

# Average Shear Rate Estimation in Conventional Stirred-Tank Bioreactor Using Non-Newtonian Fluid: An Experimental Approach

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**ABSTRACT:** An experimental approach for the estimation of average shear rate ( $\gamma_{av}$ ) in stirred tank bioreactor has been proposed for the turbulent flow regime. Based on the proposed methodology, the correlation for the estimation of  $\gamma_{av}$  was obtained as a function of agitation speed ( $N$ ), superficial gas velocity ( $V_s$ ) and the rheological properties of the non-Newtonian fluids. The  $\gamma_{av}$  estimated by the present method was found to be within the range of values calculated by correlations available in the literature. The  $\gamma_{av}$  increased with the increase of agitation and sparging in all the conditions tested. The correlation derived in the present work helps in estimation of estimation of  $\gamma_{av}$  as a function of bioreactor geometry along with physical conditions ( $N$  and  $V_s$ ), and rheological properties ( $n$  and  $K$ ) of non-Newtonian fluid in commercially available stirred tank bioreactor.

**KEYWORDS:** Average shear rate; Bioreactor; Non-Newtonian fluid; Rheology.

## INTRODUCTION

Biopharmaceutical products have revolutionized in the treatment of various diseases. These medications are manufactured using live microorganisms such as CHO cells, bacteria, yeast etc. in stirred tank bioreactor. As a result of increase in biomass, the culture broth exhibits

a pseudo plastic non-Newtonian behavior [1,2] which can be described by power-law model. To enhance adequate mixing and mass transfer in stirred tank bioreactor, selection of the type of impeller is also an important criteria [3]. The impeller geometry influences

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the hydrodynamics of the culture broth and finally affecting the batch productivity [4]. The most commonly used impeller in the bioprocess is the Rushton turbine impeller [5,6] and it provides higher gas dispersion and higher volumetric mass transfer values [7]. The increase in the biomass also increases the apparent viscosity ( $\mu_{app}$ ) of non-Newtonian fluid which can be mitigated by increasing the impeller agitation speed ( $N$ ) and specific airflow rate ( $V_s$ ). This in turn influences gas-liquid mass transfer and also causes increase in shear rate ( $\gamma_{av}$ ). Increased shear rate changes the rheological properties of the culture and thus affecting the batch performance [8-10]. Hence, need has been created to consider shear as one of the parameters during process development [11-13].

A number of correlations were available in the literature for the estimation of shear rate as a function of impeller agitation speed [14-19]. The shear rate is also effected by rheological properties of the fluids like flow index ( $n$ ) and consistency index ( $K$ ) for non-Newtonian fluids [15,16,19,20]. Correlations have been proposed for the estimation of average shear rate in laminar flow regime. Few correlations described that the average shear rate ( $\gamma_{av}$ ) in bioreactor is dependent on impeller agitation speed ( $N$ ) and  $k$  [21]. Where  $k$  is a constant with a value of 11.4 for Rushton turbine impeller. The value of  $k$  depends on the geometry of the impeller. [22] defined that  $\gamma_{av}$  is a function of impeller speed ( $N$ ) and flow index ( $n$ ), a rheological property of non-Newtonian fluids. Correlations for the estimation of  $\gamma_{av}$  in the transition flow regime has been proposed by [17,18].

The present work was aimed to develop a methodology for the estimation of average shear rate,  $\gamma_{av}$ . In this method superficial gas velocity ( $V_s$ ) was selected for the estimation of  $\gamma_{av}$ . Correlation proposed in the present work is more appropriate for the estimation of  $\gamma_{av}$  because it uses superficial gas velocity  $V_s$  (function of bioreactor geometry and specific air flow rate), impeller agitation  $N$  and rheological properties ( $K$  and  $n$ ) of the non-Newtonian fluid.

## EXPERIMENTAL SECTION

### Bioreactor configuration

The study was performed in a commercially available stir-tank bioreactor (5 L) (Sartorius, Germany). The un-baffled bioreactor had a tank diameter, ( $T$ ) of 0.160 m and a tank height ( $H$ ) of 0.32 m and was equipped with two Rushton turbine impellers ( $D_i = 0.064$  m) fixed at

a distance of one impeller diameter. Gasses were introduced into the bioreactor by using a stainless-steel ring sparger (diameter of drilled holes 1.0 mm). The ratio of diameter of impeller to diameter of tank ( $D_i:D_t$ ) was 0.4. Fig. 1a and b shows the pictorial representation of the bioreactor and the impeller used in this work.

### Fluids

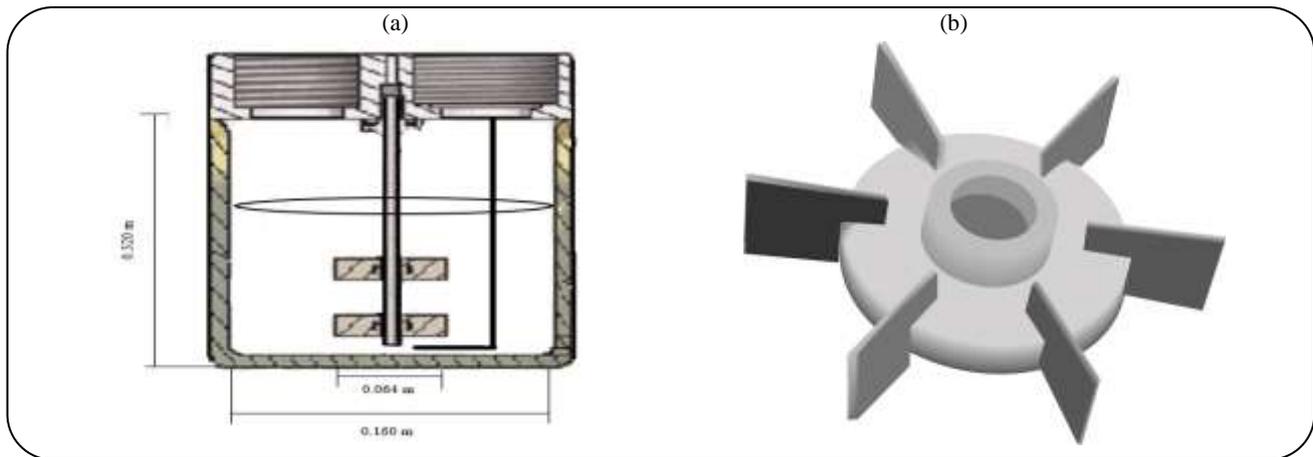
Glycerol solutions (Sigma Aldrich, St. Louis, Missouri, USA) were used as Newtonian fluid and Carboxy methyl cellulose (Sigma, C5678) was used as non-Newtonian fluid. The dynamic viscosity of the Newtonian fluid and rheological parameters of the non-Newtonian fluid were determined by using a viscometer (Bohlin Visco88 and TA-instruments AR2000) fitted with a coquette configuration. Table 1 shows the dynamic viscosity and rheological properties of the fluid used in the current work.

### Experimental determination of $K_{La}$

The determination of the overall volumetric mass-transfer coefficient value of a bioreactor is essential to establish its aeration efficiency and to quantify the effect of operating variables on the provision of oxygen. The,  $K_{La}$  estimation was done by following the static gassing-out method. The non-Newtonian fluid was deoxygenated by sparging nitrogen gas until the dissolved oxygen level reached below 5% of the saturation. The variation in the dissolved oxygen concentration, ( $C_L$ ) in the liquid phase was detected using an oxygen probe. The dissolved oxygen concentration in the bioreactor liquid phase was measured by means of an oxygen probe inserted vertically and placed at 2 cm under the liquid level, the DO probe fitted with a Teflon membrane and with an electrolytic solution of  $Na_3PO_4$  in the cell. The oxygen probe signals were measured using an A/D converter and a recorder on a PC. After that the nitrogen gas flow was turned off and the flow switched to the air flow with a specific volumetric flow rate using the rotameter. Then the dissolved oxygen concentration was recorded with respect to time as air was distributed into the bioreactor and until the water became saturated with oxygen. The dissolved oxygen was monitored until saturation,  $C^*$  was reached. Gas composition was constant. The system was isothermal, and the effect of the dynamics of the dissolved oxygen electrode was negligible.

**Table 1: Dynamic viscosity ( $\mu$ ) and rheological properties ( $K$  and  $n$ ) of the Newtonian and non-Newtonian fluid.**

Newtonian Fluids		Non-Newtonian Fluids		
Glycerol Solution (GS)	$\mu \times 10^3$ (Pa s)	Carboxy Methyl Cellulose (CMC) solution	$K$ (Pa s <sup>n</sup> )	$n$ (dimensionless)
GS-1	1.35	CMC-1	0.147	0.929
GS-2	2.72	CMC-2	0.174	0.807
GS-3	7.19	CMC-3	0.262	0.799
GS-4	9.85	CMC-4	0.346	0.715
GS-5	14.1	CMC-5	0.421	0.681

**Fig. 1: Schematic view of the (a) bioreactor and (b) impeller (Rushton turbine impeller) used in this study:**

The rate of oxygen transfer from gas to liquid phase was given by the empirical relationship [23].

$$\frac{dC_L}{dt} = K_L a (C^* - C_L) \quad (1)$$

The  $K_L a$  values were calculated from the slope of the plot of  $\ln(C^* - C_L)$  versus time,  $t$ . A turbulent flow regime is generated at higher agitation speed where the Reynolds number ( $Re$ )  $> 5 \times 10^3$ . In this study, the Reynolds number ( $Re$ ) was considered over a wide range of 25,000 to 75,000.  $K_L a$  was estimated in triplicates with the above mentioned  $Re$  range and gas flow rate ranging from  $2.0 \times 10^{-5}$  m/s to  $2.0 \times 10^{-4}$  m/s.

#### Method for estimation of average shear rate ( $\gamma_{av}$ )

The average shear rate  $\gamma_{av}$  in the stirred tank bioreactor was estimated by following the method proposed by [24] in which mass transfer coefficient ( $K_L a$ ) was used as a characteristic parameter to estimate  $\gamma_{av}$  in concentric tube airlift bioreactor. Later this methodology was modified

for stirred tank reactors by [19, 25]. The specific energy dissipation rate is dependent on shear rate  $\gamma$  and shear stress  $\tau$  [26,27]

$$\frac{P}{V} = \tau \gamma \quad (2)$$

Where  $P$  is the power input and  $V$  is the fluid volume. For Newtonian fluids, dynamic viscosity ( $\mu$ ) is the ratio of shear stress and shear rate.

$$\mu = \frac{\tau}{\gamma} \quad (3)$$

Equation (2) can be written as

$$\frac{P}{V} = \mu \gamma^2 \quad (4)$$

For non-Newtonian fluids obeying the power law [28],

$$\tau = K \gamma^n \quad (n < 1) \quad (5)$$

Where,  $K$  is the consistency index and  $n$  is the flow behavior index. Analogous to Newtonian fluids,

apparent viscosity ( $\mu_{app}$ ) of non-Newtonian fluids can be described as

$$\mu_{app} = \frac{\tau}{\dot{\gamma}} = K \dot{\gamma}^{n-1} \quad (6)$$

For non-Newtonian fluids, the equation corresponding to Equation (4) becomes

$$\frac{P}{V} = \mu_{app} \dot{\gamma}^2 \quad (7)$$

$$\frac{P}{V} = K \dot{\gamma}^{n+1} \quad (8)$$

The  $K_{La}$  values were correlated with correlation developed by [29].

$$KLa = \alpha \left( \frac{P}{V} \right)^\beta V_s^\delta \quad (9)$$

Where  $P$  is the mechanical agitation power (W),  $V$  = liquid volume ( $m^3$ ),  $V_s$  = gas superficial velocity (m/s),  $\alpha$  = is a constant,  $\beta$  and  $\delta$  are exponents. Substituting the value of equation 8 in equation 9,

$$KLa = \alpha (K \dot{\gamma}_{av}^{n+1})^\beta V_s^\delta$$

$$\dot{\gamma}_{av} = \left( \frac{KLa}{\alpha V_s^\delta K^\beta} \right)^{\frac{1}{\beta(n+1)}} \quad (10)$$

The  $K_{La}$  for the non-Newtonian fluid was estimated by following the methodology proposed by [30]

$$KLa = b N^c V_s^d K^e \quad (11)$$

Where  $N$  is the impeller,  $V_s$  = gas superficial velocity,  $b$  = is a constant,  $c$ ,  $d$  and  $e$  are exponents.

Substituting the value of equation 11 in equation 10,  $\dot{\gamma}_{av}$  is obtained as a function of agitation, gas velocity and rheological parameters of the non-Newtonian fluid

## RESULTS AND DISCUSSION

Experimental  $K_{La}$  data for Newtonian fluid was correlated according to equation 9. The experimental values of  $K_{La}$  were in the range of 0.06–0.084  $\text{min}^{-1}$ . The experimental values were fitted to the correlation in equation 9. The constant  $\alpha$  and exponentials ( $\beta$  and  $\delta$ ) were estimated by least square non-linear regression by following Marquardt's procedure [31]. The proposed correlation along with correlation coefficient ( $R^2$ ) is shown below

$$KLa = 0.055 (P/V)^{0.320} (V_s)^{0.532} \quad (12)$$

$$(R^2 = 0.999)$$

High value of the correlation coefficient  $R^2$  shows that good fits were obtained indicating that the correlation can be used for accurate estimation of  $K_{La}$ .

For non-Newtonian fluids, experimental  $K_{La}$  data were correlated according to equation 11. The experimental value of  $K_{La}$  was in the range of 0.02–0.063  $\text{min}^{-1}$ . The experimental values were fitted to the correlation in equation 11. The constant  $b$  and exponentials ( $c$ ,  $d$  and  $e$ ) were estimated by least square non-linear regression by following Marquardt's procedure [31]. The proposed correlation along with correlation coefficient ( $R^2$ ) is shown below

$$KLa = 0.9 (P/V)^{0.364} (V_s)^{0.631} K^{0.530} \quad (13)$$

$$(R^2 = 0.999)$$

Fig. 2 shows the comparison between the experimental and calculated data of  $K_{La}$  and the difference were below 7.5%. Figure 1 and high value of the correlation coefficient  $R^2$  shows that good fits were obtained indicating that the correlation can be used to accurately estimate  $K_{La}$  in agitated and aerated tanks with non-Newtonian fluids.

Effect of apparent viscosity ( $\mu_{app}$ ) on  $K_{La}$  is shown in Figure 3. It can be observed that the  $K_{La}$  increased with increase in flow index ( $n$ ) and decrease of consistency index ( $K$ ). According to Equation (6), apparent viscosity ( $\mu_{app}$ ) increases with the increase in  $K$  and decrease of  $n$ . Increase in the  $\mu_{app}$  negatively influenced the  $K_{La}$  as it generates resistance between the gas-liquid mass transfer.

After estimation of the values of parameters of equation 9 and equation 11, average shear rate correlation was equated as a function of impeller agitation ( $N$ ), superficial gas velocity ( $V_s$ ) and rheological parameters ( $K$  and  $n$ ) of the fluid.

$$\dot{\gamma}_{av} = (16.2 N^{0.364} V_s^{0.099} K^{0.210})^{1/0.320(n+1)} \quad (14)$$

As expected, the exponent of  $N$  is 3.7 times higher than  $V_s$ . Hence,  $\dot{\gamma}_{av}$  is strongly influenced by agitation speed compared to specific air flow rate. The Equation (14) is better suited for estimation of  $\dot{\gamma}_{av}$  as it uses experimental values of  $K_{La}$  and also involves superficial gas velocity which is a function of bioreactor geometry and air flow rate. To validate the correlation proposed in the present work for estimation of  $\dot{\gamma}_{av}$ , it was compared with earlier reported correlations using an intermediate CMC solution (CMC-3) as non-Newtonian fluid at different agitation speed  $N$  and  $V_s = 2.0 \times 10^{-5}$  m/s. Table 2 describes various

Table 2: Correlations for estimation of average shear rate,  $\gamma_{av}$ .

References	Proposed Correlation
[14]	$\gamma_{av} = k N$
[17]	$\gamma_{av} = 33.3N$
[18]	$\gamma_{av} = 33.1N^{1.4}$
[19]	$\gamma_{av} = 1.571 \left( \frac{2.876}{1-n} \right) K \left( \frac{0.609}{1-n} \right) N^{\frac{1.343}{1-n}}$
[22]	$\gamma_{av} = k \left( \frac{4n}{3n+1} \right)^{\frac{n}{n-1}} N$
[25]	$\gamma_{av} = \left( 3.14 N^{0.653} \phi_{air}^{-0.2} K^{0.751} n^{1.193} \right)^{\frac{1}{0.620(1-n)}}$

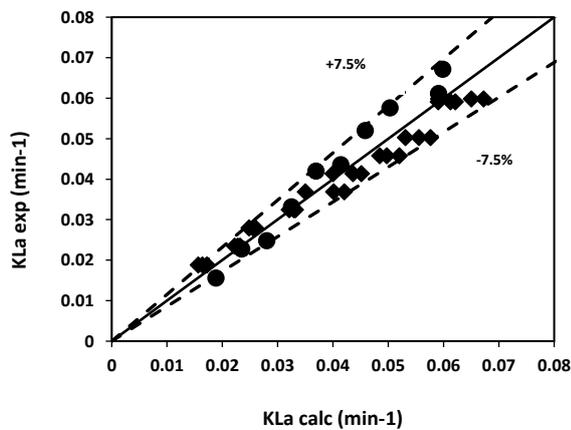


Fig. 2: Comparison between calculated and experimental data of  $KLa$  estimation at  $V_s=1.0 \times 10^{-4}$  m/s (♦) experimental values, (●) correlation values.

correlations available in the literature for evaluation of  $\gamma_{av}$  and Fig. 3 shows the profile of  $\gamma_{av}$  obtained from the correlations.

It can be observed from figure 4 that the values of  $\gamma_{av}$  calculated by the correlation proposed in the present work are within the values predicted by [17] and [18], indicating that the proposed correlation estimated  $\gamma_{av}$  is closer to the literature value. The  $\gamma_{av}$  estimated by using [14] and [22] was different as compared to other authors because these correlations were proposed for laminar flow regime while equation predicted for the estimation of  $\gamma_{av}$  by [17] and [18] were proposed for transitional flow regime. Hence the values of  $\gamma_{av}$  predicted by using these correlations were extrapolated for turbulent flow regime. A similar system set-up to that of present work was used by [19] and [25] for the estimation of  $\gamma_{av}$ , however their studies were performed in baffled bioreactor and the size of the impeller and

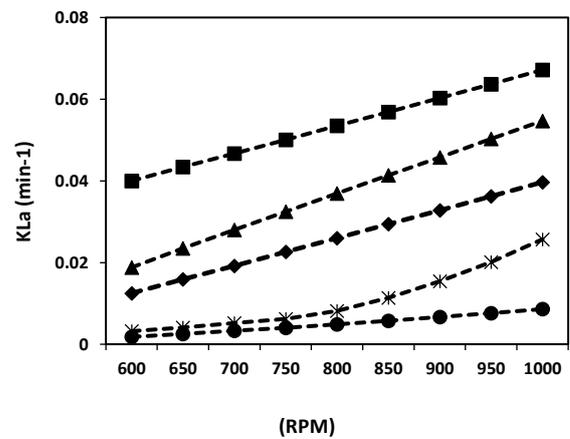


Fig. 3: Estimated  $KLa$  values at different agitation speed for non-Newtonian fluids at  $V_s= 1.0 \times 10^{-4}$  m/s. (■) CMC-1, (▲) CMC-2, (♦) CMC-3, (×) CMC-4, (●) CMC-5.

the flow rate of gas used were different as compared to the present work.

To illustrate the effect of agitation speed  $N$  and  $V_s$  on  $\gamma_{av}$ , Fig. 5 shows the  $\gamma_{av}$  as a function of  $N$  at different  $V_s$ . It can be observed that the increase in agitation speed  $N$ , positively affected shear rate  $\gamma_{av}$ . Considering the influence of  $V_s$  on  $\gamma_{av}$ , increase in gas flow rate resulted increase in the  $\gamma_{av}$ . Increase in  $V_s$  creates higher number of bubbles which further burst of the surface of the fluid and causing an increase in  $\gamma_{av}$  [32]. A similar behavior has been reported by [19] in which high values of  $V_s$  resulted in increase of  $\gamma_{av}$ .

In all the bioprocess the culture broth obeys the power law model (Eq. 6) in which the flow index ( $n$ ) decreases with the culture while and consistency index ( $K$ ) rises and increasing the apparent viscosity ( $\mu_{app}$ ) of the culture [33]. At the end of the stationary phase of the culture,

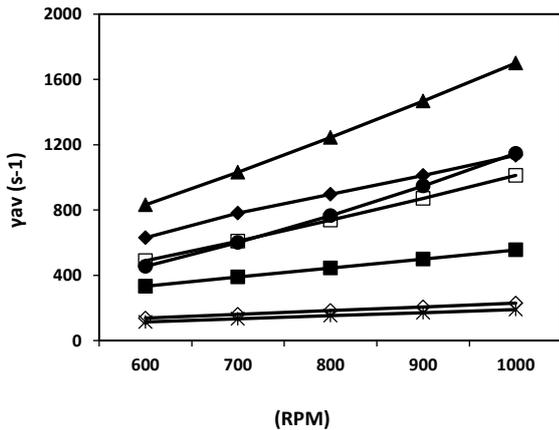


Fig. 4: Comparison of average shear rate ( $\gamma_{av}$ ) in agitated bioreactors with Rushton turbine impellers as function of agitation speed  $N$  for Carboxy methyl cellulose solution (CMC-3) aerated at  $2.07 \times 10^{-5}$  m/s. (■) [17], (▲) [18], (◆) present work, (×) [14], (◇) [22], (●) [19], (□) [25].

apparent viscosity ( $\mu_{app}$ ) decreases due to higher cell death and increasing the average shear rate ( $\gamma_{av}$ ). In this context, a similar behavior of apparent viscosity ( $\mu_{app}$ ) with average shear rate ( $\gamma_{av}$ ) has been reported by [19] while working with *S.clavuligerus* in stirred tank bioreactor.

## CONCLUSION

Good fits were obtained for the  $K_L a$  correlations derived for newtonian and non-newtonian fluids. In the present work, a correlation was derived for the estimation of average shear rate ( $\gamma_{av}$ ) as a function of agitation speed ( $N$ ), superficial gas velocity ( $V_s$ ) and rheological properties of the culture broth in the turbulent flow regime. Most of the correlations available in the literature for estimation of  $\gamma_{av}$  were developed for laminar and transient flow regime.  $\gamma_{av}$  predicted by the current method was within the range of values estimated from correlations available in the literature for turbulent flow regime, hence indicating that the present method predicts reliable values of  $\gamma_{av}$ . The choice of  $V_s$  as a characteristic parameter was appropriate for the estimation of average shear rate ( $\gamma_{av}$ ) because it considers bioreactor geometry and dimensions along with gas flow rate ( $Q$ ). Commercially available stirred tank reactors vary among themselves on the basis of different geometry and thus generating diverse hydrodynamic patterns. Considering bioreactor geometry in the evaluation will provide better understanding of the shear generated by the impellers

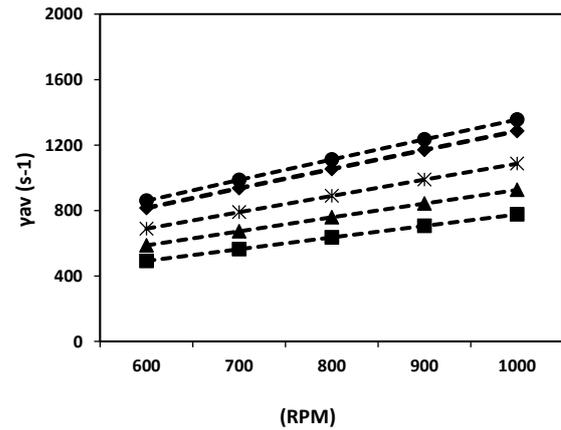


Fig. 5: Average shear rate ( $\gamma_{av}$ ) as function of agitation speed  $N$  and  $V_s$  for Carboxy methyl cellulose solution (CMC-2) aerated at (●)  $1.0 \times 10^{-4}$  m/s, (◆)  $8.03 \times 10^{-5}$  m/s, (×)  $4.01 \times 10^{-5}$  m/s, (▲)  $2.00 \times 10^{-5}$  m/s, (■)  $1.01 \times 10^{-5}$  m/s.

inside the bioreactors.  $\gamma_{av}$  increased proportionately with increase of  $N$  and  $V_s$  while increase in  $\mu_{app}$  resulted in reduction of  $\gamma_{av}$ . Hence,  $N$  and  $V_s$  needs to be optimized to protect the cells from shear generated by the impeller in stirred tank reactor.

## Nomenclature

$C^*$	Dissolved oxygen saturation concentration, mg/L
$C_L$	Initial dissolved oxygen concentration, mg/L
CHO	Chinese hamster ovary
$D_i$	Diameter of impeller, m
$H$	Tank height, m
$K_{L,a}$	Volumetric mass transfer coefficient, $\text{min}^{-1}$
$K$	Consistency index, $\text{Pa S}^n$
$N$	Agitation speed, rps
$n$	Flow behavior index
$N_p$	Power number
$P$	Power, Watts
$Q$	Gas flow rate, $\text{m}^3/\text{s}$
$Re$	Reynolds number
$T$	Tank diameter, m
$V_s$	Superficial gas velocity, m/s
vvm	Vessel volume per minute
$V$	Working volume, $\text{m}^3$
$\alpha, \beta, \delta$	Parameters of equation 9, dimensionless
$b, c, d, e$	Parameters of equation 11, dimensionless
$\gamma$	Shear rate, $\text{s}^{-1}$
$\gamma_{av}$	Average shear rate, $\text{s}^{-1}$

$\mu$	Dynamic viscosity, Pa s
$\mu_{app}$	Apparent viscosity, Pa s
$\tau$	Shear stress, Pa

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