Supporting Information

S1 The proof of the equation (21).

According to Levenspiel [1], in dimensionless form where $z = (ut + x)/L$ and $\theta = \bar{t}/t = tu/L$, the basic differential equation representing the dispersion model becomes

$$\frac{\partial C}{\partial \theta} = \frac{D}{uL}\frac{\partial^2 C}{\partial z^2} - \frac{\partial C}{\partial z} \qquad \text{(S-1)}$$

For step input of concentration C_0, $F = C/C_0$, and $\xi = \frac{z - \theta}{\sqrt{4\theta / Pe}}$, the final equation takes the following form

$$\begin{cases} \frac{d^2 F}{d\xi^2} + 2\xi \frac{dF}{d\xi} = 0 \\ F(-\infty) = F(\infty) = 0 \end{cases} \qquad \text{(S-2)}$$

It is compared to the ordinary equation y”+2xy’-2ny = 0 with n = 0 – as reported by Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables by Milton Abramowitz, Irene A. Stegun, Dover Publications, Inc., 1974 – with the solution form of

$$F(\xi) = A.erfc(\xi) + B.erfc(-\xi) \qquad \text{(S-3)}$$

where using the infinite limits of Eq.(S-2), the constants are A = ½ and B = 0. Therefore, the final equation is

$$F(\xi) = \frac{1}{2}[1 - erf(\xi)] \qquad \text{(S-4)}$$

At the outlet (z = 1), the response to the step input will be

$$F(\theta) = \frac{1}{2}[1 - erf(\frac{1-\theta}{\sqrt{4\theta / Pe}})] \qquad \text{(S-5)}$$

Differentiation of Eq. (S-5) respect to θ using $\frac{d}{dx} erf(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$, results in Eq. (21)

S2 MATLAB codes for generating Figures 2 to 7 in the manuscript.

Figure 2

```
clc
clear all
ALL_TetaMin=0.25:1e-3:0.35;
for i=1:length(ALL_TetaMin)
   TetaMin=ALL_TetaMin(i);
   x=TetaMin:1e-5:10;
   E=3*TetaMin./x.^3/(1-TetaMin).*(1-exp(-3/2*(x-
TetaMin)/...
      (1-TetaMin))).*exp(-3/2*(x-TetaMin)/(1-TetaMin));
   IntE(i)=trapz(x,E);
end

ind_1=find(round(IntE*1e2)/1e2==1);
ALL_TetaMin(ind_1)
IntE(ind_1)

Fontsize=12;
h=plot(ALL_TetaMin,IntE,'k')
xlabel('\theta_{min}','FontSize',Fontsize);
ylabel('\int E.d\theta','FontSize',Fontsize)
set(h,'LineWidth',2);
set(gca,'FontSize',Fontsize);
```

Figure 3

```
clc
clear all

TetaMin1=0.32;
x1=TetaMin1:1e-4:2;
E1=3*TetaMin1./x1.^3/(1-TetaMin1).*(1-exp(-3/2*(x1-
TetaMin1)/...
   (1-TetaMin1))).*exp(-3/2*(x1-TetaMin1)/(1-
TetaMin1));
A_log=trapz(x1,E1)

n1=6;
```

```
TetaMin2=2*n1^2/(n1+1)/(2*n1+1);
x2=TetaMin2:1e-4:2.5;
E2=2*n1*TetaMin2./x2.^3.*(1-
(TetaMin2./x2).^n1).*(TetaMin2./x2).^(n1-1);
A_Power6=trapz(x2,E2)

n_1=10;
TetaMin_2=2*n_1^2/(n_1+1)/(2*n_1+1);
x_2=TetaMin_2:1e-4:2.5;
E_2=2*n_1*TetaMin_2./x_2.^3.*(1-
(TetaMin_2./x_2).^n_1...
   ).*(TetaMin_2./x_2).^(n_1-1);
A_Power10=trapz(x_2,E_2)



Pe=1;
Sig=2/Pe;
x3=1e-4:1e-4:200;
E3=(1+1./x3)/4/pi^0.5.*(Pe./x3).^0.5.*exp(-Pe/4*(1-
x3).^2./x3);
A_D_closed=trapz(x3,E3)
E_3=1./2./(pi.*x3/Pe).^0.5.*exp(-(1-x3).^2./(4*x3/Pe));
A_D_open=trapz(x3,E_3)

N=2;
x4=1e-4:1e-4:10;
E4=N*(N*x4).^(N-1)/factorial(N-1).*exp(-N*x4);
A_Tanks=trapz(x4,E4)

x5=1e-4:1e-4:10;
ind5=find(x5>1/2);
E5(ind5)=1./2./x5(ind5).^3;
A_Laminar=trapz(x5,E5)

Fontsize=12;
h=plot(x1,E1,'k',x2,E2,':k',x_2,E_2,':b',x3,E3,'-.k',...
   x3,E_3,'-.',x4,E4,'--k',x5,E5)
xlim([0 2])
xlabel('\theta','FontSize',Fontsize);
ylabel('E(\theta)','FontSize',Fontsize)
legend('Log profile',['Power profile; n = ' num2str(n1)],...
   ['Power profile; n = ' num2str(n_1)],['Closed Pe='
num2str(Pe)],...
   ['Open Pe=' num2str(Pe)],['Tanks; N='
num2str(N)],'Laminar','Location','NorthEast')
set(h,'LineWidth',2);
set(gca,'FontSize',Fontsize);
```

Figure 4

```
clc
clear all

epsi=1e-5;
ALL_kappa=[0.3 0.5 0.7];
for i=1:length(ALL_kappa)
   kappa=ALL_kappa(i);
   lambda_2=(1-
kappa^2)/2/log(1/kappa);lambda=lambda_2^0.5;
   K=2/(1+kappa^2-2*lambda_2);
   x1=kappa+epsi:epsi:lambda-epsi;
   x2=lambda+epsi:epsi:1-epsi;

   f1=K*(1-x1.^2+2*lambda_2*log(x1));theta1=1./f1;
   E1=1./theta1.^3.*x1.^2./(1-kappa^2)./K./abs(x1.^2-
lambda_2);
   [xx1 ind]=sort(theta1);yy1=E1(ind);


   f2=K*(1-x2.^2+2*lambda_2*log(x2));theta2=1./f2;
   E2=1./theta2.^3.*x2.^2./(1-kappa^2)./K./(x2.^2-
lambda_2);
   Sorted_theta(i,1:length(xx1)+length(theta2))=[xx1
theta2];
   Sorted_E(i,1:length(xx1)+length(theta2))=[yy1 E2];
   Area(i)=trapz(xx1,yy1)+trapz(theta2,E2);
   inv_theta_min=K*(1-lambda_2*(1-log(lambda_2)));
   theta_min(i)=inv_theta_min^(-1);
end



theta_min
Area

Fontsize=15;
h=loglog(Sorted_theta(1,:),Sorted_E(1,:),'sk',Sorted_thet
a(2,:),...
   Sorted_E(2,:),'ob',Sorted_theta(3,:),Sorted_E(3,:),'*r')
xlim([0.67 1e4])
```

```
xlabel('\theta','FontSize',Fontsize);
ylabel('E(\theta)','FontSize',Fontsize)
legend(['\kappa = ' num2str(ALL_kappa(1))],...
   ['\kappa = ' num2str(ALL_kappa(2))], ...
   ['\kappa = '
num2str(ALL_kappa(3))],'Location','NorthEast')
set(h,'LineWidth',2);
set(gca,'FontSize',Fontsize);
```

Figure 5

```
clc
clear all
global Pe
ALL_Pe=[1 10 90];
Endpoint=[1.5 1.3 1.1];
x=zeros(200,length(ALL_Pe));
f=x;
for i=1:length(ALL_Pe)
   Pe=ALL_Pe(i);
   [xx,ff]=ode45('profile',[1e-2 1],Endpoint(i));
   x(1:2*length(xx),i)=[-xx;xx];
   f(1:2*length(xx),i)=[ff;ff];
end

Fontsize=12;
h1=plot(f(:,1),x(:,1),'s',f(:,2),x(:,2),'o',f(:,3),x(:,3),'*')
xlabel('u/u_m','FontSize',Fontsize);
ylabel('r/R','FontSize',Fontsize)
legend(['Pe = ' num2str(ALL_Pe(1))],['Pe = '
num2str(ALL_Pe(2))],...
   ['Pe = ' num2str(ALL_Pe(3))],'Location','West')
yticks([-1 0 1])
yticklabels({'1','0','1'})
set(h1,'LineWidth',2);
set(gca,'FontSize',Fontsize);


function fprime=profile(x,f)
global Pe
fprime=-4*(pi/Pe)^0.5*x.*f.^(5/2).*exp(Pe*(f-1).^2/4./f);
```

Figure 6

```
clc
clear all
global Pe
ALL_Pe=[1 10 100];
Endpoint=[1.5 1.3 1.1];
x=zeros(200,length(ALL_Pe));
f=x;
for i=1:length(ALL_Pe)
   Pe=ALL_Pe(i);
   [xx,ff]=ode45('profile',[1e-2 1],Endpoint(i));
   x(1:2*length(xx),i)=[-xx;xx];
   f(1:2*length(xx),i)=[ff;ff];
end

Fontsize=12;
h1=plot(f(:,1),x(:,1),'s',f(:,2),x(:,2),'o',f(:,3),x(:,3),'*')
xlabel('u/u_m','FontSize',Fontsize);
ylabel('r/R','FontSize',Fontsize)
legend(['Pe = ' num2str(ALL_Pe(1))],['Pe = '
num2str(ALL_Pe(2))],...
   ['Pe = ' num2str(ALL_Pe(3))],'Location','West')
yticks([-1 0 1])
yticklabels({'1','0','1'})
set(h1,'LineWidth',2);
set(gca,'FontSize',Fontsize);


function fprime=profile(x,f)
global Pe
fprime=-8*x.*f.^(5/2).*(pi/Pe)^(0.5)./(4+f).*exp(Pe/4*(f-
1).^2./f);
```

Figure 7

```
clc
clear all
global N
ALL_N=[1 10 100];
Endpoint=[1.5 1.3 1.1];
x=zeros(200,length(ALL_N));
f=x;
for i=1:length(ALL_N)
   N=ALL_N(i);
   [xx,ff]=ode45('profile',[1e-2 1],Endpoint(i));
   x(1:2*length(xx),i)=[-xx;xx];
   f(1:2*length(xx),i)=[ff;ff];
end

Fontsize=12;
h1=plot(f(:,1),x(:,1),'s',f(:,2),x(:,2),'o',f(:,3),x(:,3),'*')
```

```
xlabel('u/u_m','FontSize',Fontsize);ylabel('r/R','FontSize',
Fontsize)
legend(['N = ' num2str(ALL_N(1))],['N = '
num2str(ALL_N(2))],...
    ['N = ' num2str(ALL_N(3))],'Location','West')
yticks([-1 0 1])
yticklabels({'1','0','1'})
set(h1,'LineWidth',2);
set(gca,'FontSize',Fontsize);

function fprime=profile(x,f)
global N
fprime=-2*factorial(N-1)/N^N*x.*f.^(N+2).*exp(N./f);
```