

# Thermodynamic Assessment and Optimization of Performance of Irreversible Atkinson Cycle

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**ABSTRACT:** Although various investigations of Atkinson cycle have been carried out, distinct output power and thermal efficiencies of the engine have been achieved. In this regard, thermal efficiency, Ecological Coefficient of Performance (ECOP), and Ecological function (ECF) are optimized with the help of NSGA-II method and thermodynamic study. The Pareto optimal frontier which provides an ultimate optimum solution is chosen utilizing various decision-making approaches, containing fuzzy Bellman-Zadeh, LINMAP, and TOPSIS. With the help of the results, interpreting the performances of Atkinson cycles and their optimization is enhanced. Error analysis has also been performed for verification of optimization and determining the deviation in the study.

**KEYWORDS:** Atkinson cycle; Thermodynamic analysis; Power; Ecological Coefficient of Performance; Thermal efficiency; Entropy generation.

## INTRODUCTION

The main purpose of designing the cycle is to present the performance of the system by the mean of input power. In the Atkinson cycle, the intake, compression, power, and exhaust strokes of the four-step take place by one piston sweep. In the Atkinson cycle, the compression ratio is less than the expansion ratio, due to the linkage, which results in higher efficiency related to the engines utilizing the alternative Otto cycle. Also, four-stroke engines are referred to as the Atkinson cycle. In these arrangements, the intake step takes longer time to fill the intake manifold with fresh air. Its result is the reduction of efficient compression ratio, and on the condition

of combining with an increment stroke and/or decreased volume of the combustion chamber, causes the ratio of expansion to outstrip the ratio of compression, throughout the time of maintaining a regular compression pressure. So, it is favorable for enhanced cost efficiency, since in a spark-ignition engine, the octane rating ratio limits the compression. So, a longer power stroke is the result of a high expansion ratio, provides higher expansion ratios, followed by the reduction of heat associated with the waste in the exhaust [1].

A growing number of thermodynamic investigations has focused on determining the limits of performance

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in thermal systems as well as optimizing thermodynamic cycles and processes containing finite-rate, finite-time, and finite-size constraints [2-7]. Also, The performance analysis for internal combustion engine cycles by using finite time thermodynamics were also performed by other papers [8-12]. Recently, abundant performance evaluations of the Atkinson heat engine have been carried out founded on irreversible, endo-reversible, and reversible cycle arrangements with utilizing various objective functions including power output, the generated power and performance, etc [13-25]. The engine sizes' effect associated with the investment cost is not taken into account in the analyses of performance contributed to the mentioned optimization criteria. In this way, *Sahin et al.* [26,27] presented a novel optimization criteria named the maximum power density examination for the sake of including the engine size's effects. Optimum operational states of reversible [26] and irreversible [27] non-regenerative Joule-Brayton cycle have been studied by utilizing the maximum power density (MPD) criterion. In the study, the power density has been maximized and model elements at MPD states have been found. These conditions lead to more efficient and smaller setups. Many investigations have been performed the MPD method to various models of heat engines [28-38].

Added to this, *Chen et al.* [39] and *Wang and Hou* [40] performed the technique of MPD to the Atkinson cycle. They presented that the MPD efficiency is higher than the MP efficiency. Also, *Wang and Hou* [40] investigated an Atkinson cycle linked to a variable temperature heat source at MP and MPD. The analysis revealed that an engine which is designed for MPD conditions, has smaller size than a MP design based engine.

For the sake of unraveling enigma of this general category, during the whole of the mid-eighties, Evolutionary Algorithms (EA) were basically utilized [41]. Determining a cluster of answers, each of which implements the objectives on the condition of a gratifying degree without being overshadowed by any other answers is a pragmatic answer to a multi-objective puzzle [42]. Issues contributed to multi-objective optimization regularly perform as an achievably innumerable group of answers which is called Pareto frontier, where investigated vectors indicate primary feasible

interchanges in the objective function area. With respect to this, multi-objective optimization of various processes has been examined in plenty of researches [43-83].

In the present research, an irreversible Atkinson cycle is optimized in line with performance improvement of the system. In this scenario, for maximizing the thermal efficiency, ECOP and ECF parameters, a multi-objective optimization solution is applied. For the sake of evaluating eventual answers' precision in different decision-making approaches, error analysis is carried out.

### Cycle model and analysis

Fig. 1 shows an air standard Atkinson cycle diagram.

The working fluid of the most cycle models is considered to be as an ideal gas with the characteristic of fixed specific heats. However, this presumption could be authentic only on the condition of small temperature difference. So, in practical cycle, in which there is large temperature difference, the mentioned presumption cannot be implemented. As ref. [84] indicated, under the condition of temperature range between 200-1000 K, the specific heat capacity with fixed pressure is as follows:

$$C_p = (3.56839 - 6.788729 \times 10^{-4}T + 1.5537 \times 10^{-6}T^2 - 3.29937 \times 10^{-12}T^3 - 466.395 \times 10^{-15}T^4)R_g \quad (1)$$

For temperatures between 1000 and 6000 K, the  $C_p$  is calculated as:

$$C_p = (3.08793 + 12.4597 \times 10^{-4}T - 0.42372 \times 10^{-6}T^2 + 67.4775 \times 10^{-12}T^3 - 3.97077 \times 10^{-15}T^4)R_g \quad (2)$$

For temperatures between 200 and 600k, Eqs. (1) and (2) can be used which the rage is too wide for the temperature range (300-3500 K) of pragmatic engine. Thus, for describing the specific heat model a single equation has been utilized. The presumption associated with this is air must be an ideal gas.

$$C_p = (2.506 \times 10^{-11}T^2 + 1.454 \times 10^{-7}T^{1.5} - 0.4246 \times 10^{-7}T + 3.162 \times 10^{-5}T^{0.5} + 1.3303 - 1.512 \times 10^4 T^{-1.5} + 3.063 \times 10^5 T^{-2} - 2.212 \times 10^7 T^{-3}) \quad (3)$$

$$C_v = C_p - R_g \quad (4)$$

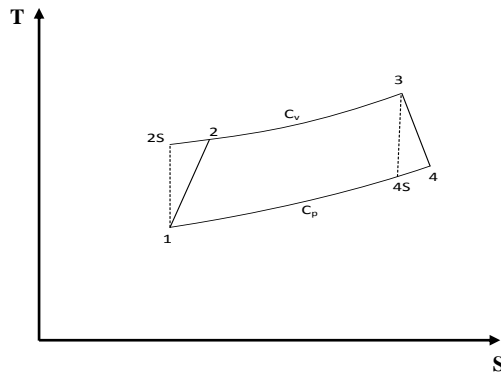


Fig. 1: Schematic of an Atkinson cycle diagram.

According to Eq (4) the  $C_v$  at fixed volume is determined as follows:

$$C_v = (2.506 \times 10^{-11} T^2 + 1.454 \times 10^{-7} T^{1.5} - 0.4246 \times 10^{-7} T + 3.162 \times 10^{-5} T^{0.5} + 1.0433 - 1.512 \times 10^4 T^{-1.5} + 3.063 \times 10^5 T^{-2} - 2.212 \times 10^7 T^{-3}) \quad (5)$$

The received heat by the working fluid in through  $2 \rightarrow 3$  is:

$$Q_{in} = M \int_{T_2}^{T_3} C_v dT = M(8.353 \times 10^{-12} T^3 + 5.816 \times 10^{-8} T^{2.5} - 2.123 \times 10^{-7} T^2 + 2.108 \times 10^{-5} T^{1.5} + 1.0433T + 3.024 \times 10^4 T^{-0.5} - 3.063 \times 10^5 T^{-1} + 1.106 \times T^{-2})_{T_2}^{T_3} \quad (6)$$

The rejected heat is calculated as follows:

$$Q_{out} = M \int_{T_1}^{T_4} C_p dT = M(8.353 \times 10^{-12} T^3 + 5.816 \times 10^{-8} T^{2.5} - 2.123 \times 10^{-7} T^2 + 2.108 \times 10^{-5} T^{1.5} + 1.3303T + 3.024 \times 10^4 T^{-0.5} - 3.063 \times 10^5 T^{-1} + 1.106 \times T^{-2})_{T_1}^{T_4} \quad (7)$$

The adiabatic compression and expansion performance of the cycle in  $1 \rightarrow 2$  and  $3 \rightarrow 4$  procedure are determined as follows [85-91]:

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \quad (8)$$

$$\eta_e = \frac{T_4 - T_3}{T_{4s} - T_3} \quad (9)$$

$\eta_c$  And  $\eta_e$  can present the internal irreversibility of the procedures. Due to the dependency of  $C_p$  and  $C_v$  on temperature, adiabatic element  $k = C_p/C_v$  varies by the changes of temperature. Thus, Eq. (10) is not valid for adiabatic stages. Nonetheless, regarding to refs. [90-101], a proper estimation for reversible adiabatic procedure with  $k$  can be carried out, *i.e.* this procedure can be split to many infinitesimally small stages that each  $k$  is considered fixed. For instance, between states  $i$  and  $j$ , every reversible adiabatic process could be considered as containing plentiful infinitesimally small processes with fixed  $k$ . On the condition of volume  $dV$  of the working fluid takes place, and an infinitesimally small change in temperature  $dT$ , for any of these processes, the equation for reversible adiabatic process with variable  $k$  can be presented as follows.

$$TV^{k-1} = (T + dT)(V + dV)^{k-1} \quad (10)$$

The heat added in constant-volume process is calculated as follows:

$$Q_{in} = C_v(T_j - T_i) = \bar{T} \Delta S_{i \rightarrow j} = \bar{T} C_v \ln(T_j/T_i). \quad \text{So}$$

one has  $\bar{T} = (T_j - T_i) / \ln(T_j/T_i)$ , where  $T$  is the equivalent temperature of the procedure. When  $C_v$  is the subordinate of temperature, the  $C_v(\bar{T})$  could be considered as mean specific heat of fixed volume.

From eq. (10), one gets

$$C_v \ln\left(\frac{T_j}{T_i}\right) = R_g \ln\left(\frac{V_i}{V_j}\right) \quad (11)$$

The temperature in  $C_v$  calculation is logarithmic  $T = (T_j - T_i) / \ln(T_j/T_i)$ . Also, the compression ratio is determined as:

$$\gamma = \frac{V_1}{V_2} \quad (12)$$

Thus, equations contributed to reversible adiabatic processes  $1 \rightarrow 2s$  and  $3 \rightarrow 4s$  are:

$$C_v \ln\left(\frac{T_{2s}}{T_1}\right) = R_g \ln \gamma \quad (13)$$

$$C_V \ln\left(\frac{T_{4s}}{T_3}\right) - R_g \ln\left(\frac{T_1}{T_{4s}}\right) = -R_g \ln \gamma \quad (14)$$

Heat transfer losses are not considered for an ideal Atkinson design. Nonetheless, heat transfer irreversibility between cylinder wall and working fluid cannot be neglected in a real Atkinson cycle. It is presumed that mean temperature of the working fluid and the cylinder wall mutually and the heat transfer via the cylinder wall (*i. e.* the heat leakage loss) are relative and the wall temperature is equal to  $T_0$  (K). If the generated heat by combustion procedure per second be  $A_1$  (kW) and the heat leakage factor of the cylinder wall be  $B_1$  [kJ/kg.K], the applied heat flow rate is calculated as follows [102-105]:

$$Q_{in} = A_1 - B_1 \frac{(T_2 + T_3 - 2T_0)}{2} \quad (15)$$

Eq. (15), shows that  $Q_{in}$  contains two terms, the first part is  $A_1$ , and the other one is the heat loss per second, which can be defined as:

$$Q_{leak} = B(T_2 + T_3 - 2T_0) \quad (16)$$

where  $B = B_1/2$ .

In order to consider the friction loss [92] we have:

$$f_\mu = \mu v = \mu \frac{dx}{dt} \quad (17)$$

Where  $\mu$  [Ns/m] is a friction parameter and  $x$  is the piston displacement.  $P_\mu$ , which represents the lost power is defined as:

$$P_\mu = \frac{dW_\mu}{dt} = \mu v^2 \quad (18)$$

The total displacement of the piston in each cycle of four-step engines is calculated as:

$$4L = 4(x_1 - x_2) \quad (19)$$

The mean velocity of the piston is as follows ( $N$  cycles):

$$\bar{v} = 4LN \quad (20)$$

Then, the power output and the cycle performance efficiency are determined as follows:

$$P = Q_{in} - Q_{out} - P_\mu \quad (21)$$

$$\eta_{th} = \frac{P}{Q_{in} + Q_{leak}} \quad (22)$$

$T_{2s}$  can be achieved by Eq. (13) while  $R_g$ ,  $T_1$ ,  $T_3$ ,  $\eta_c$ , and  $\eta_e$  are provided. Next, substituting  $T_{2s}$  into Eq. (8) leads to measure  $T_2$ ,  $T_{4s}$  with the help of Eq. (14). The last step is to determine  $T_4$  by substituting  $T_{4s}$  into Eq. (9). The power and efficiency can be obtained by Substituting  $T_2$  and  $T_4$  into Eqs. (21) and (22).

Entropy generation, ECOP and ECF (kW) of the Atkinson cycle can be determined as following as:

$$S_{gen} = \left( \frac{Q_{out}}{T_L} - \frac{Q_{in}}{T_H} \right) \quad (23)$$

$$ECOP = \frac{P}{T_0 S_{gen}} \quad (24)$$

$$ECF = P - T_0 S_{gen} \quad (25)$$

### Multicriteria optimization

For optimization purposes, Genetic Algorithms (GA) were proposed by *Holland* (1960) [44]. The evolution generally begins a population which each individuals generated accidentally. The fitness rate associated with each individual of the population is investigated. Many random individuals are developed to make a new population. At the next step, the new population is applied to the following iteration of the GA. Regularly, on the condition of achieving an favorable fitness level of the population or creating the highest value of generations, the GA stops. Previous works have been presented more details of GA [42,46].

Furthermore, during the past years with persistent investigations on multipart mathematical puzzles and on pragmatic engineering problems, MOEAs were extracted. Also, throughout the examinations, it was concluded that the complexity of conventional approaches can be excluded [42,46]. Fig. 2 depicts the construction contributed to the MOEA applied in this paper [44,46]. It is worth stating that the actual amounts of decision elements were employed instead of their binary coded.

Three objective functions are utilized in this optimization: thermal efficiency, ECOP and ECF, described by Eqs. (22), (24) and (25), respectively.

Also, five decision variables are considered: temperatures of state points 1 and 3, the Expansion

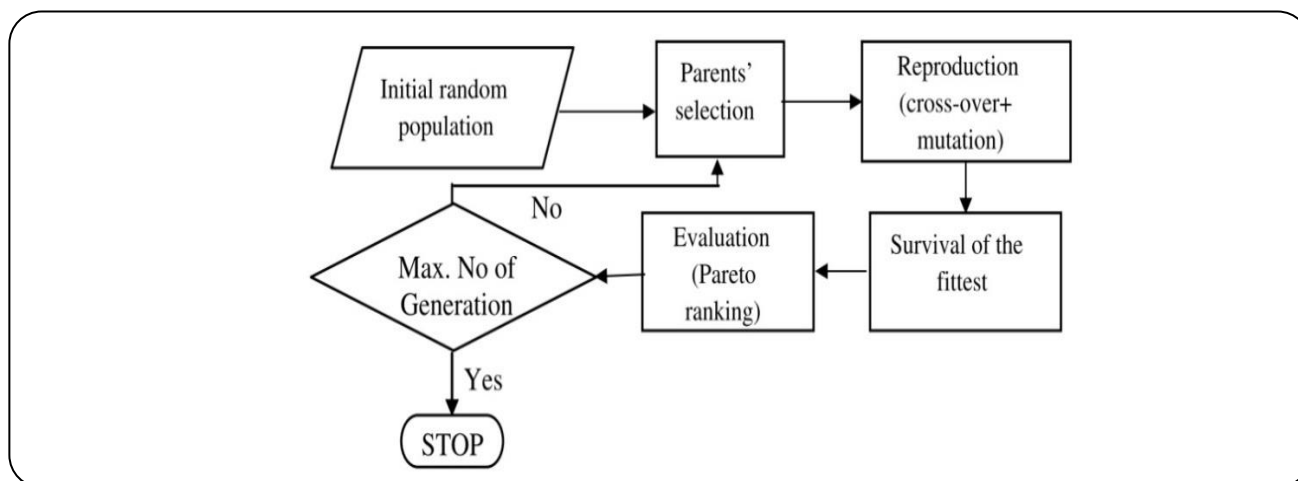


Fig. 2: Algorithm steps applied in the study [44,46].

efficiency ( $\eta_e$ ), the compression efficiency ( $\eta_c$ ) and the compression ratio  $\gamma$ .

Although the decision variables might be different in the optimizing plan, but they typically need to be fitted in a sensible range. Thus, the objective functions are determined by relative succeeding limits:

$$325 \leq T_1 \leq 400\text{K} \quad (26)$$

$$1300 \leq T_3 \leq 1900\text{K} \quad (27)$$

$$0.85 \leq \eta_e \leq 0.97 \quad (28)$$

$$0.85 \leq \eta_c \leq 0.97 \quad (29)$$

$$6 \leq \gamma \leq 12 \quad (30)$$

## RESULTS AND DISCUSSION

In this section, the sensitivity of the objective functions for decision parameters is investigated. Following Ref. [106], the following parameters are used here:  $x_1 = 8 \times 10^{-2}$  m,  $x_2 = 1 \times 10^{-2}$  m,  $\mu = 12.9$  (Ns/m),  $T_H = 2200$  K,  $T_L = 300$  K,  $T_0 = 300$  K,  $N = 30$ ,  $\gamma = 8.5$ ,  $M = 4.553 \times 10^{-3}$  kg/s,  $B = 0.2$  (kJ/kgK),  $\eta_e = 0.97$ , and  $\eta_c = 0.97$ .

According to Fig. 3a, the ECF reduced with augmenting the  $T_1$  at different values of  $T_3$ . As illustrated in Fig. 3b, the ECOP decreased considerably with  $T_1$  at different values of the  $T_3$ . According to Fig. 3c, the thermal efficiency ( $\eta_{th}$ ) reduced by augmenting the  $T_1$  at different values of the  $T_3$ . According to Fig. 3d,

the Power output ( $P$ ) reduced by increasing the  $T_1$  at different amounts of the  $T_3$ .

According to Fig. 4a, the ECF of the system reduced significantly with augmenting the  $T_1$  at different rates of the Expansion efficiency ( $\eta_e$ ). According to Fig. 4b, ECOP decreased considerably with  $T_1$  at different rates of the Expansion efficiency ( $\eta_e$ ). As depicted in Fig. 4c, the thermal efficiency ( $\eta_{th}$ ) reduced considerably with rising of  $T_1$  at different rates of the Expansion efficiency ( $\eta_e$ ). As it is seen in Fig. 4d, the Power output ( $P$ ) decreased significantly with rising of  $T_1$  at distinct rates of the Expansion efficiency ( $\eta_e$ ).

As depicted in Fig. 5a, the ECF reduced by increasing the  $T_1$  at different rates of the compression efficiency ( $\eta_c$ ). As it is illustrated in Fig. 5b, the ECOP reduced by enhancing  $T_1$  at different rates of the compression efficiency ( $\eta_c$ ). As shown in Fig. 5c, the thermal efficiency ( $\eta_{th}$ ) reduced with increasing the  $T_1$  at different rates of the compression efficiency ( $\eta_c$ ).

As shown in Fig. 5d, the Power output ( $P$ ) reduced with increasing the  $T_1$  at different rates of the compression efficiency ( $\eta_c$ ).

In this study, the thermal efficiency, ECOP and ECF of the Atkinson cycle are maximized concurrently utilizing multi-objective optimization based on the NSGA-II approach. The objective functions are illustrated by Eqs. (22), (24) and (25) and the limitations by Eqs. (26) -(30).

The decision parameters of optimization are as follow:  $T_1$ ,  $T_3$ , the Expansion efficiency ( $\eta_e$ ), the compression efficiency ( $\eta_c$ ) and the compression ratio  $\gamma$ .

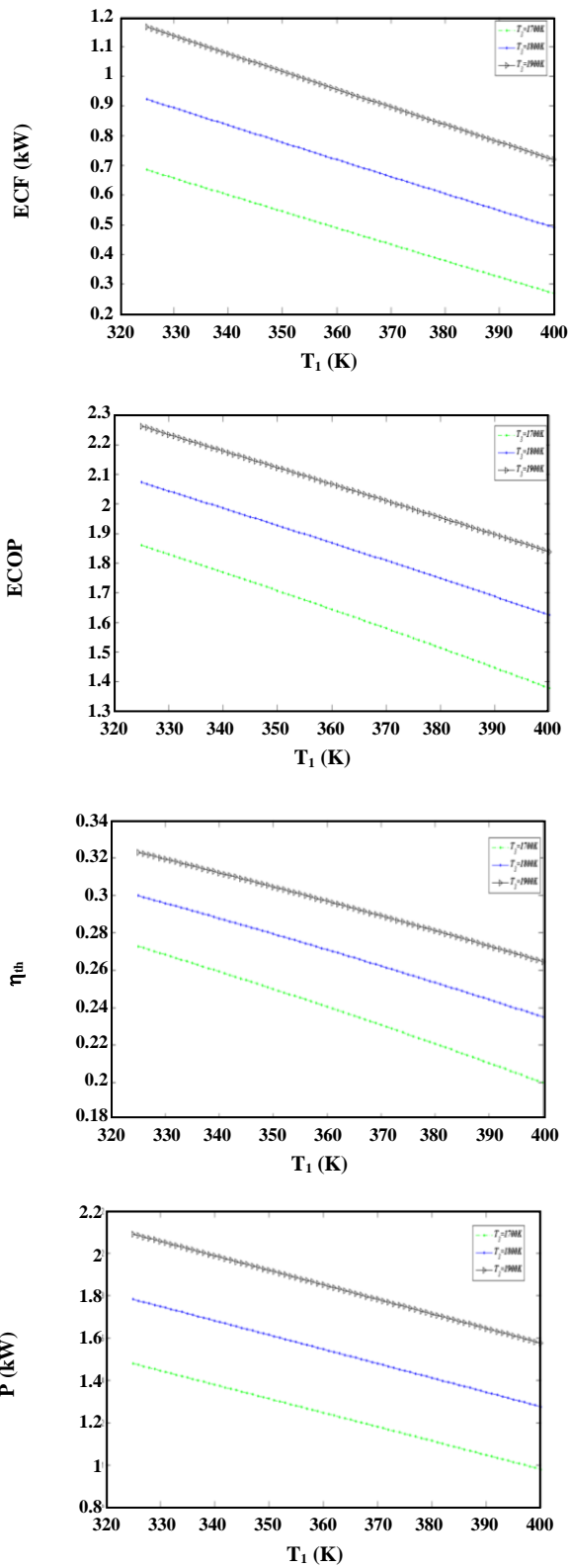


Fig. 3: Effects of  $T_1T_3$  on the (a) ECF (b) ECOP, (c) thermal efficiency, (d) Power output in  $\eta_e = 0.97$ ,  $\eta_c = 0.97$ .

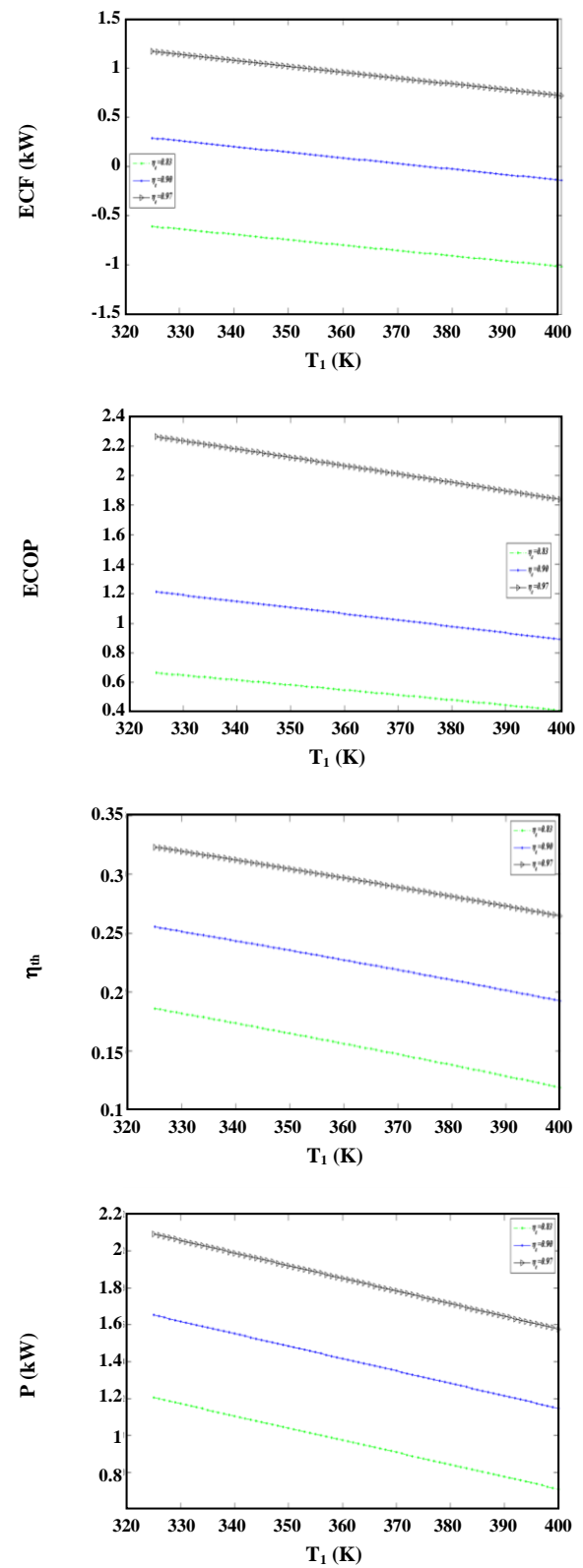


Fig. 4: Effects of the  $T_1$  and the expansion efficiency ( $\eta_e$ ) on the (a) ECF (b) the ECOP, (c) thermal efficiency, (d) Power output in  $T_3 = 1900$  K,  $\eta_c = 0.97$ .

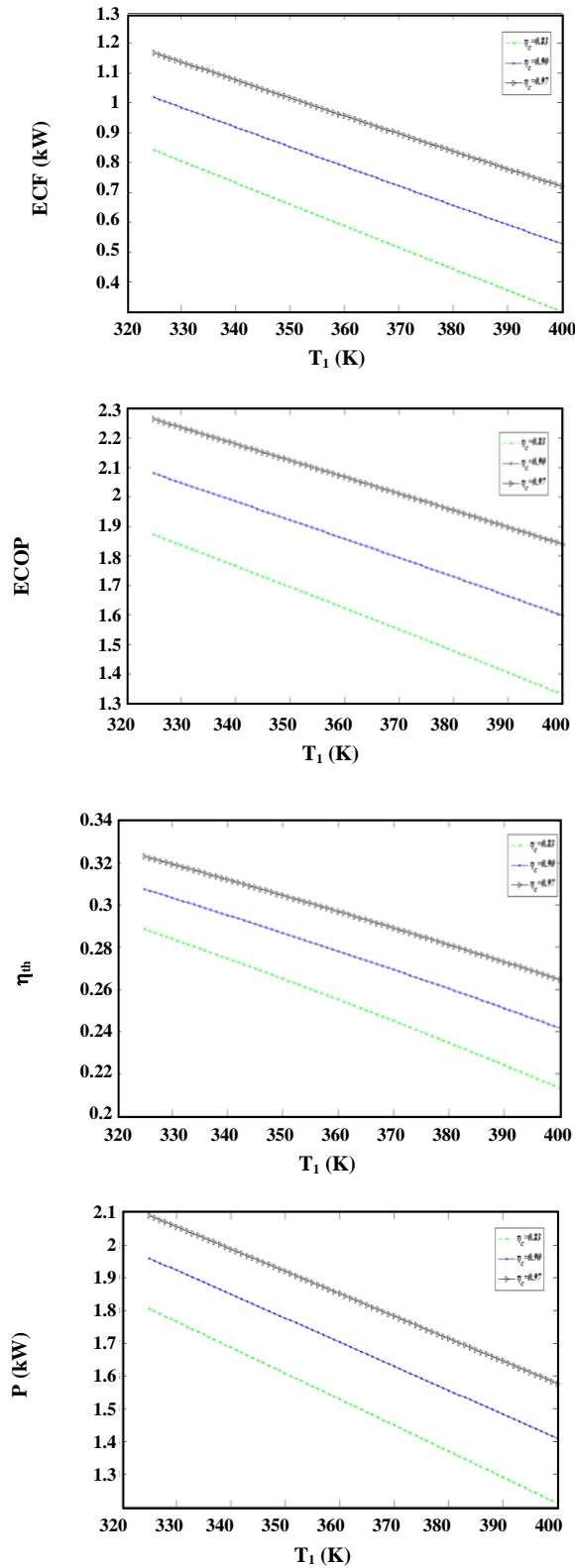


Fig. 5: Effects of the  $T_1$  and the compression efficiency ( $\eta_c$ ) on the (a) ECF (b) ECOP, (c) thermal efficiency, (d) Power output in  $T_3 = 1900$  K,  $\eta_e = 0.97$ .

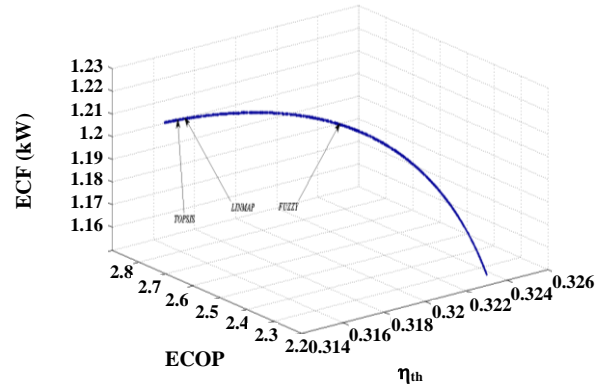


Fig. 6: Pareto optimum frontier in objective space.

The Pareto optimal frontier of objective functions (the thermal efficiency, ECOP and ECF) is depicted in Fig. 6. Selected points with different decision-making approaches are depicted, as well.

Table 1 outlines and compares the optimal results associated with decision elements and objective functions utilizing LINMAP, TOPSIS, and Bellman-Zadeh decision-making approaches. Analyses results are presented in Table 2, as well. The achieved results are favorable and it is predicted that the present investigation improves interpreting the optimum model of the Atkinson cycle. Based on the Mean Absolute Percent Error (MAPE) approach, an error analysis was performed to define the average error for thermal efficiency of solutions gathered by decision making approaches. Along with results, these errors are 0.071%, 0.079% and 0.138% for TOPSIS, LINMAP and FUZZY, respectively. This study demonstrated that the average error for the ECOP are 0.163%, 0.122% and 0.237% for TOPSIS, LINMAP and FUZZY, respectively. This study demonstrated that the average error for the ECF of solutions are 0.237%, 0.234% and 0.169% for TOPSIS, LINMAP and FUZZY, respectively.

### CONCLUSIONS

A thermodynamic optimization procedure has been employed for determining the thermal efficiency, ECOP and ECF of the Atkinson cycle. The Expansion efficiency ( $\eta_e$ ), the compression efficiency ( $\eta_c$ ),  $T_1, T_3$ , and the compression ratio  $\gamma$  are studied utilizing the NSGA-II method. For the sake of designing and evaluating the performance and robustness of Atkinson cycle, the results can be implemented. By utilizing decision making

Table 1: Decision making results of this study.

Decision Making Method	Decision variables					Objectives		
	T <sub>1</sub> (K)	T <sub>3</sub> (K)	$\eta_e$	$\eta_c$	$\gamma$	$\eta_{th}$	ECOP	ECF (kW)
TOPSIS	325/003	1900	0/970	0/970	11/888	0/314	2/696	1/217
LINMAP	325/003	1900	0/970	0/970	11/813	0/315	2/688	1/217
Fuzzy	325/004	1900	0/970	0/970	10/348	0/320	2/520	1/213

Table 2: Error analysis based on MAPE for this study.

Decision Making Method	TOPSIS			LINMAP			Fuzzy		
Objectives	$\eta_{th}$	ECOP	ECF	$\eta_{th}$	ECOP	ECF	$\eta_{th}$	ECOP	ECF
Max Error %	0.141	0.310	0.464	0.156	0.251	0.466	0.234	0.240	0.380
Average Error %	0.071	0.163	0.237	0.079	0.122	0.234	0.138	0.237	0.169

approaches (LINMAP, TOPSIS and fuzzy), a final optimum solution was chosen from Pareto frontier. Generally, the results achieved from distinct decision-making methods are very close. This investigation illustrated that all results are acceptable and propitious for this system.

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