Application of Axial Velocity Profile in Order to Develop Residence Time Distribution (RTD) for Different Laminar and Turbulent Flows

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ABSTRACT: *The famous definition of RTD is based on the volumetric flow rate but is experimentally defined using the tracer concentration. These different views have erroneously limited the application of the velocity profile for RTD evaluation to the laminar flows. In this work, a more general sense of RTD is introduced and it has been emphasized that regardless of the dispersion behavior, the velocity profile is sufficient in order to obtain the corresponding RTD. A general algorithm for RTD evaluation using axial velocity profile is developed and the relations were derived for different systems. In addition, the corresponding velocity profiles to the famous RTD models were numerically evaluated. It has been shown that the final forms are consistent compared to the previous relations for laminar flows.*

KEYWORDS: *RTD; Velocity profile; Tracer; Pipe flow.*

INTRODUCTION

The distribution of different lengths of time to pass through different routes in a vessel is called RTD denoted by E(t) function [1]. The ever-increasing amount of literature on this topic since Danckwerts' work [2] has generally followed his nomenclature [3].

The RTD concept was used in various processes such as fixed and fluidized bed reactors, two-phase stirred tanks, heat exchangers, distillation and absorption columns, chromatography columns, and trickle bed reactors beside other fields such as microfluidics, hydrology, pharmaceutical manufacturing, and mixing behavior of solid processes [4]. The RTD applications are so wide that helped the researchers with the mathematical modeling of static mixers [5].

The value of E(t).dt is the fraction of the fluid leaving the vessel with the age between t and t+dt and therefore E(t) can be defined as the response to the unit impulse function. The transfer function, G(s), is defined as the ratio of the Laplace transform of the response function to the Laplace transform of the input function [6]. Replacing the Laplace transform of the unit impulse function as unity, the RTD can be derived via the inversion of Laplace transforms of the transfer function

$$E(t) = L^{-1}\{G(s)\} \tag{1}$$

where can be used for the systems with a defined transfer function, keeping in mind that some systems do not have a transfer function, e.g. a pipe in the laminar flow

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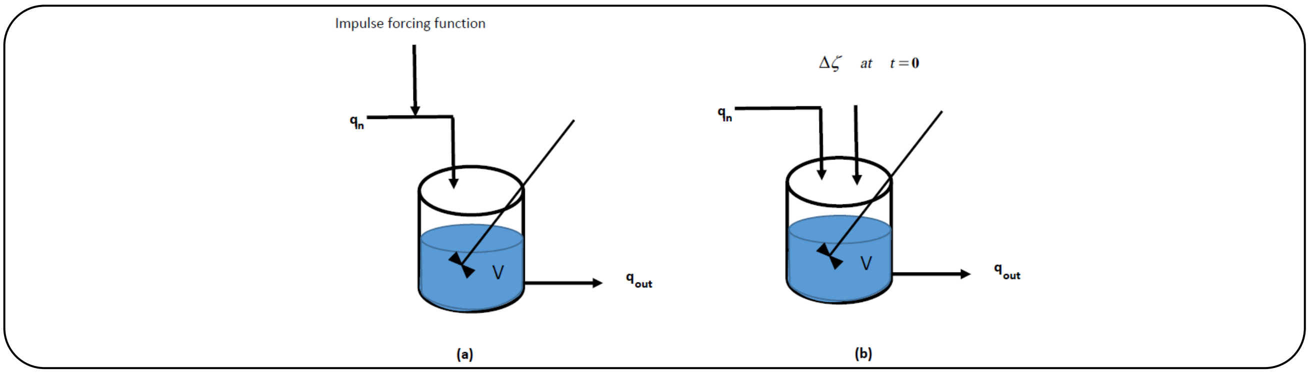

***Fig. 1** Two corresponding modeling of a pulse input to the completely mixed vessel. (a): using a new pulse input at t = 0; (b): addition of a specified value to the vessel at the instant t = 0.*

condition. The theoretical RTD of laminar flow is derived using the volumetric flowrate forcing function [3]. However, if the tracer concentration is studied, the injection and detection methods should be specified [7,8]. It has been shown that at Reynolds number (Re) <3000, a theoretical RTD for laminar flow that assumes no radial mixing provides a good approximation [9]. *Levenspiel et al.* derived the RTD of the laminar flow of liquids in various forms of vessels [1].

Application of pulse input in a laboratory as the requirement for full cross-sectional mixing typically requires the injection to occur some distance upstream from the monitoring position. Recently the up-and downstream concentration profiles are measured and the data are deconvolved to give the system's RTD [9]

Basically, the forcing function should be the input volumetric flowrate, however experimentally the tracer concentration is measured and the ratio of its output/input concentration will denote RTD. Some recommendations are presented in the literature for selecting the tracer materials [4]. Likewise, other properties such as radioactivity [10,11], pH, and electrical conductivity in the tracer experiment can be investigated. For variable flow systems, the product of concentration by volumetric flowrate should be studied [12]. In this work, it is proposed that similar to the above extensions, one perhaps uses other forcing functions and measures the related responses such as temperature variation against time without using any external tracer in order to define a new suitable RTD. Nevertheless, it probably results in a relation with a different form compared to the ordinary RTD.

Suitable forcing function in the Dirac delta form

The RTD of a vessel will be derived as the response to an impulse Dirac delta function, $\delta(t)$ in the unit of inversion of time. However, the forcing function needs a suitable multiplier in order to have the correct dimension. Without loss of generality, we enter the input impulse function $\bar{\xi}_{in}(t)$ (e.g. concentration of tracer) to a completely mixed flow vessel as is shown in Fig. 1a. The response $\bar{\xi}_{out}(t)$ (e.g. the output concentration of the tracer) should be similar to the response of another view of this system in which the amount of input $\Delta\zeta$ (e.g. $\Delta M/V$ where ΔM is the mass of tracer [kg] and V is the volume of the tank [m^3]) enters the vessel at t = 0 instantaneously, as is shown in Fig. 1b.

The related balance equation (e.g. the tracer continuity equation) in Fig. 1 (a) is [6]

$$\bar{\xi}_{in}(t) - \bar{\xi}_{out}(t) = \tau \frac{d\bar{\xi}_{out}(t)}{dt} \tag{2}$$

Where $\tau = V/q_m$ and the outlet function is equal to its value in the vessel because of a completely mixed vessel assumption. Also, the function $\bar{\xi}$ is defined in the deviation form and consequently, the initial condition is $\bar{\xi}_{out}(\mathbf{0}) = \mathbf{0}$.

The Laplace transform of Eq. (2) can be rearranged to

$$\xi_{out}(s) = \frac{\xi_{in}(s)}{\tau s + 1} \tag{3}$$

The balanced equation of the corresponding system of Fig. 1 (b) is

$$0-\bar{\xi}_{out}(t)=\tau\frac{d\bar{\xi}_{out}(t)}{dt} \quad (4)$$

with the initial condition $\bar{\xi}(0)=\Delta\zeta$ and the Laplace transform of Eq. (4) is

$$\xi_{out}(s)=\frac{\tau\Delta\zeta}{\tau s+1} \quad (5)$$

The Eqs. (3) and (5) are equal unless at t = 0, therefore the input impulse function is

$$\bar{\xi}_{in}(t)=\tau\Delta\zeta\delta(t) \quad (6)$$

It is worth noting that although the form of the impulse function is derived after its entrance to a completely mixed vessel, it will have the same form for other applications as well. The various initial instantaneous input quantity $\Delta\zeta$ and the corresponding multiplier of the Dirac delta function of Eq. (6) are reported in Table 1 for different experiments. The first row of Table 1 is consistent with [1]. The required assumptions may be constant input flow rate, constant density, and constant heat capacity to satisfy Eq. (2) for different rows of Table 1.

The RTD as a function of the velocity profile

The RTD is basically defined as the fraction of the outlet stream as a function of time. Therefore, the best experiment is the determination of the outlet flowrate as the response to the impulse Δq_{in} as forcing function. Nevertheless, usually, the tracer concentration is studied (ΔC_{in} among the cases of Table 1). According to the usual derivation of RTD based on the tracer concentration, it relates to the velocity profile if and only if there is no transfer of molecules in the radial direction between streamlines [3]. Therefore, only laminar and plug flow velocity profiles can be analytically used to derive the correct RTD. Also, the injection and detection methods should be specified. However, if the outlet flowrate, q(t), is studied and a step forcing function, q_0, is used, regardless of the dispersion behavior of the stream, the RTD can be defined as

$$E(t)=\frac{d}{dt}\left(\frac{q}{q_0}\right) \quad (7)$$

and since $dq = udA_c$, where u is fluid velocity distribution and A_c, is cross-section area. For the velocity distributions with more than one independent variable the final result is complicated [13] however for axial flow where the velocity depends only on one independent variable, Eq. (7) will be rearranged to

Table 1: The entered amount at t = 0 and the multiplier of the corresponding Impulse function.

Impulse type	$\Delta\zeta$	$\tau\Delta\zeta$
$\Delta C_{in}\left[kg/m^3\right]$	$\Delta M/V[kg/m^3]$	$\Delta M/q_{in}[kg.s/m^3]$
$\Delta T[K]$	$\frac{\Delta Q}{\rho V C_p}[K]$	$\frac{\Delta Q}{\rho q_{in} C_p}[K.s]$
$\Delta q_{in}[m^3/s]$	$\frac{\Delta V}{\tau}[m^3/s]$	$\Delta V[m^3]$

$$E(t)=f(x)\left|\frac{dA_c/A_c}{dt}\right| \quad (8)$$

where $f(x)=\frac{u}{u_m}$, u_m is mean velocity (q_0/A_c), x is a dimensionless position variable, and the absolute operator warrants a positive value of E. The final term of Eq. (8) could be written as a function of x like

$$\left|\frac{dA_c/A_c}{dt}\right|=g(x)\left|\frac{dx}{dt}\right| \quad (9)$$

Where $g(x)$ is a function determined based on the problem geometry. If the length of the conduit is L, the flow-through whole cross-section, A_c, passes it during the mean residence time (τ) and the flow through the area element, dA_c, passes it during the time t. Therefore

$$\frac{\tau}{t}=\frac{1}{\theta}=f(x) \quad (10)$$

Where θ is the dimensionless time variable. Differentiation of Eq. (10) will result in

$$\left|\frac{dx}{dt}\right|=\frac{\tau}{t^2}\frac{1}{|df/dx|} \quad (11)$$

According to Eqs. (8) to (11) and E(θ) = τE(t), RTD can be derived through

$$E(\theta)=\frac{1}{\theta^3}\frac{g(x)}{|df/dx|} \quad (12)$$

Where Eq. (12) together with Eq. (10) result in E versus θ for any specified velocity profile regardless of laminar or turbulent regimes. The velocity profile usually is expressed as $f = u / u_m$ or in the form of u / u_{max} where assuming $u_{max} = L / t_{min}$ it can be written as $f(x) = \frac{1}{\theta_{min}}\frac{u}{u_{max}}$ or equivalently $\frac{u}{u_{max}} = \frac{\theta_{min}}{\theta}$ with $\theta_{min} = t_{min}/\tau$. The value of θ_{min} may be determined via the definition of mean velocity as

$$\int_{x_{min}}^{x_{max}} f(x) g(x) dx = 1 \tag{13}$$

where x_{min} and x_{max} were specified based on the geometry of the system; or the condition of

$$\int_{\theta_{min}}^{\infty} E(\theta) d\theta = 1 \tag{14}$$

Sometimes both Eq. (13) and (14) are needed.

The RTD of the logarithmic velocity profile

The logarithmic law is used for turbulent flow in pipes like power-law profile, [14]

$$u^+ = \frac{1}{\kappa} \ln y^+ + B \tag{15}$$

Where κ and B are constants, $u^+ = u / u^*$, $u^* = \sqrt{\tau_w / \rho}$, $y^+ = y / \delta^*$ and $\delta^* = v / u^*$. According to the definition of y= R-r, the maximum velocity at the center of the pipe ($u^+_{max} = u^+\big|_{y^+ = R^+}$) and Eq. (10) after some rearrangement

$$\theta_{min} f(x) = \frac{1}{\kappa u^+_{max}} \ln(1 - x) + 1 \tag{16}$$

where x = r/R and may change in the range of $0 \le x \le 1$. The cross-section element in the cylindrical coordination is $dA_c = 2\pi r dr$ that besides total cross-section as $A_c = \pi R^2$ and Eq. (9), it can be manipulated to give $g(x) = 2x$. Finally replacing in Eq. (12) and Eq. (10), the final RTD is derived as

$$E(\theta) = \frac{3\theta_{min}}{\theta^3 (1 - \theta_{min})} \times \tag{17}$$

$$\left\{1 - \exp\left[-\frac{3}{2}\left(\frac{\theta - \theta_{min}}{1 - \theta_{min}}\right)\right]\right\} \exp\left[-\frac{3}{2}\left(\frac{\theta - \theta_{min}}{1 - \theta_{min}}\right)\right]$$

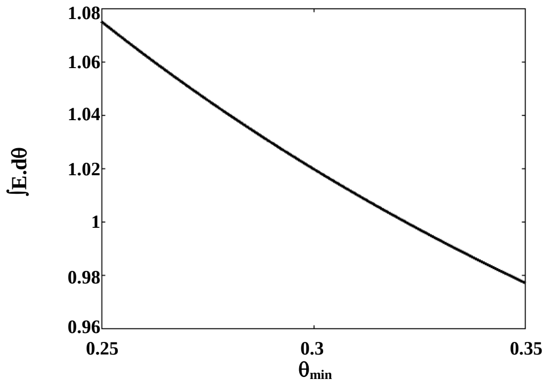

Fig. 2: The integral of Eq. (17) against θ_{min} were only the value of θ_{min} = 0.32 satisfies Eq. (14).

However, the value of θ_{min} should be determined using Eq. (14). The related integration is evaluated numerically for different values of θ_{min} as is shown in Fig. 2.

According to Fig. 2 the best value that satisfies the Eq. (14) with two decimal places is θ_{min} = 0.32. Also, Eq. (16) and Eq. (13) will result in a new equation as

$$\theta_{min} = 1 - \frac{3}{2\kappa u^+_{max}} \tag{18}$$

Where immediately arises $\kappa u^+_{max} = 2.2$. The mean velocity based on Eq. (15) is

$$u^+_m = \frac{1}{\kappa} \ln R^+ + 2B - \frac{3}{\kappa} \tag{19}$$

After some manipulations, Eq. (19) beside Eq. (15) for maximum velocity and Eq. (18) imply B and κ are not independent but relate together *via*

$$B = \frac{3}{2\kappa} \tag{20}$$

Eq. (20) is derived in this work based on the unity area of RTD and limits the general form of Eq. (15). Especially Eq. (19) is simplified $u^+_m = \frac{1}{\kappa} \ln R^+$. If the value of κ = 0.41 is chosen as reported in fluid mechanics literature, the other parameters will be B = 3.66; $u^+_{max} = 5.4$; $u^+_m = 1.7$.

Usually the dispersion model and tanks-in-series model apply to turbulent flow in pipes and laminar flow in very long tubes [15]. Here is the RTD of the dispersion model

Table 2: The RTD corresponding to the velocity profile for various type of flow.

Type of flow	Velocity profile	x	g(x)	f(x)	RTD	parameters
Laminar pipe flow	$u/u_{max} = \left[1-(r/R)^2\right]$	$x = r/R$ $0 \le x \le 1$	2x	$(1/\theta_{min})(1-x^2)$	$E(\theta) = \frac{\theta_{min}}{\theta^3}$	$\theta_{min} = \frac{1}{2}$
Couette flow	$u/u_{max} = 1 - y/\delta$	$x = y/\delta$ $0 \le x \le 1$	1	$(1/\theta_{min})(1-x)$	$E(\theta) = \frac{\theta_{min}}{\theta^3}$	$\theta_{min} = \frac{1}{2}$
Falling film flow	$u/u_{max} = 1-(y/\delta)^2$	$x = y/\delta$ $0 \le x \le 1$	1	$(1/\theta_{min})(1-x^2)$	$E(\theta) = \frac{1}{2}\frac{\theta_{min}}{\theta^3}\left(1-\frac{\theta_{min}}{\theta}\right)^{-1/2}$	$\theta_{min} = \frac{2}{3}$
Laminar film flow	$\frac{u}{u_{max}} = \left[1-\frac{y}{\delta}\right]\left[1+(1-\beta)\frac{y}{\delta}\right]$	$x = y/\delta$ $0 \le x \le 1$	1	$(1/\theta_{min})(1-x)\left[1+(1-\beta)x\right]$	$E(\theta) = \frac{\theta_{min}}{\theta^3}\left[\beta^2 + 4(1-\beta)\left(1-\frac{\theta_{min}}{\theta}\right)\right]^{-1/2}$	$\theta_{min} = \frac{4-\beta}{6}$
Power law profile	$u/u_{max} = \left[1-(r/R)\right]^{1/n}$	$x = r/R$ $0 \le x \le 1$	2x	$(1/\theta_{min})(1-x)^{1/n}$	$E(\theta) = 2n\frac{\theta_{min}}{\theta^3}\left[1-\left(\frac{\theta_{min}}{\theta}\right)^n\right]\left(\frac{\theta_{min}}{\theta}\right)^{n-1}$	$\theta_{min} = \frac{2n^2}{(n+1)(2n+1)}$
Logarithmic profile	$u^+ = \frac{1}{\kappa}\ln y^+ + B$	$x = r/R$ $0 \le x \le 1$	2x	$\frac{1}{\kappa\theta_{min}u^+_{max}}\ln(1-x) + \frac{1}{\theta_{min}}$	$E(\theta) =$ $\frac{3\theta_{min}}{\theta^3(1-\theta_{min})}\left\{1-\exp\left[-\frac{3}{2}\left(\frac{\theta-\theta_{min}}{1-\theta_{min}}\right)\right]\right\}\times$ $\exp\left[-\frac{3}{2}\left(\frac{\theta-\theta_{min}}{1-\theta_{min}}\right)\right]$	$\theta_{min} = 0.32;$ $u^+_{max} = 5.4$ $\kappa = 0.41;$ $B = 3.66$
Flow through annulus	$u(r) = \frac{\Delta P.R^2}{4\mu L}\left[1-\left(\frac{r}{R}\right)^2 - \right.$ $\left. - \frac{1-\kappa^2}{\ln(1/\kappa)}\ln\left(\frac{R}{r}\right)\right]$	$x = r/R$ $\kappa \le x \le 1$	2x	$K\left[1-x^2+2\lambda^2\ln x\right]$	$E(\theta) = \frac{1}{\theta^3}\frac{x^2}{(1-\kappa^2)K\left\|x^2-\kappa^2\right\|}$ $\frac{1}{\theta} = f(x)$	$2\lambda^2 = \frac{1-\kappa^2}{\ln(1/\kappa)};$ $\theta_{min} = \frac{1}{f(\lambda)}$ $K = \frac{2}{1+\kappa^2-2\lambda^2}$

for closed-closed boundary conditions is derived by differentiation of the response to the step function of inert tracer concentration [16] as

$$E(\theta) = \frac{1+1/\theta}{4\sqrt{\pi}}\sqrt{\frac{Pe}{\theta}}\exp\left[-\frac{Pe}{4}\frac{(1-\theta)^2}{\theta}\right] \quad (21)$$

Where *Pe* is Peclet number [1]. The RTDs corresponding to different velocity profiles is represented in Table 2.

Among various relations, the RTDs of laminar flows including the first four rows of Table 2 were previously developed [15] and have precisely the same forms as derived in this work that verified the general formula developed here. Since $\theta_{min} = u_m / u_{max}$, all velocity profiles will have a nonzero θ_{min}. The range of x determines the limits of Eq. (13). The parameters are evaluated using Eq. (13) or Eq. (14) or both of them.

Fig. 3 shows different possible RTD for pipe flow including logarithmic profile, Eq. (17); power law profile [17] with n = 6 and 10; dispersion model with open-open boundary conditions [3] at Pe = 1; dispersion model with closed-closed boundary condition, Eq. (21), at Pe = 1; tanks-in-series model with N = 2 and laminar flow.

The RTD of flow through an annulus

As the last example, the velocity profile of a flow-through annulus [18] is used to derive the corresponding RTD. The flow regime is assumed in the range of $\kappa \le r/R \le 1$ and the maximum velocity will be at $r = \lambda R$

Where $\lambda = \frac{1}{2}\left(\frac{1-\kappa^2}{\ln(1/\kappa)}\right)^{1/2}$. The ratio of $u/u_m = f$ will be

$$f(x) = K\left[1-x^2+2\lambda^2\ln x\right] \quad (22)$$

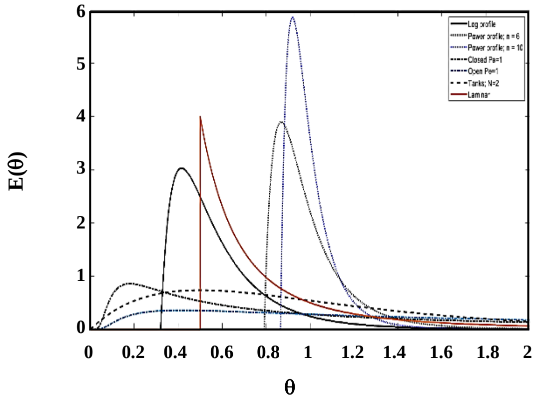

Fig. 3: The RTD of different models for pipe flow including logarithmic profile, power law (n=6 and 10), dispersion model (closed and open boundary condition), and tank-in-series model (N=2), and laminar flow.

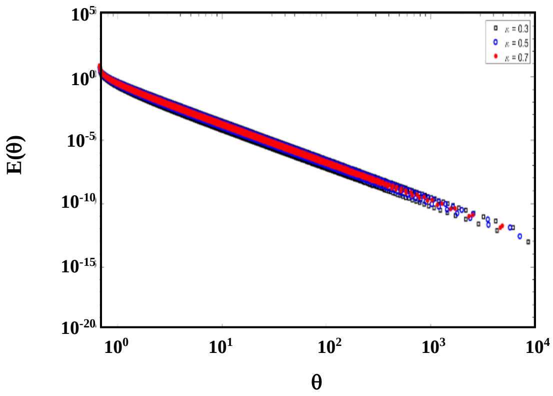

Fig. 4: The RTD of annular flow for different radius ratios of the inner to outer pipe (0.3, 0.5, and 0.7).

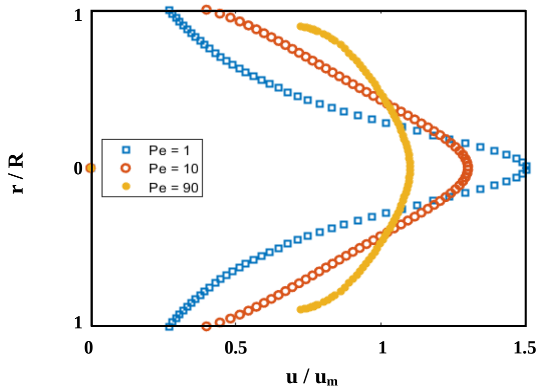

Fig. 5: The velocity profile corresponding to the dispersion model with open-open boundary conditions for different values of Pe (1, 10, and 90).

Where $K = 2/(1+ \kappa^2 - 2\lambda^2)$. Eq. (12) leads to

$$E(\theta) = \frac{1}{\theta^3} \frac{x^2}{(1-\kappa^2) K \left| x^2 - \kappa^2 \right|} \tag{23}$$

Where E is a function of θ beside Eq. (10). However, evaluation of E versus θ is more straightforward if x is specified at first and then θ and E are calculated. However, for testing Eq. (14) it is worth noting that as x increases from κ to λ, the value of θ decreases. Therefore, for numerical integration purposes, the values of θ and E should be sorted at first. Fig. 4 shows the RTD of annular flow for different values of κ.

According to Fig. 4 the RTD is nearly independent of κ.

The velocity profile corresponding to the famous RTD models

If the vessel is in a cylindrical geometry- as usually occurs for pipe flow, packed column, etc. - the Eq. (12) will take the following form

$$E(\theta) = -\frac{1}{\theta^3} \frac{2x}{df/dx} \tag{24}$$

Eqs. (24) and (10) may be used to develop the velocity profiles for the famous RTD models. Using the dispersion model with open-open boundary conditions [3] the following differential equation should be solved numerically to derive f(x) versus x

$$\frac{df}{dx} = -4\sqrt{\frac{\pi}{Pe}}\, x f^{5/2} \exp\left[\frac{Pe(f-1)^2}{4f}\right] \tag{25}$$

The final result using the function ode45 in MATLAB is depicted in Fig. 5. The shape of the curve especially for Pe = 1 is completely different compared to the power-law or logarithmic velocity profiles.

A similar equation based on Eq. (21) for closed-closed boundary conditions is

$$\frac{df}{dx} = -8 x f^{5/2} \frac{\sqrt{\pi/Pe}}{4+f} \exp\left[\frac{Pe(f-1)^2}{4f}\right] \tag{26}$$

The final result is shown in Fig. 6.

The theory of the dispersion model is based on tracer concertation. Therefore, the derived velocity profiles

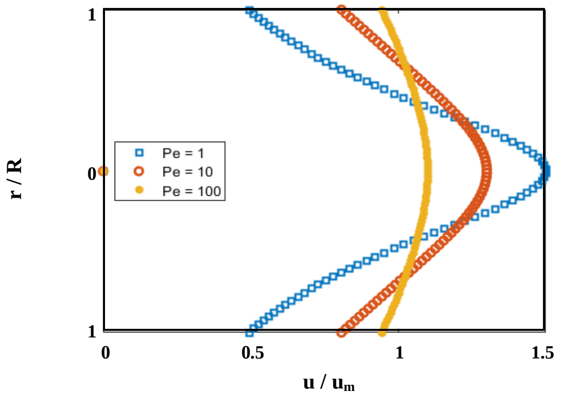

Fig. 6: Fig. 6 The velocity profile corresponding to the dispersion model with closed-closed boundary conditions for different values of Pe (1, 10, and 90).

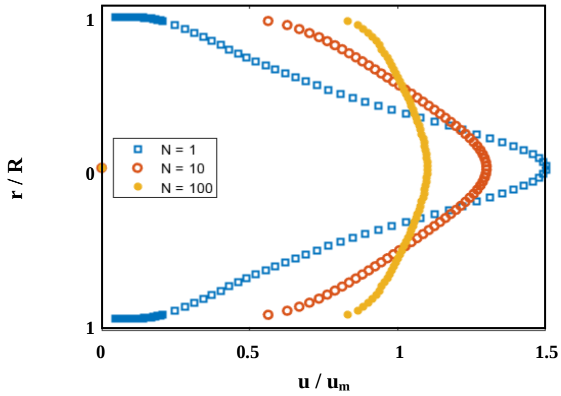

Fig. 7: The velocity profile corresponding to the tanks-in-series model for a different number of the tanks (1, 10, and 90).

Eqs. (25) and (26) are correct if and only if the velocity profile coincides with the concentration profile.

Finally, the velocity profile of the tanks-in-series model can be derived as the solution of the following differential equation

$$\frac{df}{dx} = -\frac{2(N-1)!}{N^N} x f^{N+2} e^{N/f} \quad (27)$$

The numerical solution leads to Fig. 7 as the velocity profile corresponding to the tanks-in-series model for RTD.

The case of N = 1 denotes the velocity profile of a pipe in a completely mixed flow condition if the velocity distribution is considered a one-dimensional radial function. It does not have any physical sense. Therefore, the parameter N in the velocity profile derived based on tanks-in-series should be considered as a parameter of the model without any physical meaning.

CONCLUSIONS

In this work, it was shown that each axial velocity profile corresponds to an individual form of RTD and vice versa. This conclusion is not restricted to laminar flows as reported in previous works. In addition, various senses of RTD can be introduced based on the applied forcing function. Based on these findings, the following cases are mentioned as the conclusions:

- A theoretical basis for the definition of some useful RTDs was established.
- The RTDs corresponding to logarithmic velocity profile, power-law velocity profile, and laminar flow through an annular pipe were derived for the first time.
- The general form of the logarithmic velocity profile has two constants that are related together.
- The velocity profile corresponding to famous RTD models is numerically derived.

Supporting Information

The proof of the equation (21), and MATLAB codes for generating Figs. 2−7(PDF).

Numencluture

A_c	Cross section, m^2
B	Constant
C	Concentration, kg/m^3
C_p	Specific heat capacity, j/kg.K
E(t)	RTD function
$f(x) = u / u_m$	Dimensionless velocity
g(x)	A geometric function
G(s)	Transfer function
M	Mass, kg
N	Number of tanks
Pe	Peclet number
Q	Heat, j
q	Flow rate, m^3/s
R	Radius, m
R	Radial position, m
T	Temperature, K
t	Time, s
V	Volume, m^3
x	Dimensionless position variable
y	Position variable, m

Subscript

in	Input
max	Maximum
min	Minimum
out	Output
w	Wall

Superscript

*	Characteristic base on the viscous scale
+	Dimensionless base on the viscous scale

Greek letters

δ	Characteristic length scale, m
$\delta(t)$	Impulse Dirac delta function, 1/s
ζ	Denotes some suitable quantity
$\theta = t/\tau$	Dimensionless time
κ	Constant, radius ratio of the inner to outer pipe
λ	Constant
$\nu = \mu / \rho$	Kinematic viscosity, m^2/s
ξ	Denotes some suitable function
ρ	Density, kg/m^3
τ	Time constant, space time, stress

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