Estimating Aqueous Nanofluids Viscosity *via* GEP Modeling: Correlation Development and Data Assessment

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ABSTRACT: *This paper focuses on developing a new method that represents user-accessible correlation for the estimation of water-based nanofluids viscosity. For this, an evolutionary algorithm, namely Gene Expression Programming (GEP), was adapted based on a wide selection of literature published databanks including 819 water-based nanofluids viscosity points. The developed model utilized the base fluid viscosity as well as volume fraction and size of the nanoparticles as the inputs of the model. Several statistical parameters integrated with graphical plots were employed in order to assess the accuracy of the proposed GEP-based model. Results of the evaluation demonstrate fairly enough accuracy of the developed model with statistical parameters of AARD%=11.7913, RMSE=0.3567, and SD=0.1851. Furthermore, the trend analysis indicates that the GEP calculated points satisfactorily follow the trend of the nanofluid viscosity variation as a function of different model inputs. To provide more verification, the proposed GEP model was compared with some literature theoretical and empirical correlations leading to the supremacy of the developed model here. The applied sensitivity analysis reveals that the highest impact value is assigned to the volume fraction of the nanoparticle. Moreover, the outlier's detection by Williams' technique illustrates that about 96.5% of the GEP estimates are in the applicability domain resulting in the validity of the proposed model in this study. At last, the results of this study demonstrate that the new method here outperform other literature-published correlations from the standpoint of accuracy and reliability.*

KEYWORDS: *Nanofluids; Viscosity; Gene expression programming; Correlation; Outliers detection; Sensitivity analysis.*

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INTRODUCTION

During nanotechnology advancement, development of the nanoparticles has led to the establishment of a new generation of heat transfer liquids termed Nanofluids. The aforesaid novel liquid-solid materials are suspensions of nanometer-sized particles (average size <100 nm) in various base fluids. This colloidal dispersion of nanoparticles in the base fluids offer intriguing changes in the different thermo-physical properties of the base fluids such as cooling capacity, lubricating efficiency, electrical properties, and viscosity behavior [1]. Accordingly, nanofluids have been widely employed in various branches of chemical and petroleum engineering such as improvement of oil recovery [2, 3], fines migration [4], inhibiting calcium sulphate scale precipitation/deposition [5], performance enhancement of heat exchangers [6, 7] and wettability alteration of reservoir rock [8]. Nanofluids can be classified into two major categories, namely single material and hybrid nanofluids [9]. The single material nanofluids [10] are conventional mixtures that are made of a single type of nanoparticle including metals (e.g. Al, Fe, Cu, and Ni), oxides (e.g. Al_2O_3, TiO_2, Fe_2O_3, Sio_2, CuO, and Cu_2O), metal nitrides (e.g. AlN, TiN, and SiN), and carbon materials (e.g. diamond, carbon nanotubes, and graphite) [9, 11, 12]. The second class of the nanofluids, namely hybrid mixtures, is formed by employing a combination of nanoparticles such as CNT-Cu (Carbon Nanotube-Copper) and CNT-Au (Carbon Nanotube-Gold) in a base fluid [13]. On the other hand, the traditional coolant liquids including water, Ethylene Glycol (EG), ethanol, refrigerants, and engine oils are utilized as the base fluids [14].

Understanding the thermo-physical properties of the aforementioned nanofluids such as thermal conductivity and viscosity are essential for designing a thermal system in which a nanofluid performs as a working fluid [11]. Viscosity is one of the most important characteristics of nanofluids which represents the tendency of the fluid to resist the flow. Nanofluids demonstrate significantly higher viscosity compared to their base fluids [15]. Additionally, several critical parameters including pumping power, flow behavior, pressure drop, and convective heat transfer are found to be direct functions of the viscosity of working fluid [16]. Accordingly, the experimental/mathematical measurements of viscosity are essential to describe the thermo-fluidic behavior

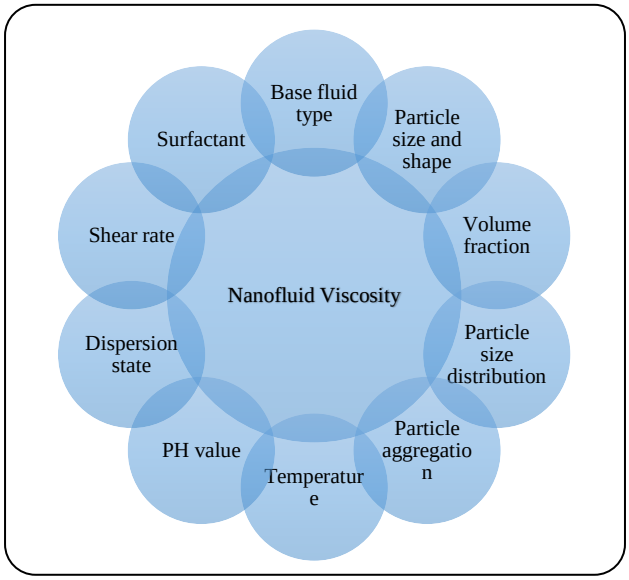

Fig. 1: Factors influencing the viscosity of nanofluids.

of nanofluids. Thereby, several pieces of research have been carried out regarding the factors affecting the rheological behavior of nanofluids (i.e., Newtonian and non-Newtonian) [15]. Results of their evaluation demonstrate the dependency of nanofluids viscosity to various entities including the type of base fluid, particle size and shape [17], volume concentration [18], particle size distribution [19], presence of particle aggregation [20], temperature [21], PH value [22], dispersion state [23], shear rate [24] and presence of surfactant [25]. Factors influencing the viscosity of nanofluids are indicated in Fig. 1.

Although the effect of some of the aforementioned parameters on the viscosity of nanofluids seems to be obvious, there are discrepancies between the experimental reports regarding other factors. For example, approximately all evaluations demonstrate that nanofluid viscosity increases by increasing the base fluid viscosity and the concentration of nanoparticles in the base fluid [18, 26]. However, some contradictions respecting the effect of nanoparticle size have been reported in the literature. According to *Nguyen et al.* [27] and *He et al.* [28], the viscosity of Alumina-water and TiO_2-distilled water nanofluids increases with an increase in nanoparticle size. In spite of previous research, *Namburu et al.* [29], *Chevalier et al.* [30], *Lu* and *Fan* [31], and *Anoop et al.* [17] have indicated the reducing trend of SiO_2-ethanol, CuO-water, and Al_2O_3-water viscosity with nanoparticle size. Accordingly, experimental/mathematical determination of nanofluids' viscosity is a sophisticated procedure.

It is widely accepted that the experimental techniques are the most appropriate methods for accurate measurement of the viscosity of nanofluids. Although, employing the aforesaid techniques is associated with limitations from the standpoints of time and expenses. Accordingly, several researchers have attempted to predict this parameter by employing three different techniques including theoretical models, empirical correlations, and computer-aided approaches [32-34]. Einstein [33] was the first one that developed theoretical expression in order to predict the viscosity of nanofluids as a function of base fluids viscosity and the volume fraction of the nanoparticles. This formula was limited to the spherical particles suspended in the base fluid at a volume fraction less than 0.02. Compensating these limitations, several power-law-based models, as well as extensions of Einstein's model, were proposed in the literature [34-36]. However, the researchers found the foregoing models inapplicable for estimating nanofluids' viscosity [37, 38]. Furthermore, numerous empirical correlations have been developed in order to predict the viscosity of different nanofluids as a function of various parameters such as base fluids viscosity, the volume fraction of the nanoparticles, nanoparticles aggregates, size of the nanoparticles, and temperature [27, 39-41]. The majority of these empirical expressions have been correlated based on a limited experimental condition, namely, nanofluids type, size and shape of the nanoparticles, and the concentration of particles. Accordingly, this approach also suffers from high deviations from the experimental points [42].

In the last decade, computer-aided approaches have grabbed the attention of researchers as the recent technique for estimation of different parameters in petroleum and chemical engineering [43-46]. For this purpose, several smart models have been developed by employing various soft computing algorithms including Multilayer Perceptron (MLP) neural network, Radial Basis Function (RBF) neural network, and Least Square Support Vector Machine (LSSVM) [47, 48]. According to the literature [32], the aforementioned approaches have outperformed both theoretical and empirical expressions from the accuracy and reliability points of view. Moreover, smart modeling approaches have been developed by employing a wide range of experimental data, which demonstrates the universality of these models. However, most of the foregoing models do not represent the user-accessible symbolic expressions, and computers are required for executing the models. In other words, algorithms such as LSSVM, MLP neural network, and RBF neural networks establish the black-box models which cannot be applied in future studies easily; even though the so-called strategies termed Gene Expression Programming (GEP) and Group and Group Method of Data Handling (GMDH) can provide white-box models [49-52].

In the current study, an evolutionary algorithm entitled Gene Expression Programming (GEP) was exploited in order to satisfactorily predict the viscosity of Al_2O_3, TiO_2, SiO_2, and CuO nanofluids as a function of volume fraction and size of the nanoparticles together with base fluid viscosity. To this end, an extensive range of databanks that encompass various operational conditions was adopted from the literature [17, 23, 26, 40, 53-63]. The accuracy and reliability of models were evaluated using Absolute Relative Deviation (ARD), Average Absolute Relative Deviation (AARD), Standard Deviation (SD), Root Mean Square Error (RMSE), and Determination Coefficient (R^2). In the end, the proposed model was compared to the other literature-published models in order to represent further assessment regarding the GEP model. Sensitivity analysis and William's technique were also employed to explore each variable impact value on output estimation, and validity analysis, respectively. In continuum, an explanation of data collected from open literature is given. Then, the GEP mathematical strategy will be presented in detail by giving the modeling procedure. In the results and discussion section, the developed correlation will be shown and evaluated using numerous statistical parameters and diagrams. Additionally, many correlations in the literature are used for comparison with the proposed GEP model here. Validation analysis of the used database is implemented by using William's plot. Investigating the applied controlling variable over the adopted database is conducted by utilization of Pearson's technique [64] for sensitivity analysis. Finally, the main achievements of this study will be indicated.

THEORETICAL SECTION

Data collection

Employing a wide range of datasets is one of the significant requirements of developing a universal intelligent model. Accordingly, a cumulative number of 819 viscosity

Table 1: Details of the employed experimental data sets.

Parameters	Types/Ranges
Base fluid	Water
Nanoparticles	Al_2O_3, TiO_2, SiO_2, and CuO
Base fluid viscosity range (mPa.s)	0.39307-1.306
Nanoparticle size range (nm)	10-150
Nanoparticle volume fraction range (%)	0-13
Nanofluid viscosity range (mPa.s)	0.4120-13.2003

points have been gathered from the experimental results published in the literature [17, 23, 26, 40, 53-63]. The foregoing data points belong to four different water base nanofluids including Aluminum oxide (Al_2O_3), Titanium dioxide (TiO_2), Silicon dioxide (SiO_2), and Copper oxide (CuO). The viscosity points change as a function of nanoparticle size (nm), the volume fraction of the nanoparticles, and viscosity of base fluid (i.e., water). A detailed description of the foresaid datasets employed in the current study is represented in Table 1. As it is evidenced, the datasets encompass a wide distribution of input values which brings about a more reliable and universal model.

Details of the employed intelligent model

For more than a decade, Evolutionary Algorithms (EA) have generated imprints in a vast portion of the science branches. To date, various EAs such as Genetic Algorithm (GA) [65] and Genetic Programming (GP) [66] have been introduced, the common denominator of which is adopting the genetic mechanisms and natural selection operators for working with complex problems. The integration of the GA and GP, the Gene Expression Programming (GEP) [67] is another generation of the population-based heuristic algorithms that have recently shed light on the application of soft computing approaches in different fields of science including chemical and petroleum engineering. The linear chromosomes (strings of fixed length) and expression trees (ramified nonlinear expressions with various shapes/sizes) constitute the cornerstone of the GEP. The chromosomes or so-called Genotypes comprise multiple genes consisting of mathematical operators (the head section) and independent/constant parameters (the tail section). The expression trees are translations of the chromosomes in the form of tree-like structures, the branches of which are linked through mathematical operators' so-called linking function [68]. Fig. 2 portrays an instance of the GEP chromosome and the corresponding expression tree.

A typical GEP modeling commences with the generation of a random population of chromosomes, followed by translating them to the expression trees. Subsequently, the program is implemented and the chromosomes are evaluated from the accuracy and precision points of view. The assessment is executed by means of a pre-defined statistical measure such as Mean Square of Error (MSE). The GEP algorithm is ceased if the stopping conditions are fulfilled. Otherwise, the natural biological operators such as recombination, transportation, and mutation are applied to the befitting parent chromosomes recognized in the previous step. Hence, a recent population of the child chromosomes is established and the algorithm repeats until the meeting the termination threshold. The flowchart of a typical GEP modeling procedure is demonstrated in Fig. 3. The interested reader is pointed to the reliable literature for more detailed descriptions of the GEP approach [43, 45, 68, 69].

RESULTS AND DISCUSSION

Model Development

In the current study, a cumulative of 819 data points encompass the water-based viscosity values of Aluminum oxide (Al_2O_3), Titanium dioxide (TiO_2), Silicon dioxide (SiO_2), and Copper oxide (CuO) nanofluids have been gathered from the literature. GEP approach has been manipulated in order to develop a robust correlation for estimation of the viscosity of the foregoing water-based nanofluids. According to the literature, the viscosity of the aforementioned nanofluids is affected by various factors including the nanoparticle size, the volume fraction of the nanoparticles, and the viscosity of the base fluid. Hence, these three parameters have been contemplated as the inputs of the model are as follows:

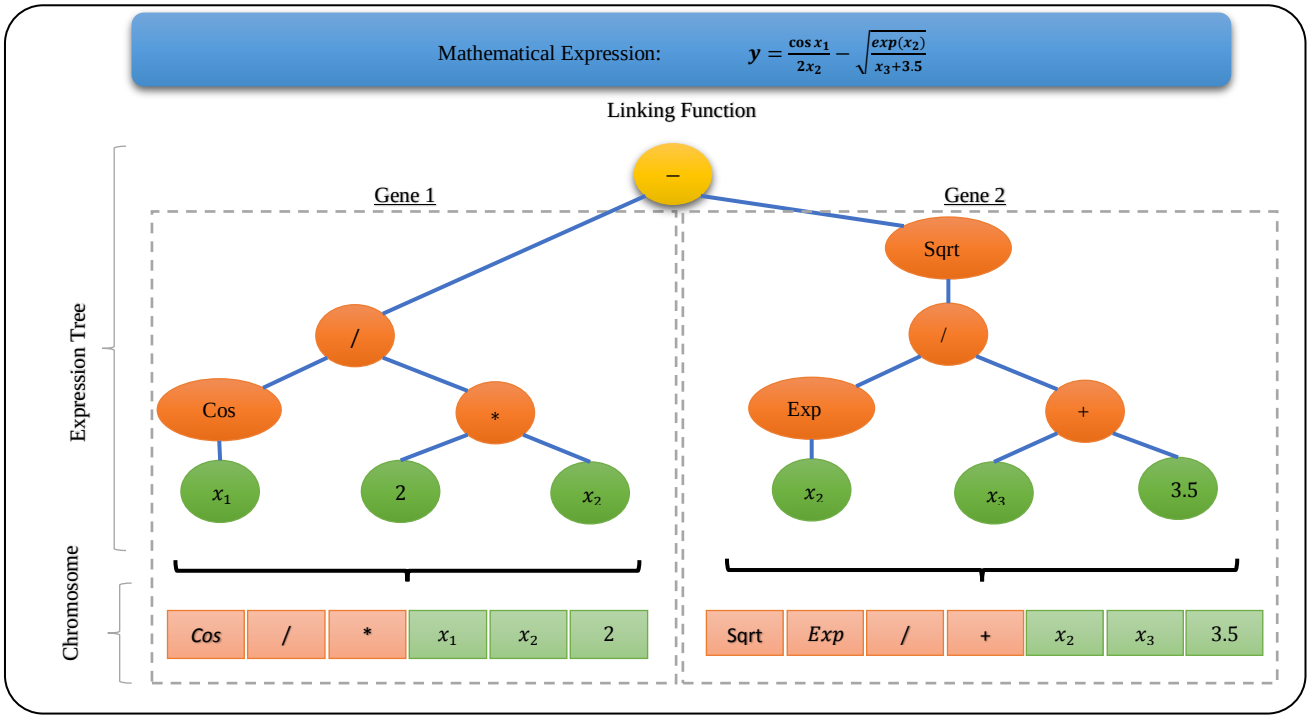

Fig. 2: General structure of the GEP model.

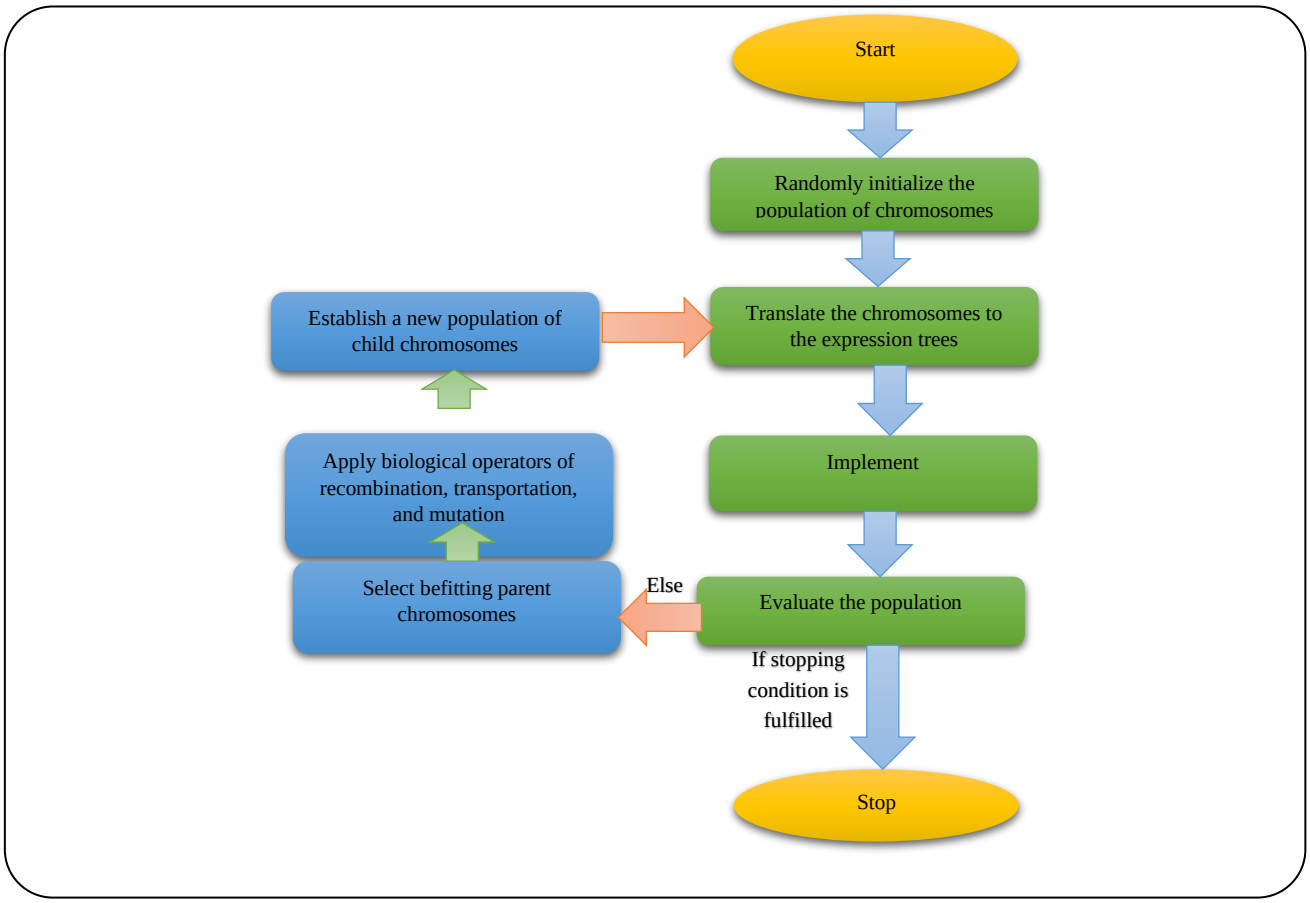

Fig. 3: GEP flowchart.

$$\mu_{eff} = f\left(\mu_b, S, \varnothing\right) \quad (1)$$

where μ_{eff} and μ_b represent the nanofluids (effective) and base fluid viscosity, respectively; and the symbols S and $\emptyset$ stand for nanoparticle size and the volume fraction of the nanoparticles, respectively. In order to develop the model, about 80% and 20% of the databank have been randomly assigned for training and validation/test steps, respectively. The GEP model was executed according to the procedure represented in the previous section. The setting parameters employed in the modeling process are reported in Table .

The GEP model was led to the following correlation for the estimation of nanofluids viscosity:

$$\mu_{eff} = A + B + C \quad (2)$$

where *A*, *B*, and *C* entities are μ_b (mPa.s), S (nm), and $\emptyset$ (%v/v) dependent expressions which can be calculated as follows:

$$A = \exp\left[\frac{a\,\mu_b \varnothing \ln S}{S}\right] \quad (3)$$

$$B = \frac{b\,\mu_b}{\left\{\left[c/\exp\left(\varnothing/S\right)\right] - 2\right\}^{\frac{1}{9}}} \quad (4)$$

$$C = -\frac{\left\{d\left[\exp\left(\varnothing/S\right) - e\right]\right\}^{0.04}}{\left\{\left[\exp\left(\varnothing/S\right)\right]^{5} - \mu_b - f\right\}^{0.2}} \quad (5)$$

where *a*, *b*, *c*, *d*, *e*, and *f* are the tuning parameters of the correlation which have the values of 1.75432848, 0.78736037, 2.72977870, 77.5730483, 1.39895300, and 3.38030970, respectively. The variation ranges of the *A*, *B*, and *C* expressions that have been calculated in the training step are equal to 1.00 to 13.02, -1.11 to 1.57, and -3.84 to 1.27, respectively.

Sensitivity Analysis

The sensitivity analysis was carried out for investigating the effect of each input variable, namely nanoparticle size (S), the volume fraction of the nanoparticles ($\emptyset$), and viscosity of the base fluid (μ_b) on the viscosity of water-based nanofluids. The relevancy factor (r) is the parameter that represents the relative impact of the aforesaid input parameters on the output of the rendered model is as follows [64, 70]:

Table 2: Setting parameters of developed GEP model in this study.

GEP algorithm parameters	Value
Number of chromosomes	30
Head size	7
Number of genes	3
Linking function	Addition
Generations without change	2000
Fitness function	Root Mean Square Error
Mutation	0.00138
Inversion	0.00546
IS transposition	0.00546
RIS transposition	0.00546
One-point recombination	0.00277
Two-point recombination	0.00277
Gene recombination	0.00277
Gene transposition	0.00277
Constants per gene	10
Permutation	0.00546
Random chromosomes	0.0026
Random cloning	0.00102
Data type	Floating point
Employed operators	+, -, ×, /, √, EXP, X^2, X^3, X^4, X^5, $X^{1/3}$, $X^{1/4}$, $X^{1/5}$, INV, LN

$$r = \frac{\sum_{i=1}^{n}\left(\overline{x_k} - x_{k,i}\right)\left(\overline{y} - y_i\right)}{\sqrt{\sum_{i=1}^{n}\left(\overline{x_k} - x_{k,i}\right)^2\left(\overline{y} - y_i\right)^2}} \quad (6)$$

where n represents the number of datasets; k denotes the type of the input namely S, $\emptyset$, and μ_b; $\overline{x_k}$, $x_{k.i}$ are average value and the *i-th* value of the *k-th* input,

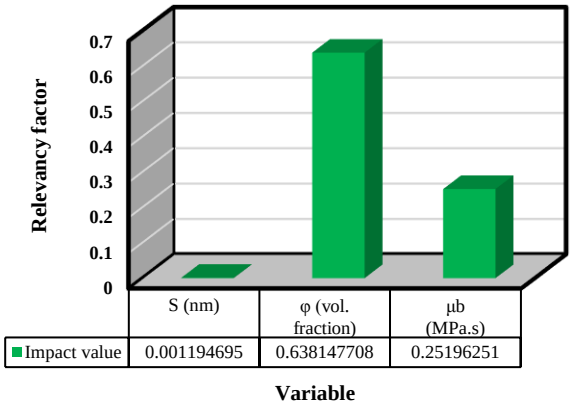

Fig. 4: The relative influence of inputs on the viscosity of nanofluids.

respectively; $\bar{y}$ and y_i stand for average value and the *i-th* value of the output, namely nanofluids (effective) viscosity. Eq. (6) prepares a normalized value for Fig. 4, which indicates the quantitative effect of each input parameter on the effective viscosity. As can be seen, the direct effect of all three variables on the effective viscosity is evidenced; however, the volume fraction has the greatest impact on μ_{eff} due to its higher relevancy factor.

Validity analysis of the developed model

The reliability and accuracy of the developed GEP model for the prediction of nanofluid viscosity have been assessed by utilizing various statistical parameters including Absolute Relative Deviation (ARD), Average Absolute Relative Deviation (AARD), Standard Deviation (SD), Root Mean Square Error (RMSE), and Determination Coefficient (R^2). The definitions of the foregoing statistical quality measures are as follows:

$$ARD\% = \frac{100}{N}\sum_{i=1}^{N}\left(\frac{\mu_i^{exp}-\mu_i^{pred}}{\mu_i^{exp}}\right) \quad (7)$$

$$AARD\% = \frac{100}{N}\sum_{i=1}^{N}\left|\frac{\mu_i^{exp}-\mu_i^{pred}}{\mu_i^{exp}}\right| \quad (8)$$

$$SD = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{\mu_i^{exp}-\mu_i^{pred}}{\mu_i^{exp}}\right)^2} \quad (9)$$

$$RMSE = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\mu_i^{exp}-\mu_i^{pred}\right)^2} \quad (10)$$

$$R^2 = 1-\frac{\sum_{i=1}^{N}\left(\mu_i^{pred}-\mu_i^{exp}\right)^2}{\sum_{i=1}^{N}\left(\mu_i^{pred}-\overline{\mu_i^{exp}}\right)^2} \quad (11)$$

In the above equations, the superscripts *pred*, *exp*, and *N* indicate predicted and measured water-based viscosity values and the size of the database used for modeling, respectively. It is noteworthy that the determination of the coefficient parameter represents dubious results regarding the accuracy of the non-linear problems. However, it was employed habitually in the current study.

The values of the aforesaid statistical quality measures for training and test steps are represented in Table 3. As can be seen, the accuracy of the proposed GEP-based correlation is fairly enough for both training and test data as well as all of the data points.

Moreover, this can be evidenced in the graphical descriptions represented in Fig. 5 to Fig. 8. An assessment regarding the deviation of the predicted nanofluid viscosity points from the actual measured data is represented in Fig. 5. As it is manifested, there is an excellent consistency between the experimental and predicted points for the entire databank including training and test data.

The cross plot of Fig. 6 represents another perceptible comparison between the experimental data and estimated points using the GEP-derived correlation. As it is depicted in this figure, all the training and test data points are allocated in the vicinity of the unit slop line which reveals the high accordance of the predictions with the corresponding target points. Fig. 7 represents another visual tool in order to survey the performance of the proposed GEP model.

This distribution plot shown in Fig. 7 demonstrates the range of the relative errors corresponding to the training and test data. It is noticeable that the relative error can be defined as follows:

$$RE = \left(\frac{\mu_i^{exp}-\mu_i^{pred}}{\mu_i^{exp}}\right)\times 100 \quad (12)$$

The horizontal line indicates the relative error of zero. As it is evident, the majority of the training and test data points are accumulated in the vicinity of the desirable line, namely the horizontal line. This demonstrates the high accuracy and statistical validity of the GEP-derived

Table 3: Statistical parameters of ARD, AARD, RMSE, and R2 for various data points employed for training and testing procedures and also for all of the data points.

Statistic Parameters	Training	Test	All of Data
ARD%	-0.998975	-1.515955	-1.103112
MINARD%[a]	0.008321	0.050404	0.008321
AARD%	11.504055	12.929752	11.791284
MAXARD%[b]	167.899329	113.146388	167.899329
SD	0.180211	0.203254	0.185084
RMSE	0.346619	0.392598	0.356752
R^2	0.908742	0.850196	0.897774

[a]MINARD refers to minimum absolute relative deviation; [b]MAXARD refers to maximum absolute relative deviation.

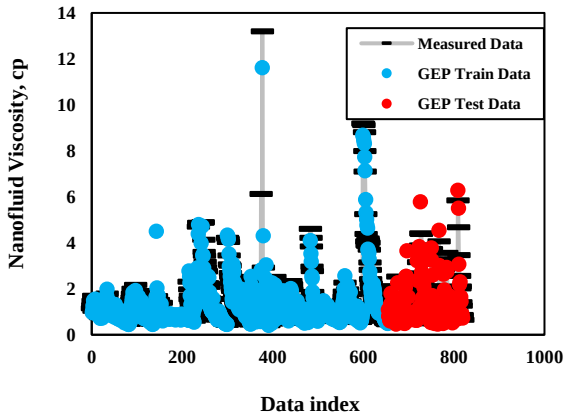

Fig. 5: The comparison between the predicted values of the nanofluid viscosity using the GEP model and the real values of the viscosity for both training and testing data.

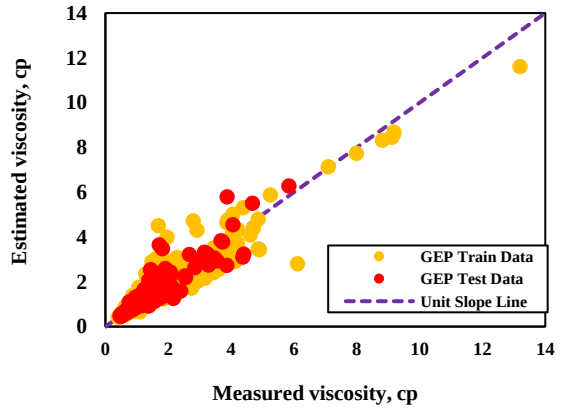

Fig. 6: Cross plot of the established GEP model for both training and testing data.

correlation. However, some observation points are located at an abnormal distance concerning the majority of data points. Although, it is noteworthy that the existence of the aforesaid outliers is unavoidable while dealing with a large number of experimental data points.

Furthermore, the histogram plot in Fig. 8 depicts the frequency of the relative deviations of the predicted data points from the targets. As it is demonstrated, the relative deviations are distributed in a relatively symmetric manner, the majority of the relative deviations are situated in the limited range of -50% to 50%, and the most frequent relative deviation is equal to zero. This represents another testimony regarding the accuracy of the GEP model developed in this study.

Outlier detection for the suggested GEP model

Recognition of points that are remarkably different in value from the rest of the observations, the outlier detection is an underlying step for corroborating the statistical assessments. The outliers could significantly influence the majority of statistical measures such as standard deviation and mean values. Accordingly, these undesirable anomalies can directly impact various analyses carried out for reflecting the accuracy of the models. Moreover, detecting and appropriately treating the outliers would enhance the reliability and accuracy of the developed models [71-73]. The literature is replete with approaches introduced for working with anomalies [71, 73]. In this study, William's plot of Standardized Residuals versus Hat indices (leverage's values) was employed for this purpose. Fig. 9 portrays the forgoing plot for the developed GEP-based model in this study.

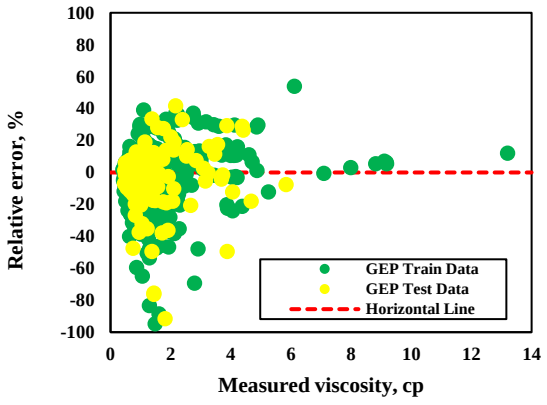

Fig. 7: Relative deviation of the proposed GEP model from the measured values.

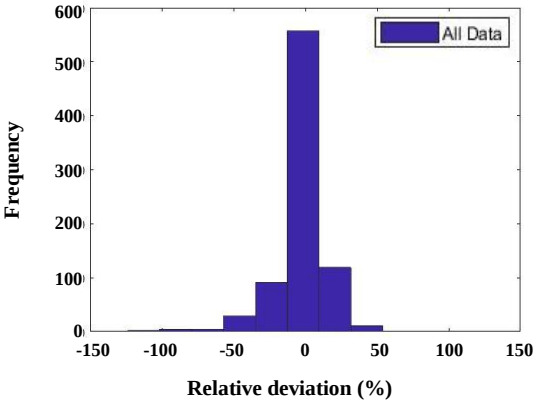

Fig. 8: The frequency of the relative deviations of the predicted data points from the targets.

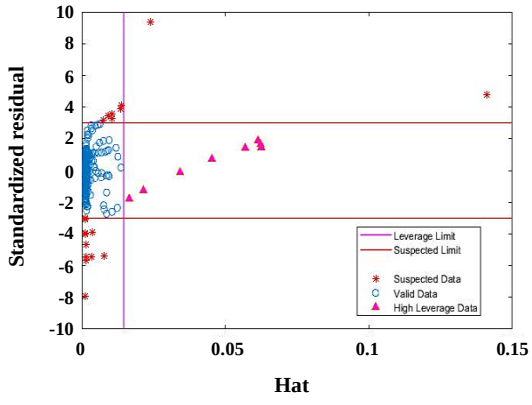

Fig. 9: The Williams plot of the established GEP model.

William's plot is split into three domains of (I) $R > 3$ or $R < -3$, (II) $-3 \leq R \leq 3$, and $H > \hat{H}$, and (III) $-3 \leq R \leq 3$ and $H < \hat{H}$ that correspond to the Bad High Leverage (Outliers), Good high leverage, and applicability domain of the model, respectively.

The parameter $\hat{H}$ is known as the critical Leverage value that can be calculated as follows:

$$H = \frac{3f + 1}{p} \tag{13}$$

in which, f is the number of model parameters and p denotes the number of data points.

According to Fig. 9: the majority of the points (96.5%) are valid data situating within the desirable domain of $-3 \leq R \leq 3$ and H <0.0144. There are only 1.06 % of the points allocated at the Good High Leverage area. Additionally, only about 2.44% of points are detected as unenviable outliers. The results provide further support regarding the correctness of the generated GEP model and database validity used in this study.

Comparison with Other Literature Published Models

A comparison was carried out between the proposed GEP-derived model and other theoretical and empirical correlations including *Hatschek* [35], *Nielsen* [36], *Thomas* and *Muthukumar* [74], *Tseng* and Lin [39], *Sundar et al.* [37], and *Meybodi et al.* [1] models. It is worth noting that the highest performance of *Meybodi et al.* [1] has been reported as compared to the eight literature published correlations including *Einstein* [33], *Brinkman* [34], *Ward* [75], *Lundgren* [76], *Batchelor* [77], *Wang, et al.* [23], *Chen, et al.* [41], and *Abedian et al.* [78]. Hence, the aforementioned correlations have been excluded from the comparison in the current study. Furthermore, it is noticeable that the traditional correlations have been generated for the particular nanofluids and experimental conditions (except *Meybodi et al.* correlation). For avoiding the immense overfitting problems, the regression analysis was carried out by employing the Gauss-Newton and Levenberg-Marquardt algorithms [79]. Accordingly, the traditional models were modified by assigning their most appropriate tuning parameters (constants). Table 4 represents the correlations that have been employed for comparison in this study as well as their set parameters. As it is evident, the default tuning parameters of each model highly deviate from the values obtained from the curve fitting analysis.

Table 4: The literature published models employed for comparison in the current study.

Literature Correlations	Model	Default tuning parameters	Modified tuning parameters
Hatschek (1913)	$\mu_{beff} = \mu_b(1 + a\emptyset)$	a=4.5	a=0.316025
Nielsen (1970)	$\mu_{beff} = \mu_b(1 + a\emptyset)e^{\left(\frac{\emptyset}{1-\emptyset_m}\right)}$	a=1.5	a=0.004704
Thomas and Muthukumar (1991)	$\mu_{beff} = \mu_b(1 + a\emptyset + b\emptyset^2 + c\emptyset^3)$	a=2.5, b=4.83, c=6.4	a= 0.0162984, b= 0.0762487, c= -0.0042447
Tseng and Lin (2003)	$\mu_{beff} = \mu_b a e^{b\emptyset}$	a=13.47, b=35.98	a= 1.09081, b= 0.14173
Sundar et al. (2013)	$\mu_{beff} = \mu_b(1 + \frac{\emptyset}{a})^b$	a=12.5, b=6.356	a= 9.7294, b= 2.1195
Meybodi et al. (2016)	$\mu_{beff} = \mu_b \frac{a + be^{\left(\frac{\emptyset}{S}\right)} + c\left(e^{\left(\frac{\emptyset}{S}\right)}\right)^2 + d\left(e^{\left(\frac{\emptyset}{S}\right)}\right)^3}{f + g\frac{\ln S}{T} + h\frac{(\ln S)^2}{T}}$	a= 133.54064976, b= –343.82413843, c= 290.11804759, d= –78.993120761, f= 0.91161630781, g= 32.330142333, h= –11.732514460	-

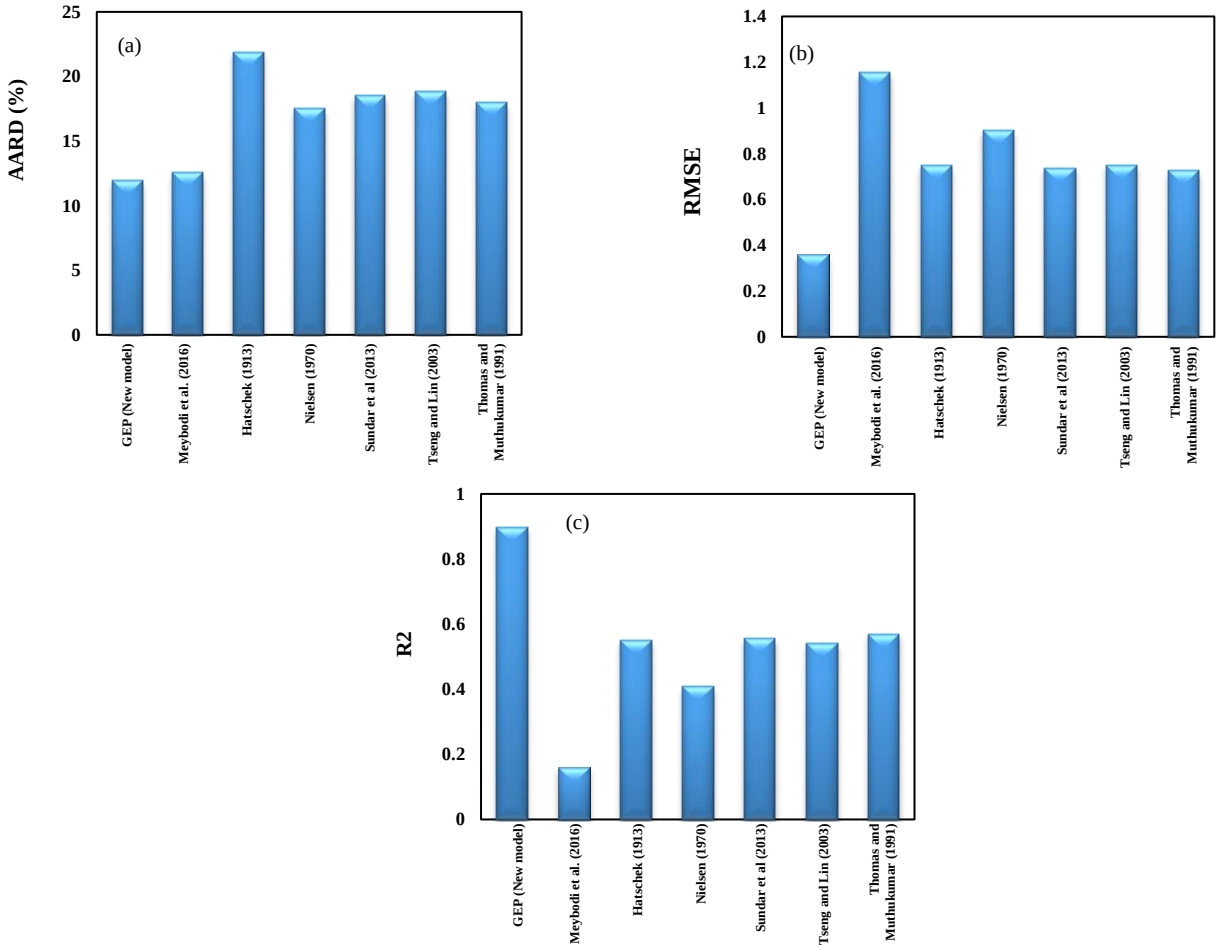

Fig. 10: A visual comparison between the values of the statistical parameters of (a) AARD%, (b) RMSE, and (c) R^2 for different nanofluid viscosity models.

Accordingly, one of the crucial shortcomings of the aforesaid models is a sophisticated procedure of tuning for obtaining a high agreement with the experimental data. The accuracy and reliability of the models have been compared by employing the statistical parameters of ARD, AARD, SD, RMSE, and R^2 [Eqs. (7) to (11)].

Additionally, Fig. 10 (a to c) depicts visual comparisons between the GEP-derived model and the aforesaid literature published models by utilizing the AARD, RMSE, and R^2 quality measures. As can be seen, the proposed GEP-based model in this study benefits from the lowest values of AARD, SD, and RMSE as well as the highest value of R^2.

As it is exhibited, the proposed GEP-based model provides more reliable results compared to other literature-published correlations. The accuracy of the aforementioned models can be ranked according to their AARD parameter as follows:

GEP< *Meybodi et al.* [1]< *Nielsen* [36]< *Thomas* and *Muthukumar* [74]< *Sundar et al.* [37]< *Tseng* and *Lin* [39]< *Hatschek* [35]

It should be noted that employing the reported default tuning parameters for the traditional correlations leads to objectionable values of the statistical parameters, namely AARDs of 551.4349, 3.37e47, 26688.1861, 4.69e263, and 181.7073 for *Hatschk* [35], Nielsen [36], *Thomas* and *Muthukumar* [74], *Tseng* and *Lin* [39], and *Sundar et al.* [37] models, respectively.

Moreover, other evaluations are executed regarding the consistency between the experimental data and the estimated viscosities, and the relative error distribution analysis of the various literature models studied here. As indicated in Fig. 11, the calculated points using the established GEP model are the most contiguous data cloud to the unit slop line. As a result, more agreement between GEP model with the measured data is exhibited.

In accordance with Fig. 12, the majority of the relative errors of the GEP model for estimation of the nanofluid viscosity are accumulated nearby the zero horizontal line. However, the relative errors of the other literature models are situated in a more spacious range. Accordingly, the proposed GEP-derived model outperforms other theoretical and empirical correlations from the standpoints of universality, accuracy, and reliability.

In order to provide more verification, the cumulative frequency of the Absolute Relative Deviation (ARD%) corresponding to the foregoing models is sketched in Fig. 13. In accordance with the figure, about 65.5% of the predictions

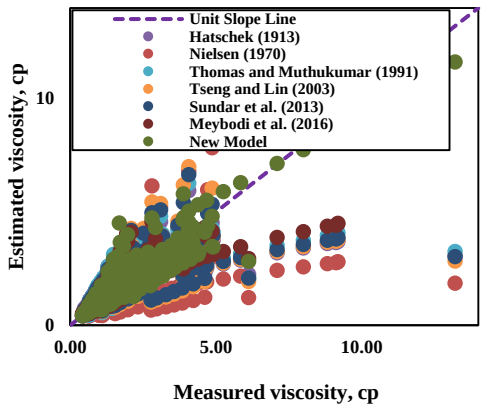

Fig. 11: Cross plot comparison of the established GEM model with different literature published model.

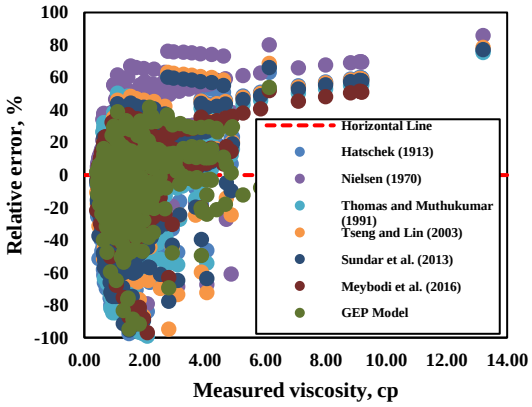

Fig. 12: Comparison between the relative deviation of the proposed GEM model with the other literature published models.

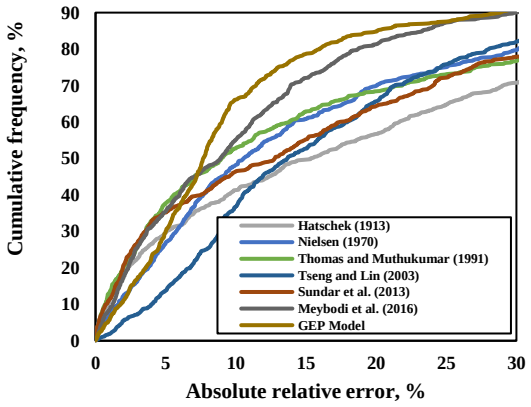

Fig. 13: Cumulative frequency of absolute relative error for various nanofluid viscosity models.

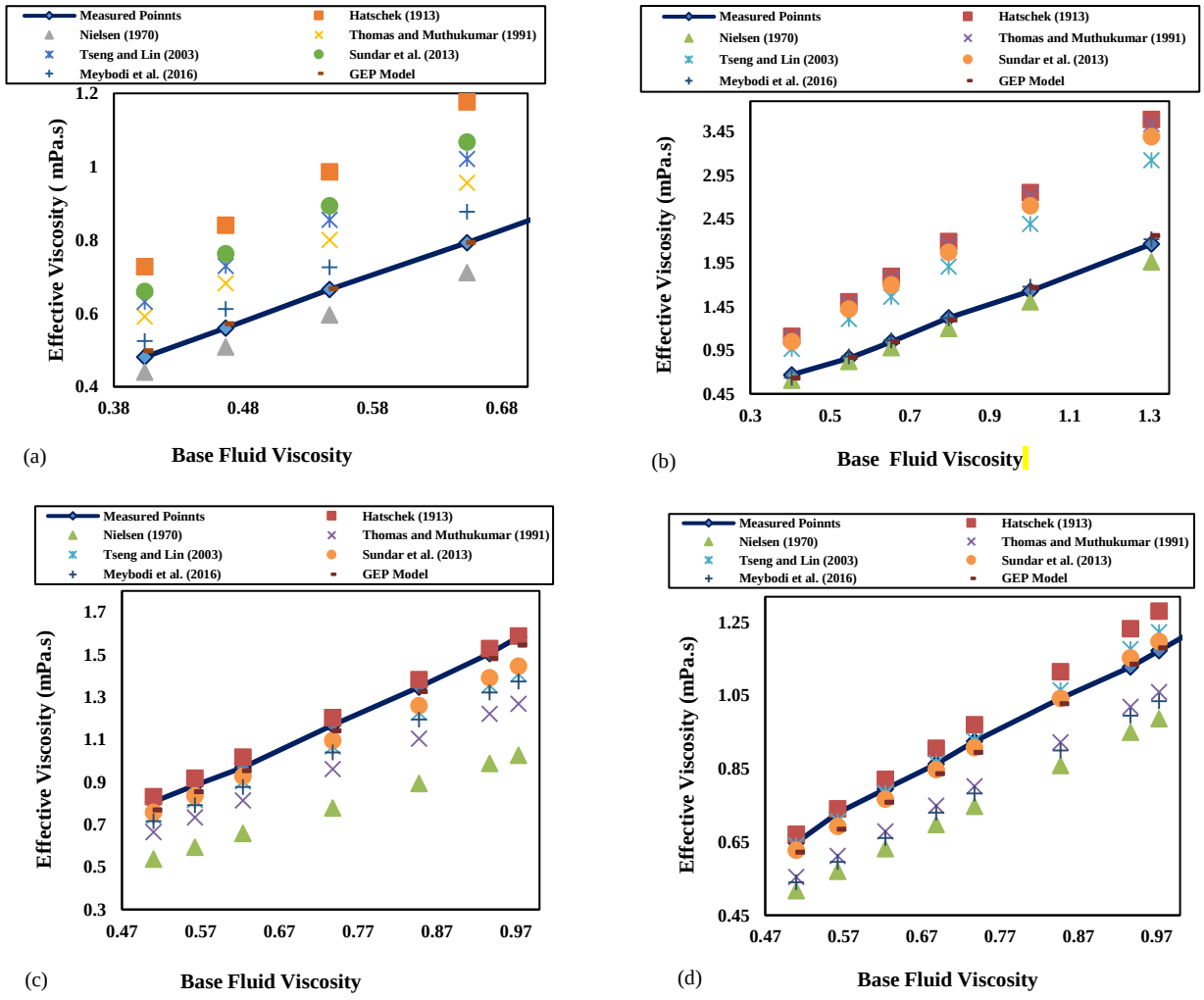

Fig. 14: Analysis of the trend of effective viscosity variation with a change in base fluid viscosity in four different conditions: (a) S=76 nm, Ø=2.54%; (b) S=76 nm, Ø=5.54%, (c) S=21 nm, Ø=2%, and (d) S=21 nm, Ø=1%.

of the GEP model have the absolute relative deviations equal to or less than 9.8%, and about 80% of them have the ARD% equal to or less than 16%. Among the literature published models, *Meybodi et al.* (2016) is the most appropriate model, in which about 65.5% and 80% of their estimations correspond to the errors equal to or less than 12.5% and 18.8%, respectively. Accordingly, the GEP model is the most reliable approach for the prediction of the viscosity of nanofluids.

Trend Analysis

A trend analysis was carried out in order to evaluate the ability of the GEP model and considered literature models for following the trends of nanofluid viscosity variation with a change in the base viscosity and volume fraction parameters. For this purpose, a wide variety of data points were adopted from the diverse literature data sets with different ranges of the inputs [53, 54, 59, 60, 62].

The variation of the effective viscosity as a function of base fluid viscosity is sketched in two different nanoparticle sizes and volume fractions, including (a) *S*=76 nm, Ø=2.54%; (b) *S*=76 nm, Ø=5.54%, (c) *S*=21 nm, Ø=2%, and (d) *S*=21 nm, Ø=1%. Additionally, the effective viscosity changes with the volume fraction of the nanoparticles are plotted in four various conditions, that are, (a) S=47 nm, μ_b=0.89008 cp; (b) S=43 nm, μ_b=0.89008 cp, (c) S=11 nm, μ_b=1.30530 cp, and (d) *S*=11 nm, μ_b=0.54685 cp. Results of the trend analysis with respect to the change in the base fluid viscosity and volume fraction of nanofluid are represented in Fig. 14 and Fig. 15, respectively. As it is obvious, the proposed model

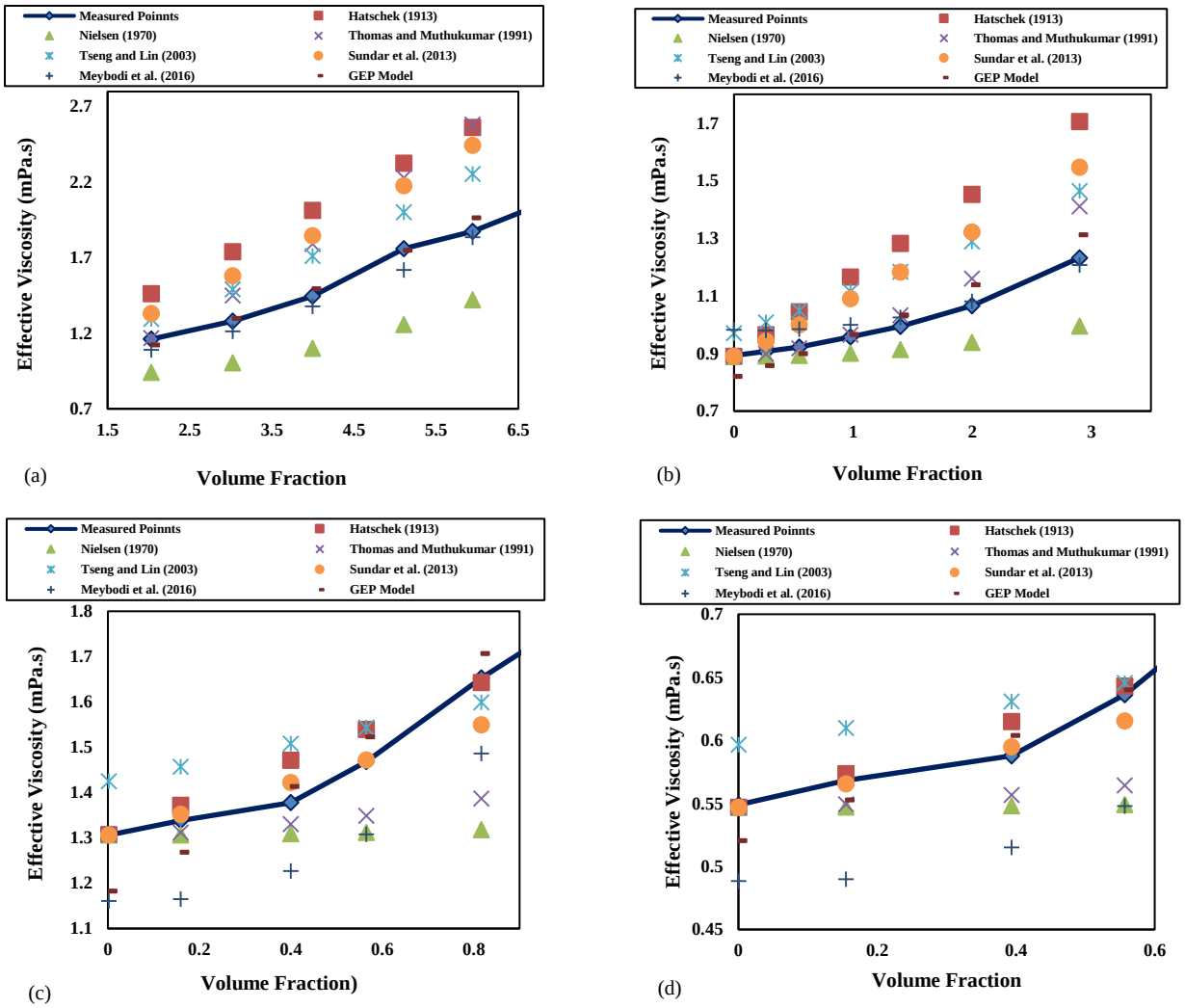

Fig. 15: Analysis of the trend of effective viscosity variation with a change in the volume fraction of nanoparticles in four different conditions: a) S=47 nm, μ_b=0.89008 cp; (b) S=43 nm, μ_b=0.89008 cp, (c) S=11 nm, μ_b=1.30530 cp, and (d) S=11 nm, μ_b=0.54685 cp.

represents fair enough agreement with the measured data points in all of the aforesaid conditions. However, the literature published models suffer from oscillation at accuracy in various experimental conditions.

Comparing the GEP model proposed here with the correlation proposed by *Meybodi et al.* [1] shows the higher potential and greater accuracy of the GEP-empirically derived model. Moreover, GEP correlation has fewer input variables. In other words, GEP model does not use temperature as the input; however, *Meybodi et al.* [1] correlation is in the need for temperature. Additionally, the number of tuning coefficients for GEP and *Meybodi et al.* [1] correlation are equal to 6 and 7, respectively. The lower the number of tuning coefficients means the lower the risk of overfitting in regression analysis. The main drawback of GEP is the intrusion of more complicated operators and functions in its mathematical equation in comparison to the correlation of *Meybodi et al.* [1].

CONCLUSIONS

In the current study, a robust mathematical strategy, termed Gene Expression Programming (GEP), was utilized to propose a symbolic empirical correlation in order to accurately estimate the water-based nanofluid viscosity as a function of base fluid viscosity, nanoparticle size, and volume fraction of the nanoparticles. For this, a source of data involving 819 data points encompassing widespread experimental conditions was adapted from

the reliable literature. The validity and applicability of the developed model were assessed using various statistical parameters, as well as graphical plots. The results of the evaluation are as follows:

1- The developed GEP model in this study demonstrates an appropriate agreement with the experimental points.

2- A comparison analysis was carried out between the proposed GEP-derived correlation and other literature published models. A regression analysis was implemented in order to modify the traditional models by obtaining the best tuning parameters for them. Notwithstanding, the established GEP correlation shows the superior accuracy with AARD% = 11.8% and R^2 = 0.90 in comparison to the previously published models. It is worth mentioning that the GEP model has less number of input variables as compared to the developed model by *Kalantari-Meybodi et al.* [1].

3- The developed GEP-based model outperforms the previous literature models from the standpoint of following the expected physical trends of the nanofluid viscosity variation with a change in the input variables.

4- Implementing sensitivity analysis reveals that the nanofluid volume fraction has the greatest impact on water-based viscosity estimation.

5- Additionally, the validity of the database and the accuracy of the GEP-based model proposed here are proven *via* a well-known outliers detection technique named Williams' plot. Only 3.5% of the database are detected as the outliers.

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