Finite-Time Sliding Mode Control for Continuous Stirred Tank Reactors Based on Disturbance Observer

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ABSTRACT: *This paper aims to investigate the robust control problem of Continuously Stirred Tank Reactors (CSTR). A CSTR is one of the most essential pieces of equipment in chemical processes, whose effects of highly nonlinear dynamic and external disturbances make it very difficult to control. Firstly, a novel finite-time sliding mode control is introduced that eliminates disturbance effects and ensures finite-time tracking. Secondly, to better compensate for disturbances and to improve controller performance, a finite-time disturbance observer is developed. Finally, an adaptive robust control method is introduced based on the proposed sliding mode control and the disturbance observer. Stability analysis is performed to investigate the finite-time tracking of the closed-loop system under the proposed controllers. Besides, to enhance the performance of the proposed controllers, the design parameters are tuned by the genetic optimization algorithm. Simulation results are produced to confirm the efficiency of the proposed methods in terms of tracking errors and convergence rates. The proposed finite-time sliding mode control and the adaptive finite-time sliding mode control with settling times of 1.73s and 1.71s as well as IAE of 0.509 and 0.4843, respectively, showed more desirable performance than other controllers.*

KEYWORDS: *CSTR; Disturbance estimation; Robust control; Finite-time convergence; Genetic algorithm.*

INTRODUCTION

A Continuous Stirred Tank Reactor (CSTR) is an important system used for converting reactants to products [1]. We can consider a CSTR as the opposite of an idealized batch or reactor [2]. Due to the complexity of this system and the effects of concentration and temperature, the control of this system is very challenging and crucial. Researchers have put a lot of effort into designing controllers able to keep the system stable in the presence of disturbances and uncertainties. The next subsection provides a detailed literature review.

In many industrial applications, CSTRs work under specific operating conditions in which linear controllers are utilized to control the system around operating points [3]. For example, in a study [4], a PID controller was designed to stabilize a CSTR. In this study, a local linear model of the system was identified by a lazy learning algorithm, with a conventional PID introduced for the linearized system. In several studies, intelligent methods such as fuzzy logic were used as well in designing controllers. Another study introduced a fuzzy PID controller

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for a CSTR, in which a nonlinear system was divided into three different linear regions and a PID controller was designed for each region [5]; the parameters of this controller were tuned by the integration of a genetic algorithm and fuzzy gain scheduling. In another study [6], the CSTR system was linearized in three equilibrium points, and the PID controller was designed for the system. In this study, the cost function was defined as an integral square tracking error, with the three coefficients of this controller tuned by an artificial bee colony algorithm. However, the main problem of this research was that the effect of disturbances was not considered. A study [7] introduced an adaptive fuzzy gain scheduling PID controller for a CSTR. The results of the mentioned controller were compared to those of the fuzzy gain scheduling PID and the conventional PID. Accordingly, it was reported that despite the same level of control inputs, adaptive fuzzy gain scheduling PID controllers performed better. In the reviewed studies, linear controllers were generally designed for linear models, yet the assumption of disturbances was not taken into account. Therefore, these controllers were not effective in controlling nonlinear systems affected by disturbances. In the simulation section, it is shown that linear controllers cannot control a nonlinear CSTR properly. Accordingly, nonlinear and robust controllers are needed to control the system more efficiently. In the following section, Sliding Mode Control (SMC) is introduced as one of the major robust and nonlinear controllers.

Sliding mode control is an efficient robust control method that is widely applied to control nonlinear systems; Among the key advantages of this controller, one can allude to its rapid response, decoupling design procedure, and high robustness in the face of external disturbances [8,9]. Nevertheless, sliding mode control suffers from some major drawbacks, including infinite time convergence and chattering. Chattering is high-frequency oscillations appearing in the responses of the sliding mode control. This phenomenon is caused by the discontinuous control law of the sliding mode control, which may cause serious damage [10]. The sliding mode controller has been extensively employed in controlling industrial processes, in a study [11], a combination of sliding mode control and fuzzy logic was introduced to control nonlinear chemical processes. The proposed controller, despite its less dependence on process dynamics, showed the same performance as that of the normal sliding mode control. Despite the proper performance of the proposed controller and its less dependence on the system model, the main problem of the SMC (chattering) has not been fully investigated.

In research [12], a novel second-order sliding mode controller was presented for a polymer electrolyte membrane fuel cell. The proposed controller was implemented in a cascade structure. The results showed the high accuracy of the introduced controller and the attenuation of the undesirable chattering phenomenon. In a study [13], a terminal sliding mode control was proposed for a CSTR. In this study, finite-time stability was achieved and a state observer was developed to estimate the internal states of the system based on output measurements. Besides, it was assumed that the upper bound of the disturbance was known, which is an unrealistic assumption. Another study [14] introduced a novel event-triggered sliding mode controller for a CSTR. This study showed that the proposed method effectively reduced the computational burden of the controller. However, in this study, the chattering problem and response rates were not considered. In designing robust controllers (especially sliding mode controllers), controller performance can be significantly improved by estimating the upper bound of the disturbances [15]. Accordingly, various studies proposed Disturbance Observers (DO).

Thus use of disturbance observers has recently become common in controlling nonlinear systems, as these estimators are capable of estimating lumped disturbances [16]. Thus, different types of disturbance observers have been introduced to make an appropriate estimate of disturbances. In research [17], a disturbance observer was developed for a grid-connected doubly fed induction generator to decrease uncertainty effects on the system. Moreover, the effectiveness of disturbance observers has been validated by experimental tests. Research [18] introduced a disturbance observer for estimating fast-varying disturbances for simple linear systems. In addition, a systematic approach was introduced to tune the parameters of the observer. In the proposed DO, conservative assumptions were avoided. The integration of the sliding mode control and disturbance observers has attracted much attention from researchers [19]. In a study [20], an active Disturbance Rejection Control (ADRC) was proposed, which was isolated for a wind-diesel hybrid power system. Firstly, a disturbance observer was produced to estimate source-load disturbances of the system, and secondly, the adaptive sliding mode control was applied to the system to regulate voltage and frequency. In another study [21], a novel nonlinear disturbance observer was proposed for a wind energy conversion system to estimate uncertainties. Next, the effects of the estimated uncertainties were compensated by a sliding mode controller. A study [22] investigated the anti-disturbance speed control for a high-torque low-speed PMSM. In this study, a second-order disturbance observer was introduced to estimate load changes in a PMSM, and a second-order sliding mode controller was presented to control the speed.

CONTRIBUTIONS AND PAPER ORGANIZATION

According to the literature review presented above, in most of the studies, linear controllers were employed to control this system. However, disturbance effects were not considered in most of these studies. The main research gap in controlling CSTRs is that nonlinear robust controllers have been rarely employed in controlling this system and eliminating external disturbances. In the present study, a novel finite-time sliding mode control was introduced to control CSTRs. Moreover, external disturbances affecting the system were estimated by a novel finite-time disturbance observer. In addition, an adaptive finite-time sliding mode controller was introduced based on the disturbance observer. Contributions of this paper are listed as follows:

- A novel sliding mode control was designed for a CSTR, with its stability, proved using the Lyapunov theory. Unlike research [14], this sliding mode control ensures finite-time convergence. The main innovation in designing this controller was the proposed sliding surface, which could ensure convergence in a finite time.
- Unlike studies [13, 23], in the proposed control method, the chattering phenomenon was eliminated by designing a dynamic control input so that a smooth control input was achieved.
- A novel finite-time disturbance observer was developed to estimate the upper bound of external disturbances.
- An adaptive sliding mode control (by the integration of the proposed finite-time sliding mode control and the disturbance observer) was introduced for CSTRs, which ensures more accurate tracking.
- Upon employing the finite-time controller and the disturbance observer, the finite-time stability of the closed-loop system was proved.

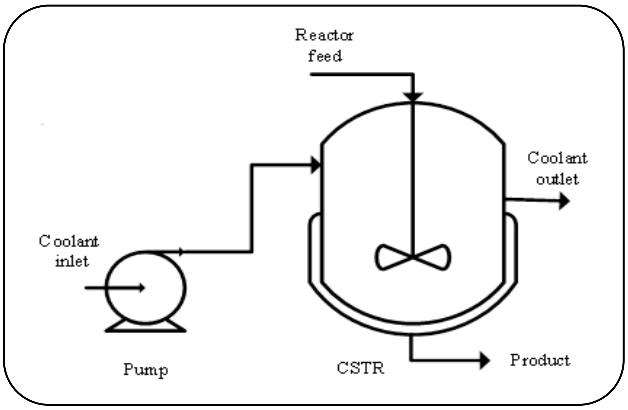

Fig. 1: Schematic of a CSTR.

- Finite-time stability of the proposed controllers made it possible to adjust the convergence time of the controller according to the limitations of the system.
- The validity of the proposed control methods was verified through comparative simulations.

The rest of the paper is organized as follows: First, the nonlinear model of CSTRs is introduced, with some important assumptions provided; next, a finite-time sliding mode controller, a finite-time disturbance observer, and an adaptive sliding mode controller are introduced for CSTRs, with their stability investigated using the Lyapunov theory. In the results and discussion section, simulations are presented to validate the theoretical results. Finally, the paper is concluded.

THEORETICAL SECTION

Problem formulation and preliminaries

The nonlinear non-dimensional model of a CSTR (as shown in Fig. 1) is presented in a state space form as follows [13]:

$$\begin{aligned} \dot{x}_1 &= -x_1 + D_a(1-x_1)e^{x_2/(x_2/\gamma+1)} - d_1 \qquad (1)\\ \dot{x}_2 &= -x_2 + BD_a(1-x_1)e^{x_2/(x_2/\gamma+1)} - \\ &\beta(x_2 - x_{2c_0}) + \beta u_T + d_2 \\ y &= x_2 \end{aligned}$$

Where $x_1, x_2 \in \mathbb{R}$ are the system states representing dimensionless composition and temperature, respectively. Besides, y is the process output and $d_1, d_2 \in \mathbb{R}$ represent the system disturbance.

Tables 1 and 2 show descriptions of other parameters.

Remark 1. In this study, the coolant temperature was selected as the control input, and the CSTR temperature as the output. The main reason for this selection was that it

Table 1: CSTR Parameters and their descriptions [24].

Description	Parameter	Unit
Reactor concentration	c_A	$kmol/m^3$
Inlet concentration	c_{Af}	$kmol/m^3$
Reactor temperature	T	K
Activation energy	E	j/mol
Universal ideal gas constant	R	j/mol.k
First-order reaction rate constant	k_0	min^{-1}
Steady- state flow rate	F	m^3/min
Reagent density	ρ	g/m^3
Specific heat capacity	C_p	cal/°Cg
Reaction heat	ΔH	cal/kmol
Coolant density	ρ_c	g/m^3
Specific heat capacity of coolant	C_{pc}	cal/°Cg
Volume of the CSTR	V	m^3
Coolant flow rate	F_c	m^3/min

Table 2. Dimensionless parameters of the CSTR [24]

Description	Dimensionless parameter
Dimensionless composition	$x_1 = (c_{Af0}-c_A)/c_{Af0}$
Dimensionless temperature	$x_2 = (T-T_{f0})/T_{f0}$
Damkohler number	$D_a = k_0\exp(-\gamma V)/V$
Dimensionless control input	$u = (T_c-T_{c0})/T_{f0}$
Activation energy	$\gamma = E/RT_0$
Dimensionless time	$t = t' (F_0 / V)$
Adiabatic temperature rise	$B= (-\Delta H)c_{Af0} / \rho c_p T_{f0}$
Heat transfer coefficient	$\beta = hA/\rho c_p F_0$
Feed composition disturbance	$d_1 = (c_{Af} - c_{Af0}) / c_{Af0}$
Feed temperature disturbance	$D_2 = (T_f - T_{f0}) / T_{f0}$

was critical to control the temperature; otherwise, a second reaction might happen in the reactor [25].

Remark 2. Due to the nonlinear dynamics of the system (1) and input disturbances, the linear controllers could not control this system efficiently. As a result, nonlinear robust controllers needed to be designed.

Remark 3. In the case of real-time situations, unmodeled dynamics or uncertainties with the input disturbance can form a lumped disturbance which can be estimated and its effect can be removed by the robust controllers [16].

Remark 4. The first equation of the dynamic Eq. (1) was a zero dynamic, so the stability of state 1 (composition) was still important and had to be checked.

Assumption 1. Process states were measurable and fully available in this study.

Assumption 2. The system disturbances and their derivatives were unknown and bounded.

$$\|d_1\|_2 \le \delta_1 \quad \delta_1 \ge 0 \tag{2}$$

$$\|\dot{d}_2\|_2 \le \delta_2 \quad \delta_2 \ge 0$$

Remark 5. According to remark 4 and assumption 2, a state feedback control makes the zero dynamics of the process stable, with the bounded disturbance (d_1) not affecting zero dynamics stability [26].

Control objective. The main objective of this study was to design a controller so that the output would track the reference trajectory in a finite time; accordingly, the error is defined as:

$$e = y - y_{ref} \tag{3}$$

It converges to zero in a finite time.

To better understand the control procedure of a system, Fig. 2 shows the schematic representation of a conventional closed-loop process control system.

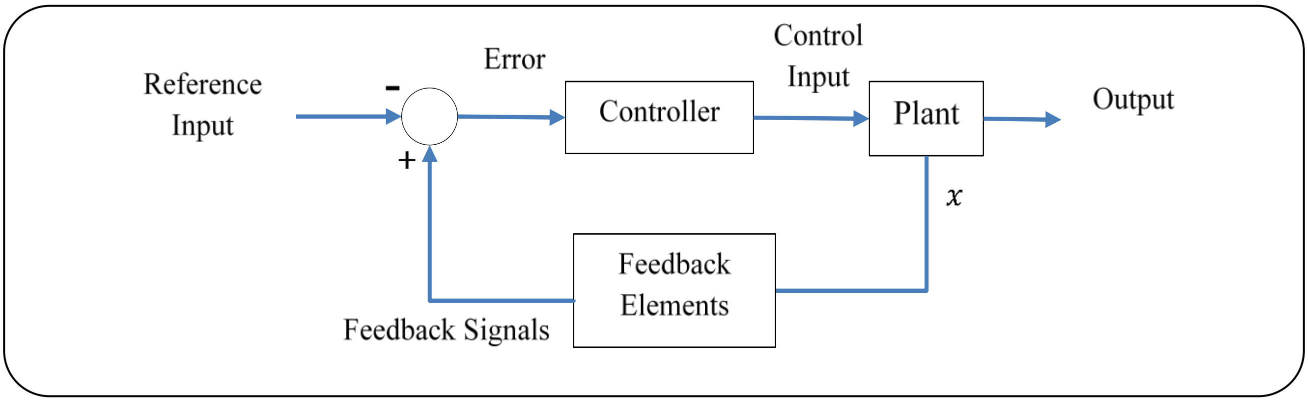

Fig. 2: Closed-loop process control configuration.

Main results

In this section, the design process of a novel sliding mode control and a disturbance observer is proposed for a CSTR to ensure tracking.

Finite-Time Sliding Mode Control (FSMC)

In this subsection, an SMC is proposed for the CSTR, with its stability and finite-time convergence studied.

Lemma 1[27]. For the system:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ &\vdots\\ \dot{x}_n &= u\end{aligned} \tag{4}$$

Next, the following control law is applied:

$$u = -k_1\,\mathrm{sgn}(x_1)|x_1|^{\alpha_1} - k_2\,\mathrm{sgn}(x_2)|x_2|^{\alpha_2} - \ldots - k_n\,\mathrm{sgn}(x_n)|x_n|^{\alpha_n} \tag{5}$$

where $\mathrm{sgn}(.)$ is the sign function defined as $\mathrm{sgn}(c) = \begin{cases}1 & c>0\\ -1 & c<0\end{cases}$ and constants k_i ($i = 1,\ldots,n$) are selected so that $p(s) = s^n + k_n s^{n-1} + \ldots + k_1 = 0$ is Hurwtiz and α_i ($i = 1,\ldots,n$) are selected as follows:

$$\begin{cases}\alpha_{n+1} = 1\\ \alpha_n = \alpha \quad \alpha \in (1-\varepsilon, 1) \quad \varepsilon \in (0,1)\\ \alpha_{i-1} = \dfrac{\alpha_{i+1}\alpha_i}{2\alpha_{i+1} - \alpha_i}\end{cases} \tag{6}$$

System (4) is finite-time stable.

Theorem 1. Considering system (1) as well as assumptions (1) and (2), the tracking error converges to zero in a finite time by designing the following control law:

$$\dot{u}_T = -\frac{1}{\beta}(\dot{F} - \dddot{y}_{ref} + k_1\mathrm{sgn}(e)|e|^{\alpha_1} + k_2\mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2} + k\,\mathrm{sgn}(s)) \tag{7}$$

where

$$F = -x_2 + BD_a(1-x_1)e^{x_2/(x_2/1+\gamma)} - \beta(x_2 - x_{2c_0}) \tag{8}$$

Proof:

The following sliding surface is designed:

$$s = \dot{e} - \dot{e}(t_0) + \int_{t_0}^{t} k_1\mathrm{sgn}(e)|e|^{\alpha_1} + k_2\mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2}\, dt \tag{9}$$

In sliding surface (9), using term $\dot{e}(t_0)$, the reaching phase is eliminated, with s being equal to zero from the beginning. The time derivative of the sliding surface is obtained as follows:

$$\dot{s} = \ddot{e} + k_1\mathrm{sgn}(e)|e|^{\alpha_1} + k_2\mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2} = \dot{F} + \beta\dot{u}_T + \dot{d}_1 - \dddot{y}_{ref} + k_1\mathrm{sgn}(e)|e|^{\alpha_1} + k_2\mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2} \tag{10}$$

The Lyapunov stability theorem is used to verify the stability of the closed-loop system using control input (7) and sliding manifold (9). Thus, a positive definite Lyapunov function is considered as follows:

$$V = \frac{1}{2}s^2 \tag{11}$$

The time derivative of the Lyapunov function is as follows:

$$\dot{V} = \dot{s}s = (\dot{F} + \beta\dot{u}_T + \dot{d}_2 - \dot{y}_{ref} + k_1 \mathrm{sgn}(e)|e|^{\alpha_1} + k_2 \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2})s \tag{12}$$

By substituting control input (7) for (12), it will yield:

$$\dot{V} = \dot{s}s = (-k\,\mathrm{sgn}(s) + \dot{d}_2)s = -k|s| + \dot{d}s \tag{13}$$

According to assumption 2:

$$\dot{V} = -k|s| + \dot{d}_2 s \le -k|s| + \delta_2 |s| \tag{14}$$

and for $k \ge \eta + \delta_2$:

$$\dot{V} \le -\eta|s| \tag{15}$$

This indicates that the time derivative of the Lyapunov function is negative definite, with the proof established. Therefore, the sliding surface remains zero.

$$s = \dot{e} - \dot{e}(t_0) + \int_{t_0}^{t} k_1 \mathrm{sgn}(e)|e|^{\alpha_1} + k_2 \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2}\, dt = 0 \tag{16}$$

Thus:

$$\dot{s} = \ddot{e} + k_1\, \mathrm{sgn}(e)|e|^{\alpha_1} + k_2\, \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2} = 0 \tag{17}$$

According to Lemma 1, the closed-loop system is finite-time stable. It is worth noting that most conventional controllers have a long convergence time. However, the convergence time in the proposed controller was finite and adjustable; therefore, it could be adjusted according to system limitations.

Remark 6. Most conventional controllers have a long convergence time. However, in the proposed controller the convergence time is finite and adjustable therefore it can be adjusted as needed according to system limitations.

Remark 7. Unlike the terminal sliding surface [28], the presented sliding manifold did not suffer from singularity problems, being one of the significant advantages of this finite-time controller.

Remark 8. An integral sliding surface was presented in the proposed control method, which could be simply differentiated from the sliding surface.

Remark 9. In the proposed control method, in addition to the finite-time convergence, the main problem of the sliding mode control method, i.e. chattering, was tackled. In this method, because of the particular form of the sliding surface, a dynamic control input was designed. Thus, by integrating it, a continuous control law was obtained, with the chattering phenomenon eliminated.

In the introduced sliding mode controller, we needed to differentiate the expression

$$F = -x_2 + BD_a(1-x_1)e^{x_2/(x_2/1+\gamma)} - \beta(x_2 - x_{2c_0}) ,$$

which affected the performance of the controller as the derivative action significantly increased the impact of the measurement noise. To solve this problem, the following two-part control law is presented by applying the control law in two steps.

The two-part finite-time sliding mode control law is presented as follows:

$$u_{T1} = -\frac{1}{\beta}(-x_2 + BD_a(1-x_1)e^{x_2/(x_2/1+\gamma)} - \beta(x_2 - x_{2c_0}) - \dot{y}_{ref}) \tag{18}$$

$$\dot{u}_{T2} = -\frac{1}{\beta}(k_1 \mathrm{sgn}(e)|e|^{\alpha_1} + k_2 \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2} + k\,\mathrm{sgn}(s)) \tag{19}$$

$$u_T = u_{T1} + u_{T2} \tag{20}$$

Equations (18) to (20) are actually presenting one equation. The control law is a combination of the two control laws (18) and (19), and none of them is an independent controller. The control law is written in the form of equations (18) to (20), for readers to have a better understanding of the controller design.

Proof:

The proof can be established in Appendix A.

Finite-time disturbance observer

In this subsection, a sliding mode disturbance observer is provided to estimate the upper bound of the disturbance affecting the system. By providing this DO and estimating the upper bound of the disturbance, a more accurate controller could be achieved.

To simplify the design, a new notation is defined as follows:

$$\mathrm{sig}^{\alpha}(x) = |x|^{\alpha}\, \mathrm{sgn}(x) \tag{21}$$

The estimation errors are defined as follows:

$$e_1 = x_2 - \hat{x}_2 \quad , \quad e_2 = d_2 - \hat{d} \tag{22}$$

Where $\hat{x}_2, \hat{d}$ are the estimations of x_2 and d, respectively. Besides, the following sliding surface is designed for designing the disturbance observer:

$$s = \dot{e}_1 + (\beta+1)e_1 + \rho_1\, sig^{\rho_2}(e_1) = 0 \tag{23}$$

Where $\rho_1 > 0$ and $0.5 < \rho_2 < 1$ are constants.

The disturbance observer is designed as follows:

$$\dot{\hat{x}}_2 = -(\beta+1)\hat{x}_2 + BD_a(1-x_1)e^{x_2/(x_2/1+\gamma)} + \tag{24}$$
$$x_{2c_0} + \beta u_T + \hat{d}$$
$$\dot{\hat{d}} = \psi(e_1)\dot{e}_1 + \lambda_1 \operatorname{sgn}(s)$$

Where λ_1 is a positive constant, and $\psi(e_1) = \rho_1\rho_2 |e_1|^{\rho_2 - 1}$ is a nonlinear function.

The error dynamic of the introduced disturbance observer is determined as follows:

$$\dot{e}_1 = -(\beta+1)e_1 + e_2 \tag{25}$$
$$\dot{e}_2 = \dot{d}_2 - \psi(e_1)\dot{e}_1 - \lambda_1 \operatorname{sgn}(s)$$

Theorem 2. considering system (1), assumptions (1) and (2), sliding manifold (23), and disturbance observer dynamic (24), error dynamic (25) is asymptotically finite-time stable, and the finite-time disturbance estimation is obtained.

Proof:

To prove the stability of error dynamic (25), the following positive definite Lyapunov function is proposed:

$$V = \frac{1}{2}s^2 \tag{26}$$

By differentiating Lyapunov function (26) with respect to the time, we have:

$$\dot{V} = s\dot{s} \tag{27}$$
$$= s[-(\beta+1)\dot{e}_1 + \dot{e}_2 + (\beta+1)\dot{e}_1 + \rho_1\rho_2 |e_1|^{\rho_2-1}(\dot{e}_1)]$$
$$= s[\dot{d}_2 - \rho_1\rho_2 |e_1|^{\rho_2-1}\dot{e}_1 - \lambda_1 \operatorname{sgn}(s) + \rho_1\rho_2 |e_1|^{\rho_2-1}\dot{e}_1]$$
$$\le (\delta_2 - \lambda_1)|s|$$

It is negative definite for $\lambda_1 > \delta_2$. Now, the finite-time estimation of the proposed DO is investigated.

Lemma 2 [29]. For the system:

$$\dot{x} = f(x), x \in \mathbb{R}^n \tag{28}$$

It is supposed that there is a positive definite and differentiable Lyapunov function with constants $p > 0, 0 < \eta < 1$ so that:

$$\dot{V}(t) \le -pV^{\eta}(t) \tag{29}$$

Next, the system is finite-time stable, and the convergence time is determined as follows:

$$T = t_0 + \frac{V^{1-\eta}(t_0)}{p(1-\eta)} \tag{30}$$

where t_0 is the initial time.

Considering inequality (27), we will have:

$$\dot{V} \le -(\lambda_1 - \delta_2)V^{1/2} \tag{31}$$

where according to (29), $p = \lambda_1 - \delta_2$. As a result, according to Lemma 2, estimation errors reach the sliding manifold in the finite time.

Lemma 3 [30]. It is supposed that a positive definite Lyapunov function satisfies the following inequality:

$$\dot{V}(t) \le -p_1V^{\eta}(t) - p_2V(t) \quad \forall t \ge t_0 \tag{32}$$

where $0 < \eta < 1$ and $p_{1,2}$ are arbitrary positive constants. Next, the Lyapunov function converges to zero in the following finite time:

$$T = t_0 + \frac{1}{p_2(1-\eta)}\ln\left(\frac{p_2V^{1-\eta}(t_0) + p_1}{p_1}\right) \tag{33}$$

According to (31), it is proved that $s = 0$, so we will have:

$$s = \dot{e}_1 + (\beta+1)e_1 + \rho_1 sig^{\rho_2}(e_1) = 0 \tag{34}$$

To check the stability of (34), the following Lyapunov function is considered:

$$V = \frac{1}{2}e_1^2 \tag{35}$$

The derivative of the Lyapunov function (35) is:

$$\dot{V} = \dot{e}_1e_1 = (-(\beta+1)e_1 - \rho_1 sig^{\rho_2}(e_1))e_1 = \tag{36}$$
$$-(\beta+1)e_1^2 - \rho_1 sig^{\rho_2+1}(e_1) = -2(\beta+1)V - 2^{(\rho_2+1)/2}\rho_1V^{(\rho_2+1)/2}$$

Due to Lemma 3, the errors converge to zero in the following finite time:

$$T_2 = T_1 + \tag{37}$$
$$\frac{1}{(\beta+1)(2-(\rho_1+1))}\ln\left(\frac{2(\beta+1)V^{1-(\rho_1+1)/2}(T_1) + 2^{(\rho_2+1)/2}\rho_1}{2^{(\rho_2+1)/2}\rho_1}\right)$$

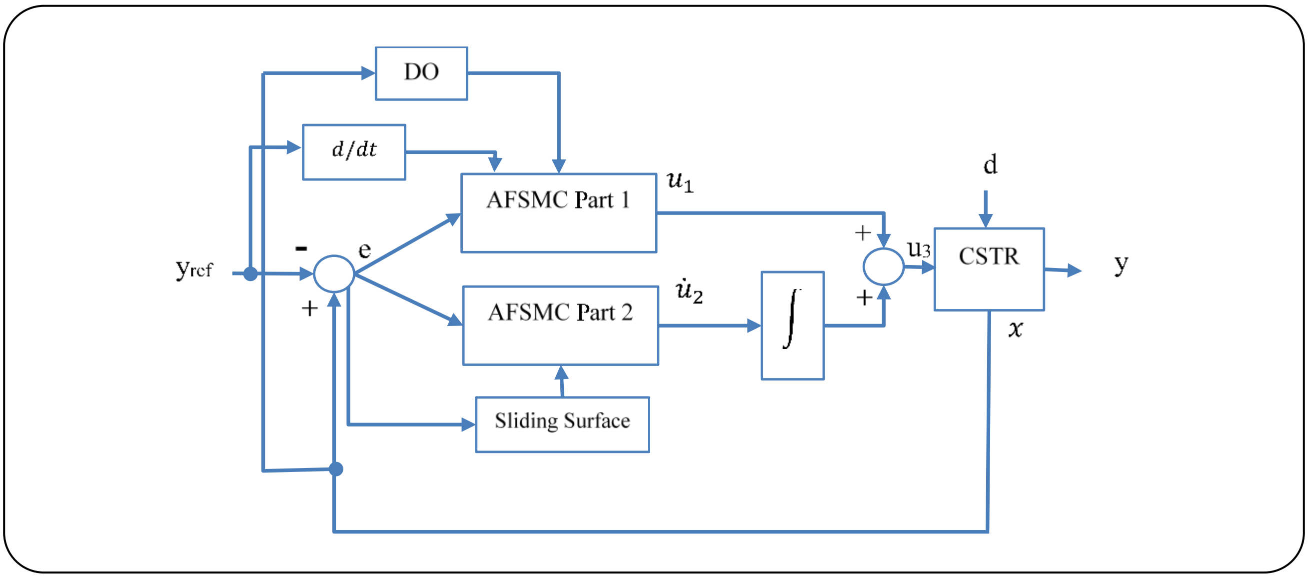

Fig. 3. Block diagram of the AFSMC for the CSTR

Accordingly, the finite-time estimation of the proposed disturbance observer is proved.

Adaptive Finite-Time Sliding Mode Control(AFSMC) based on disturbance observer

In this subsection, an adaptive control law, based on the proposed disturbance observer, is proposed, with its performance investigated. Fig.3 shows the block diagram of the proposed adaptive controller.

Theorem 3. This theorem considers system (1), sliding surface (9), and finite-time disturbance observer (24) by applying the following adaptive control laws:

$$u_{T1} = -\frac{1}{\beta}(-x_2 + BD_a(1-x_1)e^{x_2/(x_2/1+\gamma)} - \beta(x_2 - x_{2c_0}) - \dot{y}_{ref} + \hat{d}) \tag{38}$$

$$\dot{u}_{T2} = -\frac{1}{\beta}(k_1 \mathrm{sgn}(e)|e|^{\alpha_1} + k_2 \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2} + k\,\mathrm{sgn}(s)) \tag{39}$$

For positive k, the output can asymptotically track the reference in a finite time.

Proof:

The proof of theorem 3 can be found in Appendix B.

Remark 10. Similar to the finite-time sliding mode control presented in theorem 1, the adaptive finite-time sliding mode control does not suffer from an undesirable chattering phenomenon due to the dynamic form of the control law.

RESULTS AND DISCUSSION

Simulation results

In this section, the efficiency of the designed controllers is investigated by simulations and comparisons with the Sign Integral Terminal Sliding Mode Control (SITSMC) [13], PID control, and Terminal Sliding Mode Control (TSMC) [28]. A continuous stirred temperature reactor with parameters $B=8$, $\beta=0.3$, $\gamma=20$, $D_a=0.078$, and $x_{2c}=0$ is considered in this section. Besides, to achieve the best performance of the controllers and realistic comparison the parameters of all controllers are tuned by the genetic optimization algorithm. The genetic algorithm as one of the most powerful evolutionary optimization methods can ensure the optimal performance of controllers in control of this system. In addition, to facilitate the controller design for the CSTR, a flow chart is provided for the design and optimization of the proposed controller in Fig. 4.

The simulation results are presented in three sections as follows:

I. The proposed Finite-Time Sliding Mode Control (FSMC) method is implemented on the CSTR, with its performance investigated and compared to that of the SITSMC [13], PID control, and TSMC [28].

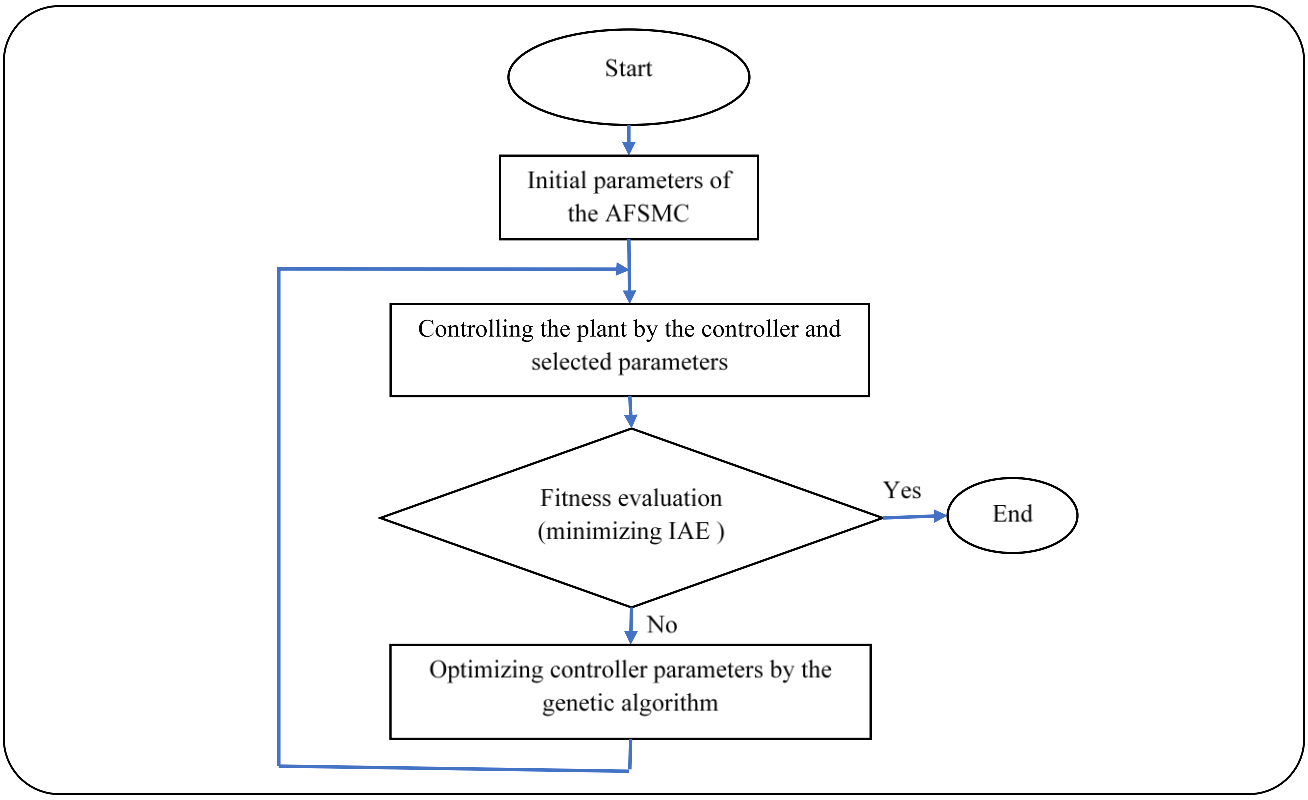

Fig. 4: Flow chart for the controller design and optimization.

II. The performance and effectiveness of the introduced disturbance observer are investigated *via* simulations.

III. The performance of the designed adaptive controller (AFSMC) is evaluated in the presence of the disturbance, with its performance compared to that of the FSMC, PID control, and TSMC.

The simulation was provided by Matlab/Simulink, and the Simulink model is illustrated in Fig. 5.

In Fig. 6, the simulation results of the CSTR system are presented under finite-time sliding mode control.

Fig. 6 shows the performance of the finite-time sliding mode control, with the results compared to those of the SITSMC, PID, and TSMC. Accordingly, x_1 is stable under the finite-time sliding mode control, PID, TSMC, and SITSMC, which converges to a small value. On the other hand, x_2 converges to the setpoint under all controllers. The convergence rate of the proposed FSMC is higher than that of the SITSMC and PID controllers and almost equal to TSMC so that the settling time for the proposed controller is 1.73s, while for controllers SITSMC, PID, and TSMC, it amounts to 4.23, 1.9, and 1.7s, respectively. Results from comparing the controllers' performance (the IAE criterion) show that the proposed controller with an error of 0.509 has a very accurate performance (Table 3 shows a detailed comparison of the controllers). According to the control input signal, it becomes clear that the proposed control method has a relatively smooth signal, while the chattering phenomenon is seen in the SITSMC case, which is very undesirable.

In the following part, the performance of the disturbance observer is investigated. Accordingly, three different signals are applied to the CSTR as disturbances, with the performance of the disturbance observer in estimating the signals evaluated. The signals affecting the system are as follows:

$$d_2a = u(t-2) - u(t-4) \tag{40}$$

$$d_2b = \sin(\pi t)\left[u(t-5) - u(t-7)\right]$$

$$d_2c = (t-8)\left[u(t-8) - u(t-9)\right]$$

Where $u(t)$ is the step signal. Fig. 7 presents the simulation results of the finite-time disturbance observer performance in estimating disturbances (40).

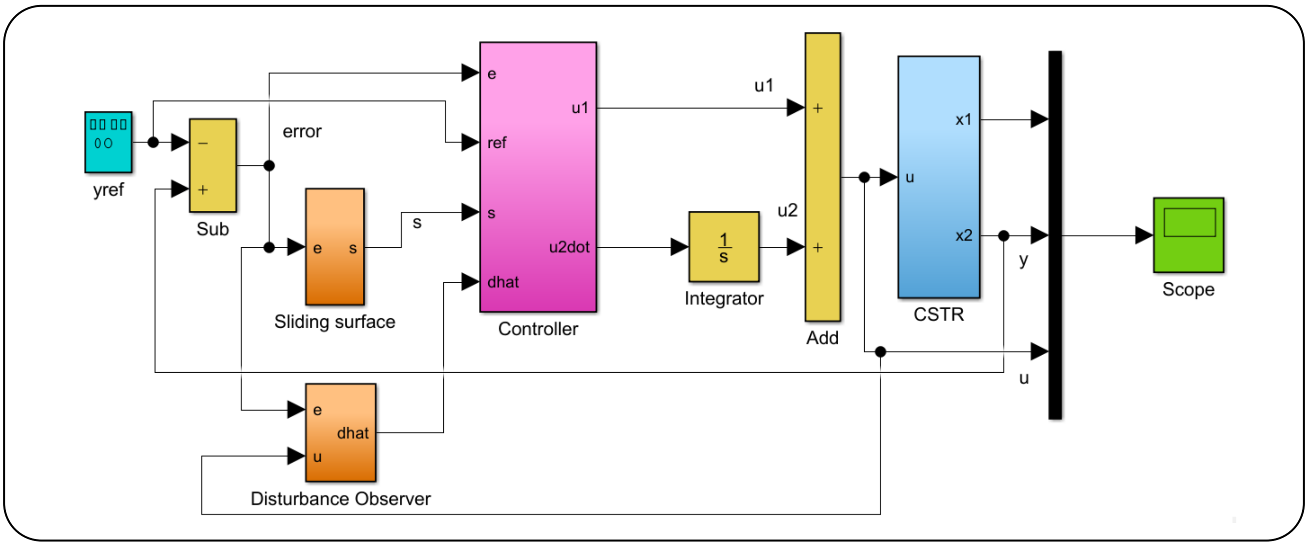

Fig. 5. The Simulink model of a CSTR under AFSMC control.

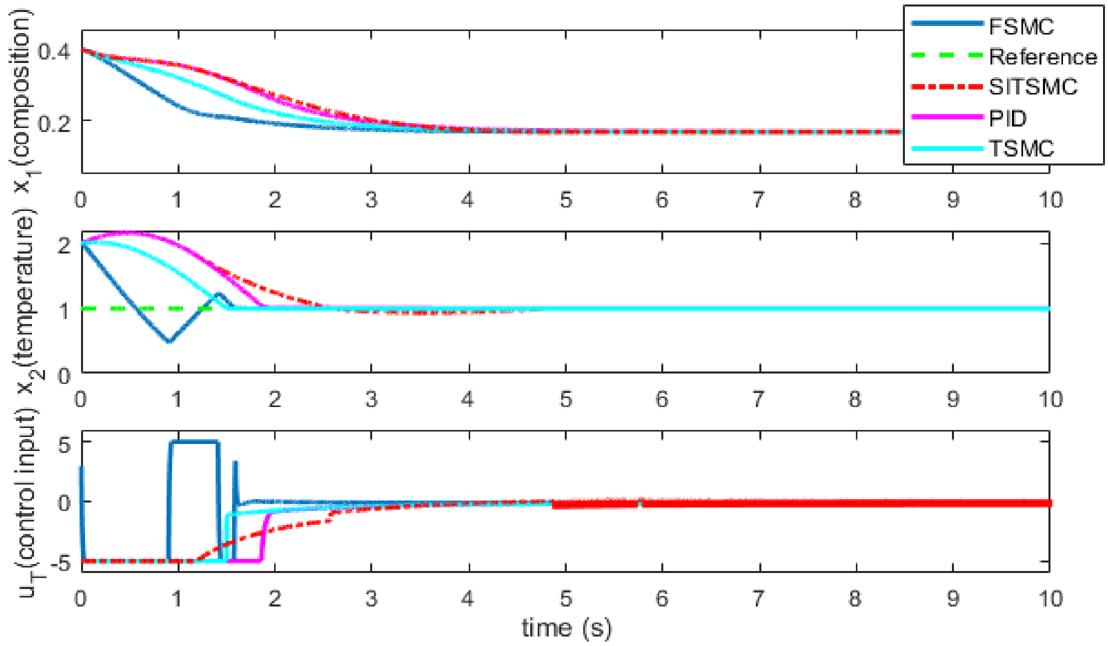

Fig. 6: Simulations results of the CSTR under finite-time sliding mode control compared to the SITSMC, PID, and TSMC.

As Fig. 7 shows, the disturbance observer is able to accurately estimate the disturbances. In addition, it verifies the effectiveness of the disturbance observer.

In the following section, the performance of the proposed adaptive finite-time sliding mode control is investigated. To this end, it is assumed that the following disturbance signal affects the system:

$$d_1 = u(t-1) - u(t-2) \quad (41)$$

$$d_2 = u(t-5) - u(t-7)$$

Simulation results of the proposed control (AFSMC) are compared to those of the finite-time sliding mode control, PID, and TSMC.

As Fig. 8 shows, as far as the disturbance is not applied to the system, the results under two controllers AFSMC and FSMC are relatively similar. However, as the zoomed part of the figure shows, the adaptive control method has a better performance in eliminating the disturbance effect. As one can see, under the finite-time sliding mode control, the output cannot track the reference signal well, while the

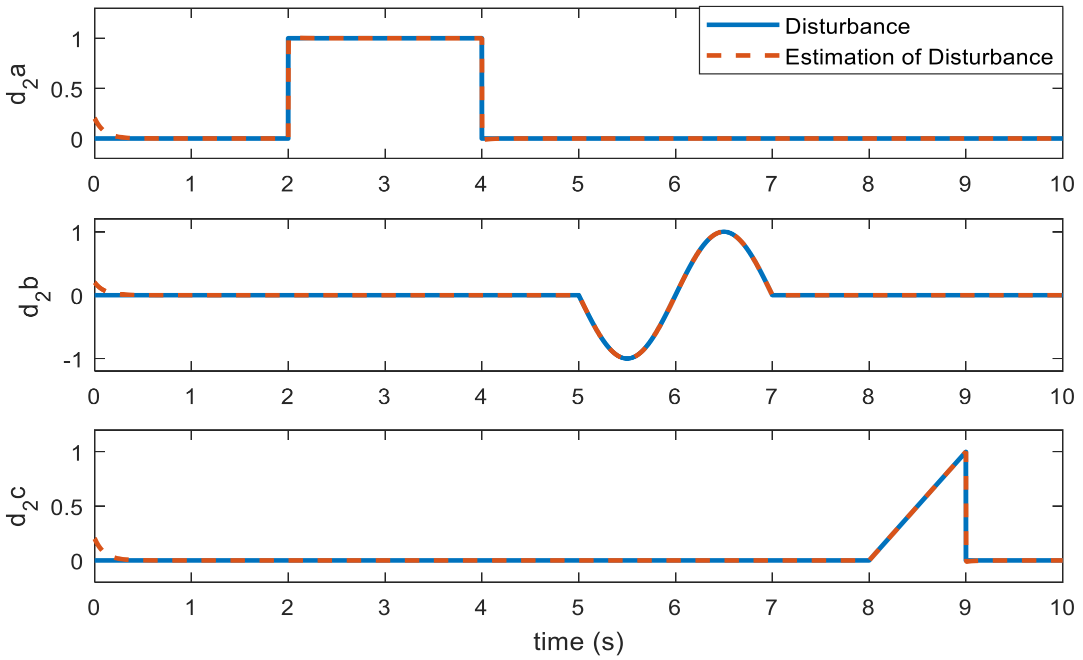

Fig. 7: Simulation results of the proposed adaptive disturbance observer.

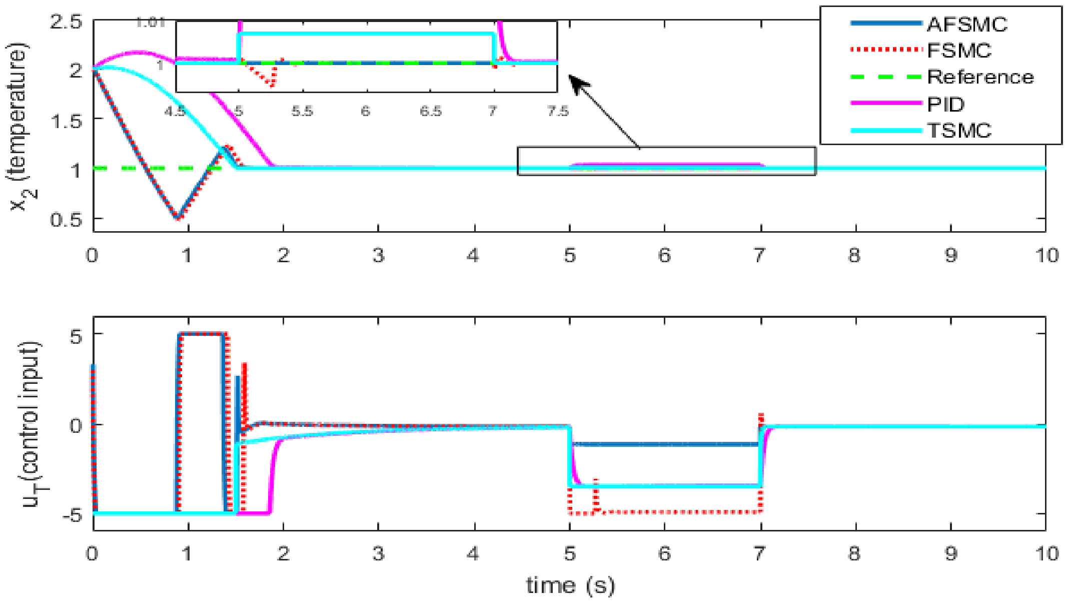

Fig. 8: Simulation results of the proposed adaptive finite-time sliding mode control compared to the FSMC, PID, and TSMC.

same system under the Adaptive Finite-Time Sliding Mode Control (AFSMC) eliminates the effect of the disturbance very well. Besides, it is quite clear that the proposed AFSMC is more efficient than the PID control and TSMC as the ITAE criterion is 0.249 for the AFSMC, but it is 1.58 and 0.65, for the PID and TSMC, respectively.

The numerical results are summarized in Table 3 to investigate the effectiveness of the proposed controllers.

In Table 3, the proposed control methods are compared in terms of speed, accuracy, and control efforts. Accordingly, the performance of the FSMC is faster in comparison with other controllers and almost equal to TSMC. But in case of accuracy, the proposed FSMC produces smaller IAE and ITAE values than other controllers which indicates the higher accuracy of this controller. On the other hand, in the presence of the

Table 3: Comparison of the performance of the proposed controllers.

Controller	Settling Time (sec)	IAE	ITAE	Control Effort
FSMC	1.73	0.509	0.265	8.9
SITSMC	4.23	1.83	1.81	12.24
PID	1.9	1.58	1.18	11.47
TSMC	1.7	1.05	0.758	9.87
AFSMC in the presence of the disturbance	1.71	0.4843	0.249	10.56
FSMC in the presence of the disturbance	1.73	0.5098	0.281	18.45
PID in the presence of the disturbance	1.98	1.648	1.58	18.08
TSMC in the presence of the disturbance	1.68	0.538	0.65	17.58

disturbance, the AFSMC follows the reference signal faster, and by properly removing the disturbance effect, a more accurate result will be achieved. As it can be seen, by comparing the control effort we can see that the proposed controllers show effective performance with less control effort in comparison with SITSMC, PID and TSMC.

Remark 11. With changes in behavior of the physical application or CSTR case, two things may happen with changes in system model, a new controller based on the new model can be designed. But it is much more likely that the system parameters will change, If the new parameters are known, the proposed control law can be modified. Otherwise these parametric uncertainties and external disturbances (lumped disturbance) can be controlled by the proposed robust controller and the effect of these uncertainties can be removed.

CONCLUSION

In this paper, a novel finite-time sliding mode controller was introduced for a continuous stirred tank reactor. Besides, a modified version of this method was presented as the two-part sliding mode control to reduce the effect of the measurement noise. Next, to accurately estimate the disturbance affecting the system, a finite-time disturbance observer was introduced. Besides, by integrating the controller and the observer, an adaptive sliding mode controller was developed. In this paper, the main drawbacks of the sliding mode control method (the infinite convergence time and chattering) were addressed. Besides, the genetic algorithm was used to optimally tune the parameters of the controllers. Furthermore, a flowchart was provided to help design and tune the parameters. Accordingly, the proposed methods were studied by numerical simulations and compared with the SITSMC, PID, and TSMC controllers. The simulation results verified the effectiveness of the proposed controllers so that the FSMC and AFSMC with the settling times of 1.73s and 1.71s and the IAE of 0.509 and 0.4843, respectively, showed their effectiveness in controlling the CSTR. In the case of future studies, designing data-based controllers, especially model-free sliding mode controllers can be suggested. Furthermore, the estimation of the nonlinear parts of the system model via machine learning algorithms and controlling systems by intelligent controllers could be among other interesting research subjects.

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Appendix A.

To investigate the stability and tracking of the proposed two-part sliding mode control, the same sliding surface (9) is considered.

One of the major features of this sliding surface is that the control input appears on the sliding surface itself, while in normal sliding surfaces, the control law appears by differentiating the sliding surface. This feature helps eliminate additional expressions before differentiating the sliding surface, thereby helping formulate a more appropriate control law.

By substituting $\dot{x}_2$ in the sliding surface (9), we will have:

$$s = F + \beta u_T + d_2 - \dot{e}(t_0) + \int_{t_0}^{t} k_1 \mathrm{sgn}(e)|e|^{\alpha_1} + k_2 \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2}\, dt \tag{42}$$

By applying (18), additional expressions are removed, and the following sliding surface is obtained:

$$s = \beta u_{T2} - \dot{y}_{ref} + d_2 - \dot{e}(t_0) + \int_{t_0}^{t} k_1 \mathrm{sgn}(e)|e|^{\alpha_1} + k_2 \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2}\, dt \quad (43)$$

Thus, the first-order time derivative of the sliding surface is obtained as follows:

$$\dot{s} = \beta \dot{u}_{T2} - \ddot{y}_{ref} + \dot{d}_2 + k_1 \mathrm{sgn}(e)|e|^{\alpha_1} + k_2 \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2} \quad (44)$$

To prove the stability of the controller, the following positive definite Lyapunov function is proposed:

$$V = \frac{1}{2}s^2 \quad (45)$$

By differentiating the Lyapunov function (45), we will have:

$$\dot{V} = \dot{s}s = (\beta \dot{u}_T + \dot{d}_2 + k_1 \mathrm{sgn}(e)|e|^{\alpha_1} + k_2 \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2})s \quad (46)$$

By applying (19), we will have:

$$\dot{V} = \dot{s}s = (-k\,\mathrm{sgn}(s) + \dot{d}_2)s \quad (47)$$

According to assumption 2:

$$\dot{V} = (-k\,\mathrm{sgn}(s) + \dot{d}_2)s = -k|s| + \dot{d}_2 s \le -k|s| + \delta_2|s| \quad (48)$$

and for $k \ge \eta + \delta_2$

$$\dot{V} \le -\eta|s| \quad (49)$$

Thus, the time derivative of the Lyapunov function is negative definite, and the proof is established.

Appendix B.

Considering sliding surface (9) and upon applying the control input (38), we will have;

$$s = \beta u_T + d_2 - \hat{d} - \dot{e}(t_0) + \int_{t_0}^{t} k_1 \mathrm{sgn}(e)|e|^{\alpha_1} + k_2 \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2}\, dt \quad (50)$$

Accordingly, additional expressions are removed, and the disturbance estimation ($\hat{d}$) is used to remove the effect of the disturbance. According to the proposed disturbance observer in the previous subsection with $\hat{d} \to d$, the sliding manifold is obtained as follows:

$$s = \beta u_T - \dot{e}(t_0) + \int_{t_0}^{t} k_1 \mathrm{sgn}(e)|e|^{\alpha_1} + k_2 \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2}\, dt \quad (51)$$

By removing the disturbance, the main problem in stabilizing the system is solved. The derivative of the sliding surface is obtained as follows:

$$\dot{s} = \beta \dot{u}_T + k_1 \mathrm{sgn}(e)|e|^{\alpha_1} + k_2 \mathrm{sgn}(\dot{e})|\dot{e}|^{\alpha_2} \quad (52)$$

Given the following positive definite Lyapunov function:

$$V = \frac{1}{2}s^2 \quad (53)$$

By differentiating the Lyapunov function (53) and applying the control input (39), we will have:

$$\dot{V} = (-k\,\mathrm{sgn}(s))s \quad (54)$$

This is negative definite for positive amounts of k, so the proof is established.

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