@kntu.ac.ir
1021-9986/08/2/39 7/$/2.70

the buoyancy force. Recently, interest for flows resulting from the combined action of both temperature and concentration has surged in view of its importance in various engineering problems. Despite this fact relatively little research work has been reported concerning double-diffusive convection in a porous medium, especially when the latter is anisotropic which is the case more often than not. In this paper natural convection as well as characteristics of mass and heat transfer in a cavity including anisotropic porous media have been studied. The equations were solved completely in numerical methods and the results are presented for wide range of effective parameters.

The results presented here are relevant to proper understanding of double-diffusive natural convection characteristics in anisotropic porous media.

THEORY AND FORMULATION

The studied system is shown in Fig. 1. Some values are assumed for height of cavity (H), width (L) and anisotropic porous media as well as permeability principal directions (K_X , K_Z) with horizontal and vertical coordinate. Different vertical walls are assumed at constant temperature and concentration, whereas horizontal walls are in isolated and impermeable form. the flow density in porous media is modeled by Boussinesq formula.

$$\rho = \rho_m [1 - \beta_T \Delta T - \beta_c \Delta C] \tag{1}$$

β_T and β_C represent for thermal concentration coefficient of expansion, respectively. The flow is considered laminar and the cavity is long enough in the direction of y so that we could consider this system as two-dimensional.

Conservation equations governing mass, momentum and energy are dimensionless as follows:

Dimensionless boundary conditions on the wall of the cavity are:

$$\frac{\partial \bar{u}}{\partial \bar{X}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \tag{2}$$

$$\bar{u} = -K \frac{\partial \bar{P}}{\partial \bar{X}} \tag{3}$$

$$\bar{w} = \frac{-\partial \bar{P}}{\partial \bar{Z}} + R_T (\bar{T} + N\bar{C}) \tag{4}$$

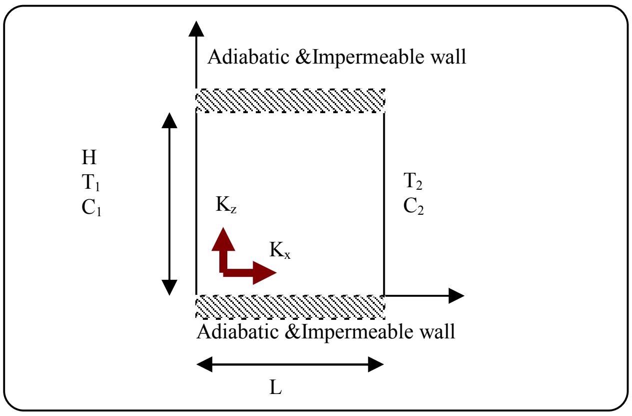

Fig. 1: Schematic of the studied system.

$$\bar{u} \frac{\partial \bar{T}}{\partial \bar{X}} + \bar{w} \frac{\partial \bar{T}}{\partial \bar{Z}} = \nabla^2 \bar{T} \tag{5}$$

$$\bar{u} \frac{\partial \bar{C}}{\partial \bar{x}} + \bar{w} \frac{\partial \bar{C}}{\partial \bar{Z}} = \frac{1}{Le} \nabla^2 \bar{C} \tag{6}$$

$$\bar{x} = 0, \quad \bar{u} = 0, \quad \bar{T} = 0.5, \quad \bar{C} = 0.5 \tag{7}$$

$$\bar{x} = \frac{1}{A}, \quad \bar{u} = 0, \quad \bar{T} = -0.5, \quad \bar{C} = -0.5$$

$$\bar{Z} = 0.1, \quad \bar{w} = 0, \quad \frac{\partial \bar{T}}{\partial \bar{Z}} = 0, \quad \frac{\partial \bar{C}}{\partial \bar{Z}} = 0$$

In the above equations dimensionless variables are determined as follow:

$$(\bar{x}, \bar{z}) = \frac{x, z}{H}, \qquad \bar{P} = \frac{P K_z}{\mu \alpha} \tag{8}$$

$$\bar{T} = \frac{T - T_m}{\Delta T}, \qquad T_m = \frac{T_1 + T_2}{2}$$

$$\bar{C} = \frac{C - C_m}{\Delta S}, \qquad C_m = \frac{C_1 + C_2}{2}$$

$$\Delta T = T_1 - T_2, \qquad \Delta \bar{C} = C_1 - C_2$$

The equations indicate five determinative dimensionless parameters. Theses parameters are thermal Rayleigh number (R_a), buoyancy ratio (N), Le number (Le), cavity dimensional ratio (A) and permeability ratio (K) that are defined as follow:

$$R_T = \frac{K_z g \beta_T \Delta T H}{\alpha \nu}, \qquad N = \frac{\beta_c \Delta C}{\beta_T \Delta T} \tag{9}$$

$$L_e = \frac{\alpha}{D}, \qquad K = \frac{K_x}{K_z}, \qquad A = \frac{H}{L}$$

The D, ν, g are gravity acceleration, cinematic viscosity, fluid and mass diffusivity coefficient and $\alpha = k/\rho C_f$ is effective thermal diffiusivity coefficient in porous media. Heat and mass rates in the direction of vertical walls are determine by heat and concentration fields. Also average Nusselt and Sherwood number is defined as follows:

$$\overline{Nu} = -\int_0^1 \left.\frac{\partial T}{\partial X}\right|_{x=0} dZ \tag{10}$$

$$\overline{Sh} = -\int_0^1 \left.\frac{\partial S}{\partial X}\right|_{x=0} dz \tag{11}$$

DIMENTIONAL ANALYSIS

Due to the effect of the different parameters of mass and heat transfer on each other, dimensional analysis of the equations (3)-(9) does not show a certain point. However in limiting conditions, mass driven or heat have main effect on the normal flow. By analyzing order of magnitude we could predict the boundary conditions.

In the boundary layer regime,most of the fluid motion is restricted to a thin layer of thickness δ_T and height *H* ($\delta_T \leq H$). In this region the continuity equation (3) requires the following balance:

$$\frac{\overline{u}}{\overline{\delta_T}} \approx \overline{w} \tag{12}$$

where $\overline{\delta_T} = \delta_T / H$ is the dimensionless thickness of the vertical boundary layer. Turning our attention to the momentum balance in the same layer, the following scales are dictated from equations (3) and (4), respectively:

$$\overline{u} \approx \overline{K}\frac{\Delta p_x}{\overline{\delta_T}} \tag{13}$$

$$(\overline{w}, \Delta p_z) \approx R_T \tag{14}$$

where Δp_x is the pressure change across the thermal layer and Δp_z the pressure difference between the horizontal boundaries. The energy equation (5), expressing a balance between convection and conduction of heat, gives:

$$(\frac{\overline{u}}{\delta_T}, \overline{w}) \approx (\frac{1}{\delta_T^2}, 1) \tag{15}$$

Combining the above equations taking into account the fact that $\overline{\delta_T} \leq 1$ results in:

$$\overline{w} \approx \frac{1}{\delta_T{}^2} \tag{16}$$

The scaling properties of the mass transfer are similar to the above equations. Limiting conditions can be expressed in relation to buoyancy ratio. N<<1 shows that heat driven value is higher than mass driven value and vice-versa, in N>>1 normal flow in environment causes from mass driven. By introducing K_{cr}, from order of magnitude of $R_T^{-1/2}$, order of magnitude of heat and mass boundary, layer thickness, Nusselt and Sherwood number can be expressed as follows:

N<<1 Heat Effective Gradient

$$Nu \sim R_T^{1/2}, \quad \overline{\delta_T} \sim NR_T^{-1/2} \qquad K \gg K_c$$

$$\overline{\delta_c} \sim (Le \cdot R_T)^{-1/2} \tag{17}$$

$$Sh \sim (Le \cdot R)^{1/2} \qquad Le \gg 1$$

$$\overline{\delta_c} \sim Le^{-1} \cdot R_T^{-1/2} \tag{18}$$

$$Sh \sim Le \cdot R_T^{1/2} \qquad Le \ll 1$$

$$Nu \sim KR_T, \quad \delta_T \sim (KR_T)^{-1} \qquad K \ll K_c$$

$$\overline{\delta_c} \sim (Le^{-1/2} \cdot R_T)^{-1} \tag{19}$$

$$Sh \sim K \cdot Le^{1/2} \cdot R_T \qquad Le \gg 1$$

$$\overline{\delta_c} \sim (KLe \cdot R_T)^{-1} \tag{20}$$

$$Sh \sim Le \cdot R_T^{1/2} \qquad Le \ll 1$$

N >> 1 Mass Effective Gradient

$$Sh \sim \frac{R_c^{1/2}}{K}, \quad \delta_c \sim R_c^{-1/2} \qquad K \gg K_c$$

$$\overline{\delta_T} \sim Le \cdot R_c^{-1/2} \tag{21}$$

$$N_u \sim \frac{Le^{-1} \cdot R_c^{1/2}}{\overline{K}} \qquad Le \gg 1$$

$$\overline{\delta_T} \sim Le^{1/2} \cdot R_T^{-1/2} \qquad Le \ll 1 \tag{22}$$

Table 1: Comparing predicted results with others.

	Le	2000	1000	400	200	100	R_T
Nu Sh	10	19.89 69.08	13.48 48.20	7.77 29.36	4.96 19.83	3.11 13.24	Present work
Nu Sh		-- --	-- --	9.69 30.73	19.83 5.61	3.27 15.61	Bejan
Nu Sh		19.90 69.26	13.47 48.32	7.77 28.41	4.69 19.86	3.11 13.25	Guio
Nu Sh	100	19.89 19.37	13.48 139.93	-- --	-- --	-- --	Present work
Bu Sh		19.90 196.62	13.47 14065	-- --	-- --	-- --	Guio

$$Sh \sim \frac{R_c^{1/2}}{\overline{K}}, \quad \overline{\delta_c} \sim (KR_c)^{-1} \qquad K << K_c \qquad (23)$$

$$\overline{\delta_T} \sim Le(KR_c)^{-1} \qquad Le >> 1 \qquad (24)$$

$$Nu \sim \frac{Le^{-1} \cdot R_c^{1/2}}{\overline{K}}$$

$$\overline{\delta_T} \sim Le^{1/2}(KR_c)^{-1} \qquad Le << 1 \qquad (25)$$

$$Nu \sim \frac{Le^{-1/2} \cdot R_c^{1/2}}{\overline{K}}$$

NUMERICAL SOLUTION

Equations (2) - (6) have been solved by finite volume (F.V). Conservation equations have been described on control volume and intergrated. Integral equations have been extended by combination method. Linear equations of conservation equations have been solved by ADI method.

Pressure speed conjugates have been analyzed by trial and error method in pressure correction (SIMPLEC Algorithm). Non-uniform mesh is applied in the programs and very fine grid is applied near walls. Maximum convergence error of all equations is subjected as 10^{-5}.

The calculation was done for a wide range of parameters. In order to control the accuracy of the calculation for isotropic case the adopted results are compared with existing results issued by the other researchers. The adopted results show a good agreement with the results of *Ni* and *Beckerman* [10] and *Goyeau* [11]. Whereas, the predicted values by Trevisan and Bejan [12] for Nusselt and Sherwood number especially R_T=400 is higher than the actual value. Table 1 shows the values.

RESULTS AND DISCUSSION

All indicated results here are for quadratic cavity (A=1). Fig. 1 shows the effect of permeability at Le=10, N=0 , $R_T = 10^5$ upon Sh and Nu.

As is shown in Fig. 2 Nusselt and Sherwood numbers, when K is low enough, go toward net diffiusive regime $(Nu \to 1, Sh \to 1)$. Also for K>>1 both numbers have asymptote. However, it is seen from Fig. 2 that both Nu and Sh reach maximum values when K $\geq$ 1 and are independent of this parameter when the latter is made larger. This situation corresponds to a porous medium with an isotropic permeability Kz.

The results shows the existence of three regimes for mass and heat transfer under problem circumstances for different values of K. These regimes are:

I) Net diffiusive for lower value of K.

II) An intermediate regime together with Nu(Sh) linear increasing based on K increase.

III) A convective regime, where the increased effect of K on Nu(Sh) is inconsiderable, these values shall have asymptotic value and shall be independent of (K) permeability ratio.

Effect of Rayleigh thermal number upon the variation of Nu(Sh) based on K is shown in the Fig. 3. As is shown transition shall start from lower values of K for higher R_T. Meanwhile for the lower amounts of Rayleigh thermal value, transition includes lower ranges of K.

In the limit N $\to\infty$, for which the buoyancy forces that drive the flow are mainly due to gradients of solute, the numerical results obtained in this study were also correlated to yield

$$Sh = 1 + 126\overline{K}^{-1} R_C^{0.42} \qquad (26)$$

$$K_{cr} = R_C^{-0.42}$$

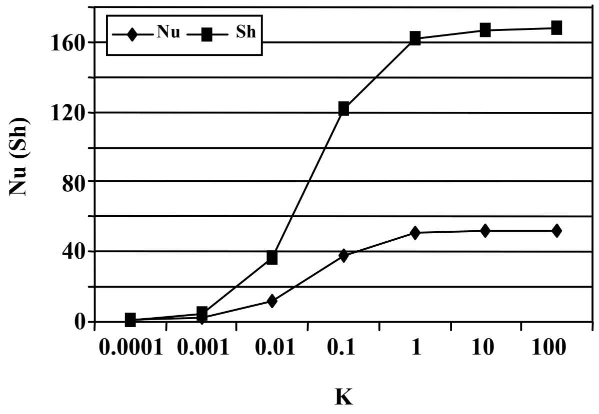

***Fig. 2:** The effect of permeability(K) upon Nu(Sh) (A=1, R_T=10^4, Le=10).*

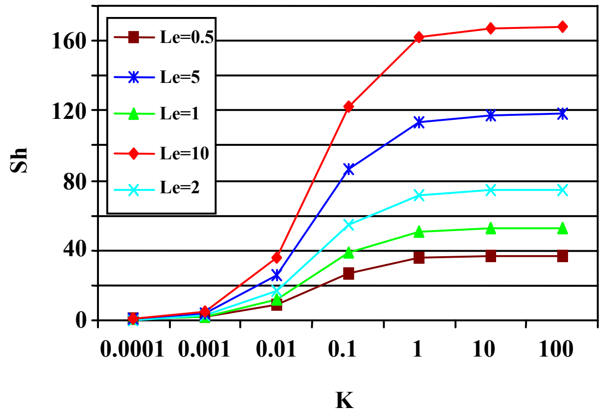

***Fig. 4:** The effect of L_e number upon Sherwood number variation based on K (A=1,R_T=10^3, N=0).*

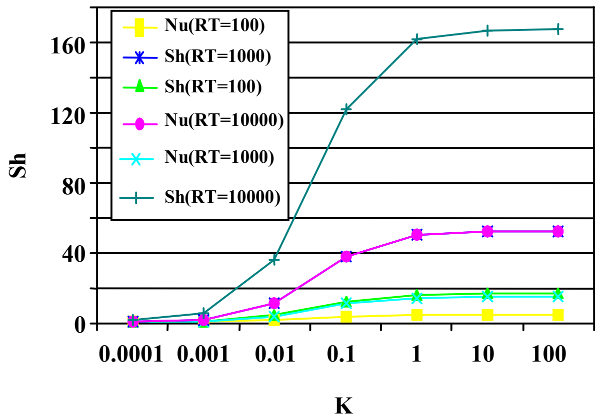

***Fig. 3:** The variation of Nu(Sh) based on K in different R_T (N=0, Le=10, A=1).*

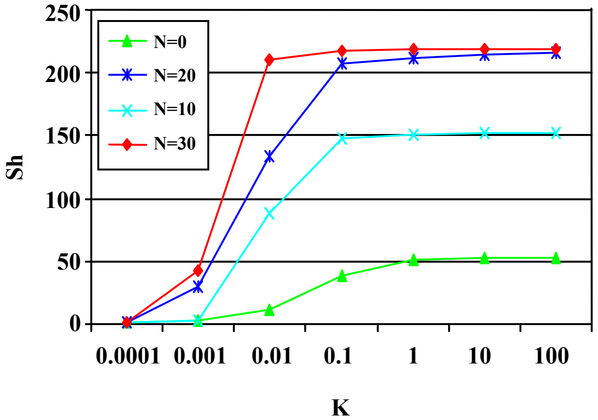

***Fig. 5:** The effect of N upon Sh number variation based on K (Le=10, $R_T$$10^3$, A=1).*

Fig. 4 shows the effect of Le number upon Sherwood number variation based on permeability ratio. Three referred regimes are clear at this point. For each definite K, Le increase shall be cause the increase of mass transfer. Also such increase causes required K for commencing transition. It is also observed from Fig. 4 that, as Le is increased, a smaller value of K is required to reach the diffusive regime. Thus, this latter is $\approx 10^{-3}$ 10^{-2} when Le =1 and for K $\approx$ reached for K when Le =10.

Finally the effect of N upon the variation of Sh number based on K is shown in the Fig. 5. By the increase of Sh amount for the higher amount of N at constant K as well as decrease of required amount of K for starting transition could be adopted from the figure.

For a given value of Le the evolution of Sh with K, i.e. the existence of three different regimes, is similar to what has been discussed in Fig. 2. For a given value of Le the evolution of Sh with K, i.e. the existence of three different regimes, is similar to what has been discussed in Fig. 2.

CONCLUSIONS

Numerical analysis in the field of mass and heat transfer simultaneously has been observed in a vertical cavity filled with anisotrop porous media where anisotrop axis are assumed adjust to coordinates axis. Order of magnitude analysis are presented to relate between parameters in limiting condition where only one of the gradients of mass or heat is effective. Numerical solution of equation are Nu, Sh diagram in terms of variation of K in different condition. These numerical results indicate the existence of three regimes, namely, a diffusive one for low values of K, a transition regime when Nu and Sh increase as the value of K is made larger and an asymptotic regime where Nu and Sh become independent of K and reach constant values as the value of K is made large enough. The transition between the different regimes depends on the thermal Rayleigh number RT, buoyancy ratio N and the Lewis number Le. The correlations proposed in this study are found to be in good agreement with the numerical results and this for a large range of the governing parameters. The boundary effects, which are expected to be important in porous media with high porosities, have also been investigated in this study on the basis of the Brinkmanextended Darcy model. The numerical results indicate that when Da is small enough the above results, obtained on the basis of Darcy's law, are valid. For intermediate values of Da the boundary frictional resistance becomes gradually important and slows down the convective motion. As a result, the effects of the anisotropic permeability of the porous medium on the convection heat transfer become less and less important.

Nomenclatures

Latin Symbols

A	Cavity dimension ratio (H/L)
D	Mass diffusivity
D_a	Darcy number (K_2/H^2)
g_H	Gravity acceleration (m/s^2)
H	Cavity height (m)
K_x	Permeability in x-direction (kg/Pa m ^{2}s)
K_z	Permeability in z-direction (kg/Pa m^2 s)
$\overline{K} = t + K_{cr}/K$	Permeability corrected ratio (kg/Pa m^2 s)
K_{cr}	Critical permeability
L	Cavity thickness (m)
Le	Lewis number
N	Buoyancy ratio
Nu	Nusselt number
P	Pressure (Pa)
R_T	Rayleigh heat number
R_c	Rayleigh concentration ratio ($R_T N L_e$)
R_{aT}	Rayleigh heat number applied for one flow (R_T / D_a)
C^0	Dimensionless concentration
Sh	Sherwood number
$\Delta C' = (C'_1 - C'_2)$	Determined concentration difference (kmol/m^3)
T	Temperature (°C)
$\Delta T' = (T'_1 - T'_2)$	Determined temperature difference(°C)
$\overline{u}, \overline{v}, \overline{w}$	Dimensionless velocity
$\overline{x}, \overline{z}$	Dimensionless coordinates

Greek Symbols

α	Effective Thermal Diffusivity
β_T	Thermal coefficient of expansion (°C^{-1})
β_C	Concentration coefficient (m^3/kmol)
δ_T	Thermal diffusion thickness (m)
δ_C	Mass diffusion thickness (m)
$\overline{\delta}_T$	Dimensionless thickness of the thermal boundary layer
μ	Dynamic viscosity of the fluid (Pa.s)
λ	Relative viscosity = μ_{eff} / μ
ν	Kinematic viscosity (m^2/s)
ρ	Density of the fluid (kg/m^3)

Received : 13th January 2007 ; Accepted : 16th September 2007

REFFERENCES

[1] Nilsen, T., Staresletten, An Analytical Study on Natural Convection in Isotropic and Anisotropic Porous Channels, Trans. ASME: *J. Heat Transfer*, **112**, 396 (1990).

[2] Nild, D., Bejan A. "Convective in Porous Media", Springer, New York, (1992).

[3] Ingham, D.B., Pop, I., "Transport Phenomena in Prous Media", Pergamon, Oxford (1998).

[4] Ni, J., Beckerman, C., Natural Convection in a Vertical Enclosure Filled with Anisotropic Porous Media, Trans. ASME: *J. Heat Transfer*, **113**, 1033 (1991).

[5] Trew, M., Mckibbin, R., Convective in Anisotropic Inclined Porous Layer, Trans P., *Porous Media*, **17**, 271 (1994).

[6] Degan, G., Vassenr, P., Natural Convection in a Vertical Stot Filled with an Anisotropic Porous Medium with Oblique Principal Axes, *Heat Transfer,* **30** A, 397 (1996).

[7] Bera, P., Eswaran, P., Singh, Numerical Study of Heat and Mass Transfer in an Anisotropic Prous Enclosure Due to Constant Heating and Cooling, *Heat Transfer*, **34** A, 887 (1998).

[8] Watson, A., The Effect of the Inversion Temp. on the Convection of Water in an Enclosed Rectangular Cavity, *J. Mech, Appl. Math.,* **25**(u), 423 (1992).

[9] Iskava, Isikava T., Hirata, S., Numerical Simulation of Natural Convection with Density Inversion in a Square Cavity, *Heat Transfer*, **37** A, 395 (2000).

[10] Chang, W. J., Yang, D. F., Transient Natural Convection of Water Near its Density in a Rectangular Cavity Filled with Porous Medium, *Heat Transfer*, **28** A, 619 (1995).

[11] Goyeau, D. Gobin, D., Heat Transfer by Thermo-solutal Natural Canvection in a Vertical Porous Cavity, *Journal of Heat Transfer*, **120**, (1998).

[12] Tervisan, E., Bejan A., Natural Convection in a Vertical Slot Filled with an Anisotropic Porous Medium, *Int. J. Heat Fluid Flow*, **18**, 334 (1997).