The Modelling of the Urea Fertilizer Dissolution Process in Finite/Infinite Volumes of Water

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ABSTRACT: *This research aims to provide a model to investigate the impact of some parameters such as impeller speed, temperature, and solid concentration on mass transfer coefficient and the dissolution rate of urea fertilizer in the water. To study the effect of solid concentration two models are presented for finite and infinite-volume fluids using mass balance. Then the urea-water mass transfer coefficient was calculated at various impeller speeds and temperatures by measuring the time to complete dissolution. To investigate the effect of impeller speed and turbulency on the mass transfer coefficient, the impeller speed and Reynolds number were set in a range of 10-50 rpm and 300-3000, respectively. The Schmidt number also was used to study the effect of temperature on the mass transfer coefficient in the range of 5-25 °C. The results show that in both finite and infinite fluid volumes, at a constant impeller speed with decreasing Schmidt number, and at a constant temperature with increasing Reynolds number, the mass transfer coefficient, and mass transfer rate increase. Furthermore, four models are presented for mass transfer coefficient in finite and infinite volume, which that shows the mass transfer coefficient and release rate in finite volume were lower than that of infinite volume at a constant impeller speed and temperature.*

KEYWORDS: *Mass transfer coefficient; Urea; Modelling; Time to complete dissolution; Finite; Infinite.*

INTRODUCTION

In agriculture, fertilizers containing inorganic nitrogen (nitrate (NO_3^-), ammonium (NH_4^+), and amide (NH_2)) are excessively consumed, especially urea fertilizer, to supply nutrients for plants even if they are in a dormant state [1]. Urea is a commonly used fertilizer due to its high nitrogen content (47%), low cost, and economical availability, it can be easily lost, owing to leaching, volatilization, and denitrification because of its high solubility in water. So the nitrate concentration in water has increased [2-5]. Humans use this water, which is converted into carcinogenic compounds in the body [6-9]. The primary problem in using urea is its rapid loss in fields, owing to leaching, volatilization, and denitrification, driven by its high solubility in water [10]. This problem is addressed

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by controlled-release urea, which is effective in regulating high dissolution. Therefore, models and the effects of some important parameters including coating thickness in controlled-release fertilizer, nutrients' release rate, contact area, granule radius, pH, and temperature on the release time of nutrients were studied [11-17]. In this way, some models for the dissolution of urea particles in a diffusion-controlled environment were obtained and the effect of the solubility number on Time to Complete Dissolution (TCD) was investigated [18-20]. When predicting the model, the TCD of fertilizer is an important parameter since this is the time required to dissolve all of the solid nutrients in the granule. Hydrodynamic conditions including rainfall intensity, wind and water speed during irrigation, system temperature, and concentration are important parameters that must be examined in the modeling of nitrate release from coated and uncoated urea granules [18, 21, 22]. Most researchers have explored release conditions in laminar flow [20, 23, 24]. However, in practical farmland, laminar flow is not applied through irrigations, and turbulent flow is formed by conditions such as wind, surface waves, and eddies, which are considered to be the Reynolds number [25]. Furthermore, environmental concentration and temperature around the granules change whether the volume is finite, infinite, or weather conditions are different.

By changing hydrodynamic conditions, such as rainfall intensity, wind and water speed during irrigation, and parameters affecting hydrodynamic conditions such as solid concentration and environmental temperature, the external resistance, mass transfer coefficient of urea, and the nitrogen solution rate subsequently changes. Therefore, it is necessary to calculate the mass transfer coefficient for different hydrodynamic conditions and evaluate its effect on the mass transfer coefficient [26, 27]. Numerous studies have reported the effect of solid concentration and impeller speed on the solid-liquid mass transfer coefficient in different systems [28-31].

Researchers have sought to calculate the mass transfer coefficient in solid-liquid systems with various methods [32-35]. For instance, the mass transfer coefficient for a mixture of sugar, citric acid, and color additives in water has been calculated using the diagram's slope of the solid remaining [32]. Another study investigated the dissolution process or mass transfer rate from urea granules and determined mass transfer coefficients using

the overall mass balance for urea in the sphere at different Rayleigh numbers [24]. However, such research has a limitation because the mass transfer coefficient was studied in an infinite volume of quiescent water. *Bong* (2013) determined the mass transfer coefficient using an electrical conductivity meter in aqueous NaOH solution and cationic ion exchange resins as the liquid and solid phases, respectively, in the solid concentration range of 0.005-0.30 v/v at various impeller speeds and a constant temperature [36]. *Bilbao et al* [37] investigated the effect of metal concentration and temperature on mass transfer for an ion exchange system. Paul and Bhattacharjee [38] also studied the effect of parameters affecting hydrodynamic conditions such as temperature and pressure on mass transfer in extracting 1,8-cineole from small cardamom seeds by supercritical carbon dioxide. According to the above discussion, most researchers have ignored the effect of some parameters affecting hydrodynamic conditions such as temperature and high concentration on the mass transfer coefficient and mass transfer rate of urea fertilizer granules. Therefore, in this study, the dissolution process of pure urea fertilizer was modeled based on a mass balance between solid-liquid phases, and the effect of solid concentration in finite and infinite volumes of water, impeller speed, and system temperature on release rate were investigated. The mass balance between the two phases, which is based on the concentration gradient in the liquid phase, is used in presenting both models. Additionally, in two boundary concentrations finite and infinite, the effect of water velocity during irrigation or rainfall and the effect of temperature on mass transfer coefficient were examined by Reynolds and Schmidt numbers, respectively. Finally, mathematical correlations were developed using the experimental data for estimating the urea-water mass transfer coefficient in both systems.

EXPERIMENTAL SECTION

In this experiment, a vessel with 12 cm diameter and 15 cm height, a vacuum pump (Rocker 300, Rocker Scientific Co., Ltd), Whatman filter paper (Cat. No. 1440-125), and a jar test (JTR90, Zag Chemie Co.) adjustable for different times and revolutions were used. The diameter of the flat blade of the agitator was 60 mm. The spherical utilized particles were urea chemical fertilizer granules (Kermanshah Razi Petrochemical Co., Kermanshah, Iran) with a mean diameter of 4 mm. Distilled water was used

as the liquid. The liquid temperature was adjusted using a thermal coil and a circulator in the range of 5-25 °C. In agricultural land under various weather conditions, the saturation concentration, which is a function of temperature, was calculated from the literature [39]. The amount of diffusion coefficient and viscosity of water in different temperatures can be found using the Stokes-Einstein equation, $D_{T_1}/D_{T_2} = T_{K_1}/T_{K_2} \cdot \mu T_{K_1}/\mu T_{K_2}$ where D is the diffusion coefficient, T_{K1} and T_{K2} are the absolute temperatures, μ is the dynamic viscosity of the water, and $\mu_{T_K} = 2.414 \times 10^{-5} \times 10^{247.8/(T_K-140)}$ (where T_K has units of Kelvin), respectively. K_{LS} is also an unknown parameter that was obtained from the present model and experimental data of the time to complete dissolution at various temperatures and impeller speeds. The granules were placed in the beaker, and the agitator was started. After several seconds of dissolution, the agitator was stopped. Next, granules were removed from the beaker with a vacuum pump and filter paper on a Buchner funnel and weighed after drying in air. This process was repeated three times at various temperatures and impeller speeds. To reach a turbulent flow, different speeds were considered according to the Reynolds number, $(Re_H = VR_H\vartheta^{-1})$, which when greater than 500 has been reported as turbulent flow [25], where V is mean velocity, R_H is the hydraulic depth, and ϑ is the kinematic viscosity of water. To introduce an irrigation stream by a Reynolds number of 500, a flow with 5 cm depth and 0.5 cm/s speed are required. However, in a field, a typical speed is more than 0.5 [cm/s], and environmental conditions would also accelerate the speed of the flow. Hence, to set the experiment close to the real state, speeds of 10, 20, 30, 40, and 50 rpm, were applied to have Re >500. This experiment was conducted for both the finite and the infinite volume in boundary concentrations 0.106 *v/v* and 0.009 v/v, respectively. At finite volume, changes in environmental concentration were more than negligible. It was assumed that the concentration of the released environment in finite volume is changeable and in infinite volume, the concentration in the environment is constant through time. Table 1 provides the physical properties of water and urea.

In this study, a finite system was considered when a large number of fertilizer granules are accumulated at one point in the field and an infinite system was considered

Table 1: The physical properties at 25 °C.

Parameter	Value
Diffusivity coefficient (D), [m²/s] [20]	1.15×10^{-9}
Urea density (ρ_s), [kg / m³]	1.32×10^3
Concentration saturated solution (C_{Sat}), [kg /m³]	1.0×10^3
Liquid density (ρ), [kg / m³]	0.99×10^3
Viscosity of liquid, (μ), [kg /m.s]	0.89×10^{-3}

when the granules are dispersed sparsely. Urea granules are also assumed to be spherical and studies have been performed in water instead of soil. These issues can be the limitations and simplifications of this study

THEORETICAL SECTION

Mathematical modeling for both finite and infinite volume systems was determined based on the number of dissolved urea granules in a specific volume of water.

Modeling the finite volume fluids

In the finite case, concentration changes over time are not inevitable because in this state the number of urea granules that dissolved in a specific volume of water is more than in the infinite state. So, it has been assumed that volume was finite and the model was presented assuming a change in the concentration of the medium over time and mass balance between the two phases.

The mass transfer rate between mixed granular urea and water is described by Eq. (1). The solid was assumed to dissolve uniformly.

$$\dot{m} = K_{SL}A\left(C_{sat} - C_L\right) \tag{1}$$

Where

K_{SL} is the mass transfer coefficient of the liquid phase, *m/s*

A is the surface area of the urea granules, m²

C_{sat} is solute concentration in the saturated phase, kg/m³

C_L is solute concentration in a liquid phase, kg/m³.

The dissolved solid mass over time is expressed as:

$$\frac{dM}{dt} = -\dot{m} = -K_{SL}A\left(C_{sat} - C_L\right) \tag{2}$$

The corresponding mass transfer balance in the liquid is:

$$V_L \frac{dC_L}{dt} = \dot{m} = K_{SL} A \left(C_{sat} - C_L \right) \tag{3}$$

Where

V_L is liquid volume, m^3.

$$V_L \frac{dC_L}{dt} = \dot{m} = K_{SL} A \left(C_{sat} - C_L \right) \tag{4}$$

Where

M_0 is a residual mass of a solid at the initial time, [kg].

C_{L0} is a concentration of solute in the liquid phase at the initial time, kg/m3.

Eq. (4) can be solved simultaneously with Eq. (2). It is assumed that mass transfer on the surface of particles is uniform; therefore, the particles' form remains spherical until the end of the test. If the radius of the spherical granule is r, the number of granules is n, and ρ_s is solid density kg/m3, the total mass of solids is obtained as:

$$M = \frac{4}{3} \pi r^3 \rho_s n \tag{5}$$

The surface area of solid particles is:

$$A = 4 \pi r^2 n \tag{6}$$

By substituting Eqs. (4, 5), and 6 into Eq. (2) for the remaining mass of the solid phase at any time:

$$\frac{dM}{dt} = -k_{LS} \left[\frac{36 \pi M^2 n}{\rho_s^2} \right]^{\frac{1}{3}} \left[C_{sat} - \left(C_{L0} + \frac{M0 - M}{V_L} \right) \right] \tag{7}$$

This equation was solved analytically.

$$t = \frac{1}{2} \frac{\left(\left(M + \left(\frac{a}{b} \right) \right) + 3 \left(\frac{a}{b} \right)^{\frac{1}{3}} M^{\frac{2}{3}} + 3 \left(\frac{a}{b} \right)^{\frac{2}{3}} M^{\frac{1}{3}} \right)}{b \left(\frac{a}{b} \right)^{\frac{2}{3}}} - \tag{8}$$

$$\frac{1}{2} \frac{\ln \left(M + \frac{a}{b} \right)}{b \left(\frac{a}{b} \right)^{\frac{2}{3}}} + \frac{\sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} \left(\frac{2 M^{\frac{1}{3}}}{\left(\frac{a}{b} \right)^{\frac{1}{3}}} - 1 \right) \right)}{b \left(\frac{a}{b} \right)^{\frac{2}{3}}} + c$$

Where a, and b are:

$$a = -k_{SL} \left[\frac{36 \pi n}{\rho_s^2} \right]^{\frac{1}{3}} \left(C_{Sat} - C_{L0} - \frac{M_0}{V_L} \right) = \tag{9}$$

$$- \frac{3 M_0^{\frac{1}{3}}}{r_0 \cdot \rho_s} \left(C_{Sat} - C_{L0} - \frac{M_0}{V_L} \right)$$

$$b = -\frac{k_{SL}}{V_L} \left[\frac{36 \pi n}{\rho_s^2} \right]^{\frac{1}{3}} = -\frac{3 M_0^{\frac{1}{3}} k_{LS}}{r_0 \cdot \rho_s \cdot V_L} \tag{10}$$

C is also an integral constant, and the initial concentration of solute in the liquid phase is zero ($C_{L0}=0$). Therefore, $\frac{a}{b} = V_L \left(C_{Sat} - \frac{M_0}{V_L} \right)$

When $M \rightarrow 0$, the Time for Complete Dissolution (TCD) was computed as:

$$t_f = -\frac{\pi \sqrt{3}}{6} \left(\frac{1}{b \cdot \left(\frac{a}{b} \right)^2} \right) \tag{11}$$

Modeling the infinite volume of fluids

In the infinite volume research, the number of dissolved granules in a determined volume of water is less than in the finite volume case, and the amount of the dissolved solute in the solvent has no significant impact on the concentration of the solvent. Therefore, the environment is infinite in the case of solute. Consequently, the case was assumed infinite, and the concentration of the solute was considered equal to its initial concentration.

By assuming no changes in water concentration, and considering Eq. (7), it can be supposed that the concentration of the liquid phase is always much lower than the saturation concentration ($C_L \ll C_{Sat}$). Thus, the liquid phase concentration is negligible. Therefore, Eq. (7) could be rewritten as:

$$\frac{dM}{dt} = -k_{LS} \left[\frac{36 \pi M^2}{\rho_s^2} \right]^{\frac{1}{3}} C_{Sat} \tag{12}$$

After the integration of this relationship, Eq. (12) is obtained, which suits the infinite condition. This equation demonstrates the relationship between time and the ratio of the remained mass of the solid phase to the initial mass.

$$\frac{M}{M_0} = \left[1 - \left(\frac{4\pi n}{3 M_0 \rho_s^2} \right)^{\frac{1}{3}} K_{LS} C_{Sat} t \right]^3 \tag{13}$$

The TCD is calculated from:

$$t_f = \frac{r_0 \cdot \rho_s}{K_{SL} C_{Sat}} \tag{14}$$

RESULTS AND DISCUSSION

Sherwood number models in finite and infinite volume case

Eqs 11 and 14, respectively, show that in finite and infinite volumes, the mass transfer coefficient (k_{SL}) was calculated using TCD dissolution of urea granules in water and at various impeller speeds and temperatures. The Schmidt and Reynolds numbers were also calculated from Eqs. (15) [40].

$$Sc = \frac{\mu \rho}{D} \qquad Re = \frac{\rho_{solution} N T^2}{\eta_{solution}} \tag{15}$$

where $\rho_{solution}$ is the solution density kg/m³, the correlation is shown in Eq. (16), T is the impeller diameter m, N is impeller speed, X is the mass fraction of the solid phase for which the correlation is shown in Eq (17), and ρ_S and ρ_L are the densities of solid and liquid phases, respectively.

$$\rho_{solution} = \frac{1}{\dfrac{X}{\rho_S} + \dfrac{1-X}{\rho_L}} \tag{16}$$

$$X = \left(\frac{\rho_S}{\rho_L} \right)\left(\frac{C_v}{1 - C_v} \right) \tag{17}$$

$\eta_{solution}$ is the apparent solution viscosity [*Pa.s*], for which the correlation is shown in Eq. 18.

$$\eta_{solution} = \eta_r \left(1 + \frac{1.25 C_v}{\varphi_m - C_v} \right) \tag{18}$$

Where η_r is the viscosity of the water and ϕ_m is the maximum volume fraction to which the particle can be packed. The amount of ϕ_m is 0.63 [36, 40].

Finally, Eqs. (19) and (20) were presented using curve fitting (Matlab software and Excel Solver) for Sherwood numbers in finite and infinite volume cases for $N{\geq}30$ [*rpm*]

and $N{<}30$ [*rpm*], respectively. It can be observed that the Reynolds and Schmidt number power in the infinite model (Eq. (20)) at high speed ($N \geq 30$) is greater than in the finite case (Eq. (19)), which indicates the greater effect of the Reynolds and Schmidt number on the mass transfer coefficient or Sherwood number. So, the mass transfer coefficient or Sherwood number in finite volume is lower than the infinite volume case at constant impeller speed and temperature. This is in agreement with the other investigators [36, 41]; in constant impeller speed temperature and high solid concentration, the mass transfer coefficient decreases with increasing solid concentration, and this coefficient is lower than infinite volume, which has low solids concentration.

$$Sh = 2.8 \times 10^{-5} Re^{1.026} Sc^{0.6} \qquad N \geq 30 \tag{19}$$

$$Sh = 0.00020 Re^{0.65} Sc^{0.59} \qquad N \leq 30$$

$$Sh = 2.018 \times 10^{-5} Re^{1.032} Sc^{0.74} \qquad N \geq 30 \tag{20}$$

$$Sh = 0.0120 Re^{0.4} Sc^{0.52} \qquad N \leq 30$$

The effect of temperature on mass transfer coefficient and dissolution rate of urea fertilizer in finite and infinite volume

Figs. 1a and 1b show the effect of the Schmidt number on the mass transfer coefficient in Sherwood models (Eqs. (19) and (20)) and experimental data indicate that the increase in temperature at a specific impeller speed leads to a decrease of the Schmidt number, whereas the mass transfer coefficient increases due to the increase of the diffusivity of the solid into the liquid. This behavior is in good agreement with another investigation [42]. This behavior is due to the reduction of viscosity and surface tension with increasing temperature, whereas the diffusivity of the solid in the liquid increases. Therefore, the Schmidt number, which is the ratio of kinematic viscosity and mass diffusivity, decreases [43]. But the mass transfer coefficient which is proportional to the diffusivity to the power 0.5-1 (based on combination film-surface renewal theory) [44, 45] increases with increasing temperature. However, as can be observed in Figs. 2a and 2b, increasing temperature, in other words, decreasing the Schmidt number, decreases the Sherwood number values for urea granules in Sherwood models and experimental data.
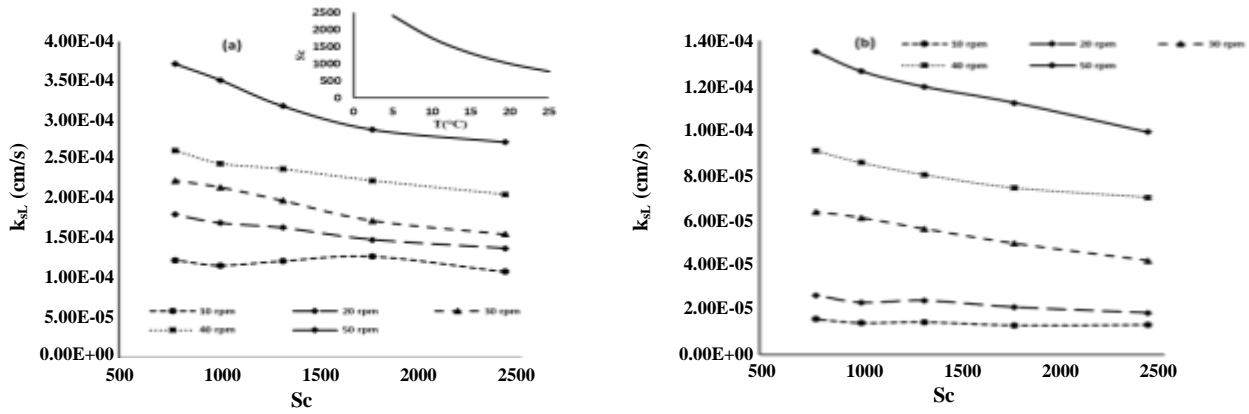
Fig. 1: The effect of Schmidt number on the mass transfer coefficient of the urea-water system at various liquid temperatures in (a) finite (Eq. 19) and (b) infinite (Eq. 20) volume fluids for data and Sherwood models.
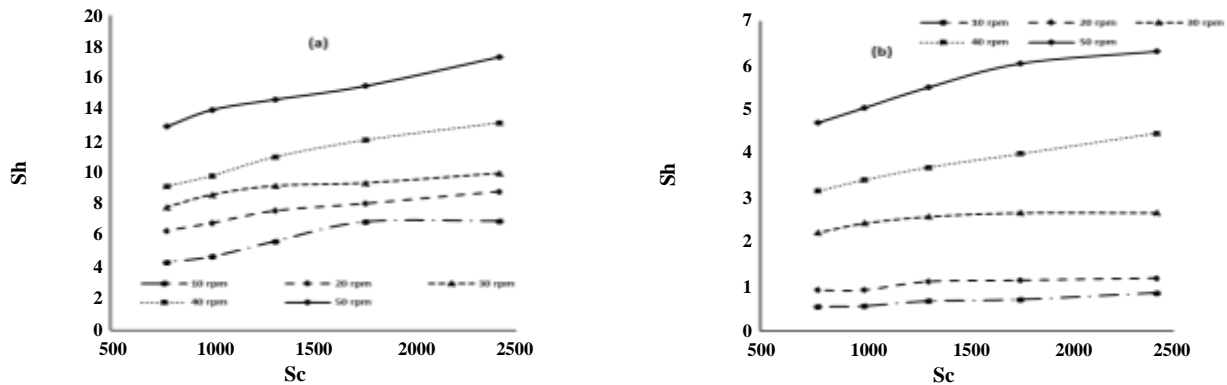

Fig. 2: The effect of Schmidt number on Sherwood number of the urea-water system at various liquid temperatures in (a) finite (Eq. 19) and (b) infinite (Eq. 20) volume fluids for data and Sherwood models.

This behavior can be related to the Sherwood correlation ($Sh=2K_{SL}r_0.D^{-1}$). In this equation, the mass transfer and diffusivity increase ($ks_L=D^n$, $n=0.5-1$) but the Sherwood number decreases due to the effect of increasing the diffusivity by temperature increase on the reduction of the Sherwood number is more than the effect of the increase of k_{SL} due to the increase of temperature.

The obtained mass transfer coefficient was substituted in the relevant prediction model. Then, the diagram of the residual mass of solid proportion, M/M_0, versus time was plotted in Figs. 3a and 3b for different liquid phase temperatures at agitation speed, 10 [*rpm*]. It can be observed that the dissolution rate increases as the liquid temperature increases because the mass transfer coefficient rises (see Fig. 1). In other words, the natural condition of agricultural lands, such as temperature changes, affects the mass transfer coefficient; thus, the urea dissolution rate would increase.

The effect of impeller speed on mass transfer coefficient and dissolution rate of urea fertilizer in finite and infinite volume

In Figs. 4a and 4b, the mass transfer coefficient (k_{SL}) values are plotted against the impeller speed for various temperatures for experimental data and Sherwood models (Eqs. (19) and (20)). The values of the individual mass transfer coefficients in solid-liquid agitated systems depend on the thickness of the fluid boundary layers, which, in turn, depends on flow velocity and fluid properties, such as viscosity and diffusivity [46, 47].

It is also seen in Fig. 5a and 5b (for experimental data and Sherwood models) that, at a given temperature, the Sherwood number increases with the increase of the Reynolds number (turbulence) due to the diffusivity is constant at a constant temperature, and the Sherwood number depends only on the mass transfer coefficient,
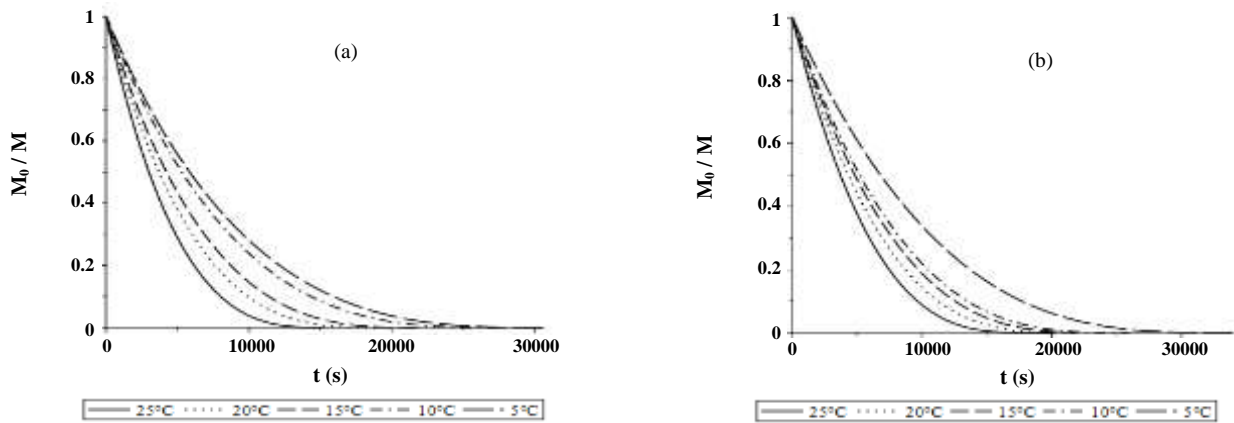
Fig. 3: The temperature effect on the mass transfer rate of the urea-water system in the present model in
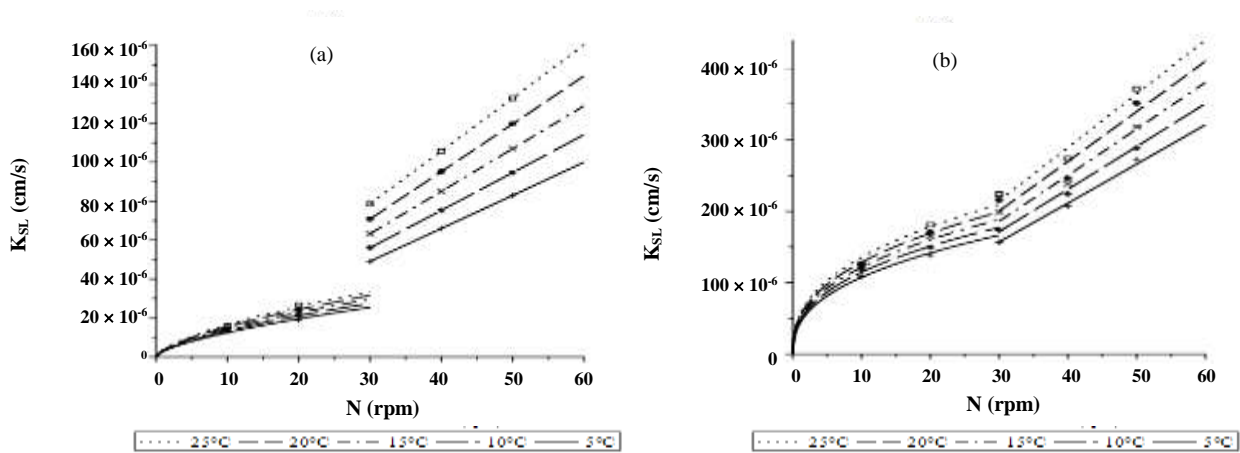*(a) finite (Eq. 8) and (b) infinite (Eq. 13) volume fluids at 10 [rpm].*

Fig. 4: The impeller speed effect on the mass transfer coefficient of the urea-water system at various liquid temperatures in
(a) finite (Eq. 19) and (b) infinite (Eq. 20) volume fluids for data and Sherwood models.

therefore with increasing the Reynolds number, the mass transfer coefficient, and the Sherwood number increase. In Figs. 6a and 6b, it can be observed that the dissolution rate increases as the agitator speed increases, because of the increase in the mass transfer coefficient. In other words, the natural condition of agricultural land affects the mass transfer coefficient; thus, the urea release rate would increase.

On the other hand, according to Eq. (2), in constant operating conditions, the maximum nitrogen transfer rate occurs when the subtraction of saturation concentration from liquid concentration $(C_{sat} - C_L)$ is the highest. It reaches its maximum when the concentration of dissolved C_L is zero. However, the driving force and release rate decrease during the release of nitrogen into the environment, due to increasing environmental concentration in finite volume. Fig. 6a shows the reduction

in the release slope. However, in the infinite volume case, due to the constant concentration driving force, the mass flux increases as the fluid velocity increases due to the reduction of laminar sub-layer thickness in turbulent water and resistance of the mass transfer. The dissolution rate in this layer is seen in Fig. 6b.

Effect of solids concentration on urea-water mass transfer in finite and infinite volume

To study the solids concentration effect on urea-water mass transfer coefficient, the mass balance between solid and agitated liquid was considered, and two models are presented for finite (Eq. (8)) and infinite volume (Eq. (13)), which assumed concentration changes through time in the liquid phase in the finite volume case, and which does not change in the infinite volume case. Fig. 7 shows the results of comparing two models at an impeller speed of 10 rpm and
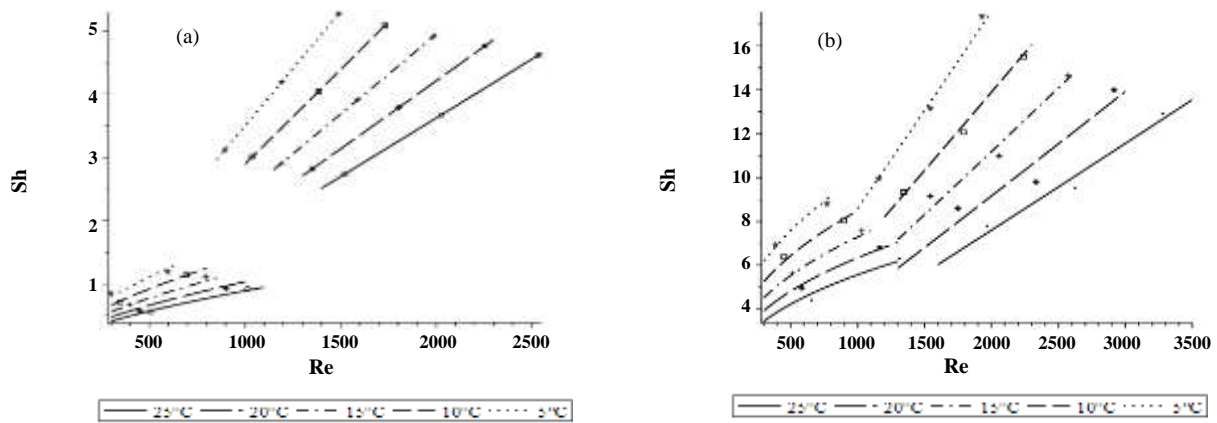
Fig. 5: The effect of Reynolds number on Sherwood number of the urea-water system at various liquid temperatures in (a) finite (Eq. 19) and (b) infinite (Eq. 20) volume fluids for experimental data and Sherwood models.
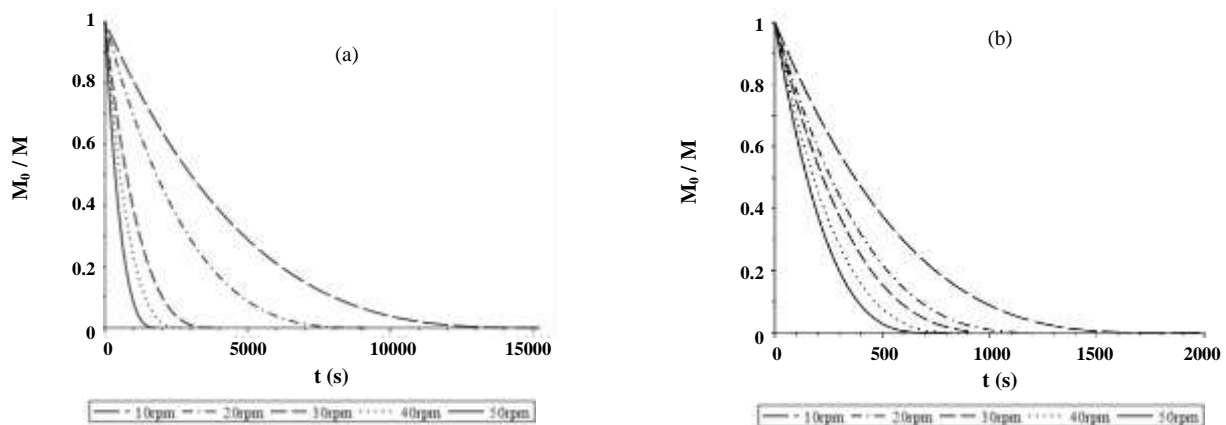

Fig. 6: The impeller speed effect on the mass transfer rate of the urea-water system in the present model in (a) finite (Eq. 8) and (b) infinite (Eq. 13) volume fluids at 25 [°C].

a temperature of 20 °C. It can be seen that the mass transfer rate for infinite volume is greater than that for finite volume. In finite volume, the concentration of the liquid phase is not stable, as a result of the changes in the concentration gradient between liquid and solid phases. Hence, by decreasing the concentration gradient over time, the mass transfer rate decreases. This result can be seen in Fig. 7, which shows a steep slope in the first stage, which decreases over time. On the other hand, in the infinite volume case, the slope of the graph is almost uniform and steep due to the constant concentration gradient; in turn, there is no change in the concentration of the liquid phase.

Several studies in the literature investigated the effect of solids concentration on the mass transfer coefficient. *Harriott* (1962) studied the effect of solids concentration on the mass transfer coefficient in an ion-exchange particle system [48]. Harriott observed that solids concentration

does not affect the mass transfer coefficient in the concentration range of 0.001-0.053 v/v. Several researchers reported that increasing the solid concentration is ineffective on the mass transfer coefficient up to a solids concentration of 0.10 v/v [49-53]. *Cline* (1978) investigated the effect of solids concentration on the mass transfer coefficient in an ion-exchange resin system in the concentration range of 0.05-0.4 v/v and a constant impeller speed. He observed that with increasing solids concentration the mass transfer coefficient decreases [41]. *Bong* (2015) studied the effect of high solids concentration and impeller speed at a constant temperature on the mass transfer coefficient [36]. He observed that increasing solid concentration does not affect the mass transfer coefficient in the range of 0.05-0.065 v/v at just-suspended speed. Although, in high solids concentration (*Cv = 0.08-0.30* v/v) with increasing solid concentration, the mass transfer
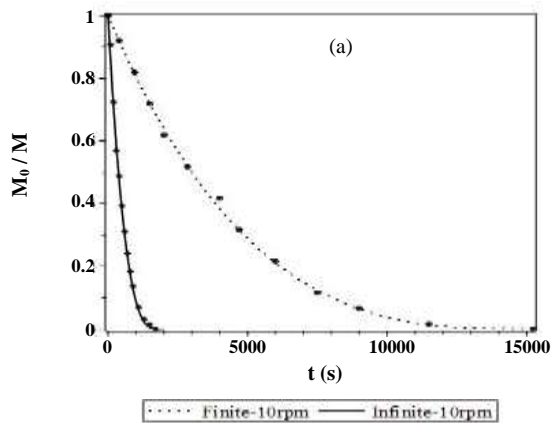
Fig. 7: Comparison of models for the release of urea from common urea into finite (Eq. (8)) and infinite (Eq. (13)) volumes of water.

Fig. 8: Comparison of the model (Eq. (13)) for the dissolution of common urea into the water with others' experimental data [12, 23].

coefficient increases and decreases after effective solid concentrations. In this study, as the concentration of the fluids increases, the mass transfer coefficient decreases; therefore, the result of this study is in agreement with the findings of *Celine* (1979) and *Bong* (2013) and disagrees with *Harriott* (1962) and some other investigators [49-54].

Verifying the mass transfer models with other experimental data

To validate the model's credibility of urea release in water, a comparison was done with other experimental data. The standard error of the estimate (*SEE*) was used to calculate the validity between the introduced model and others' experimental data. In this case, some experiments about urea release in water were conducted in quiescent water. To investigate the validity of the infinite model (Eq. (13)), experimental data from the *Xiaouy et al* (2013) study were used. They utilized 12 g of commonly used urea in 1 L of distilled water (12000 ppm). Fig. 8 depicts the results of data fitting and shows that the model for infinite volume predicts well the experimental data by SEE = 0.038. Furthermore, *Fernández-Pérez et al* (2008) used 150 mg of commonly used urea in 100 [*mL*] of distilled water (1500 ppm) in infinite volume. These data are described well in the infinite model Eq. (13) by SEE = 0.044 (see Fig. 8). Based on our review of the literature, no experimental data appear to exist regarding the release rate of commonly used urea in finite volume case, different speeds of agitation, and different temperatures; hence, we were unable to compare our model against those of others. However, our obtained experimental data proved these two models at 10 rpm and 25 °C (see Fig. 7).
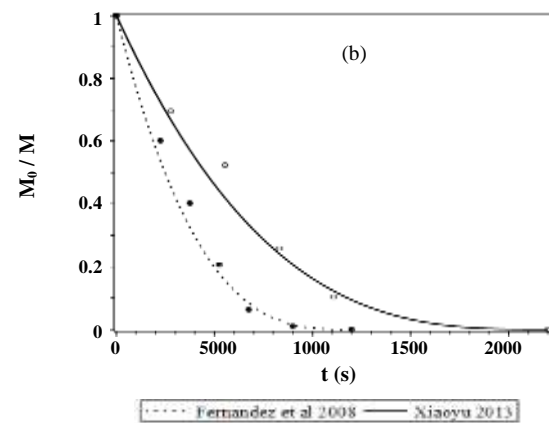
CONCLUSIONS

In this study for finite and infinite volume cases, two models with special emphasis on the effect of turbulence on release rate are presented for nitrogen release from urea fertilizer using mass balance between fertilizer granules and fluid phase Then the mass transfer coefficient was calculated using TCD at different agitator speeds (10-50 rpm) and different temperatures (5-25 °C). The proposed models were studied for different hydrodynamic conditions. The results show that these models are responsive at different temperatures and agitator speeds, so can be generalized for different hydrodynamic conditions. It was also observed that by increasing temperature or reducing the Schmidt number at a constant impeller speed, the mass transfer coefficient and release rate increase, but the Sherwood number decreases. This occurs because of the reduction of liquid viscosity and surface tension, and the increase in diffusivity of the solids in the liquid. On the other hand, this coefficient increases with increasing impeller speed (Reynolds number) in constant liquid temperature because of the greater turbulence, distribution of granules, introduction of the eddy, and decreased thickness of the laminar sub-layer. The mass transfer rate of nitrogen release depends on the solid concentration of the environment. In other words, in finite volume, liquid concentration increases; therefore, the concentration gradient between solid and liquid phases, which is a reason for transferring active agents, decreases. As a result, in a constant Reynolds and Schmidt number, the mass transfer coefficient in the finite volume case is lower than in the infinite volume.

Nomenclature

A	Area of urea granules, m^2
C_L	Concentration of solute in the liquid phase, kg/m
C_A	Concentration of urea, kg/m^3
C_{Sat}	Concentration of solute in the saturated phase, kg/m^3
D	Diffusion coefficient, m^2/s
K_{LS}	Mass transfer coefficient of the liquid phase, m/s
M	Residual mass of solid at any time, kg
M_0	Residual mass of solid at the initial time, kg
m^0	Mass transfer rate of solid phase to liquid, kg/s
n	Number of urea fertilizer granules
N	Velocity of the agitator, rpm
r	Radius of granules at any time, m
R	Initial radius of urea granules, m
R_H	Hydraulic depth, m
$Re=T^2.N.\vartheta_{solution}^{-1}$	Reynolds number (refers to the agitator)
$Re_H=V. R_H.\vartheta^{-1}$	Reynolds number hydrodynamics
$Sh=2.K_{LS}.R.D^{-1}$	Sherwood number
t	Time, s
t_f	Time complete dissolution, s
T	Stirrer diameter, m
V_L	Liquid volume, m^3
V	Mean velocity of water, m/s

Greek letters

μ	Dynamic viscosity, kg/m.s
ρ	Liquid density, kg/m^3
ρ_s	Solid density, kg/m^3
ϑ	Kinematic viscosity of water, m^2/s
$\vartheta_{solution}$	Kinematic viscosity of the solution, m^2/s

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