Prediction of Time of Capillary Rise in Porous Media
Using Artificial Neural Network (ANN)

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ABSTRACT: An Artificial Neural Network (ANN) was used to analyse the capillary rise in porous media. Wetting experiments were performed with fifteen liquids and fifteen different powders. The liquids covered a wide range of surface tension (15.45-71.99 mJ/m²) and viscosity (0.25-21 mPa.s). The powders also provided an acceptable range of particle size (0.012-45 µm) and surface free energy (25.54-63.90 mJ/m²). An artificial neural network was employed to predict the time of capillary rise for a known given height. The network’s inputs were density, surface tension, and viscosity for the liquids and particle size, bulk density, packing density, and surface free energy for the powders. Two statistical parameters namely the product moment correlation coefficient (r²) and the performance factor (PF/3) were used to correlate the actual experimentally obtained times of capillary rise to: i) their equivalent values as predicted by a designed and trained artificial neural network; ii) their corresponding values as calculated by the Lucas-Washburn’s equation as well as the equivalent values as calculated by its various other modified versions. It must be noted that for a perfect correlation r²=1 and PF/3=0. The results showed that only the present approach of artificial neural network was able to predict with superior accuracy (i.e. r² = 0.91, PF/3 = 55) the time of capillary rise. The Lucas-Washburn’s calculations gave the worst correlations (r² = 0.11, PF/3 = 1016). Furthermore, some of the modifications of this equation as proposed by different workers did not seem to conspicuously improve the relationships giving a range of inferior correlations between the calculated and experimentally determined times of capillary rise (i.e. r² = 0.24 to 0.44, PF/3 = 129 to 293).

KEY WORDS: Imbibition, Porous media, Lucas-Washburn’s equation, Artificial Neural Network (ANN), Time prediction of capillary rise.

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INTRODUCTION

Capillary rise in porous media is a frequently occurring phenomenon in both natural and industrial systems. Industrially, they occur in such diversities as oil recovery, civil engineering, dyeing of textile fabrics, ink printing, and a variety of other fields.

The first approach to analyse imbibition was reported by Bell et al. [1] who found a square root dependency of time for the height of imbibition. In 1921, Washburn [2] derived the time dependency of the height rise of a liquid into a single, straight line capillary of radius r. The derivation of this equation was based on eq.(1). The capillary pressure difference ($\Delta P_c$) causing the wetting phenomenon may be related to the liquid-vapor interfacial tension ($\gamma_{lv}$) and the contact angle ($\theta$) by what is known as the Laplace’s equation:

$$\Delta P_c = \frac{2\gamma_{lv} \cos \theta}{r} \quad (1)$$

where $r$ is the capillary radius. The Washburn’s equation (eq.(2)) was then derived by equating the capillary pressure difference to the pressure loss due to internal friction (Hagen-Poiseuille’s equation [3]):

$$h^2 = \frac{r\gamma_{lv} \cos \theta}{2\eta} t \quad (2)$$

where $h$ is the height to which the liquid has risen, $\eta$ is the liquid viscosity, and $t$ is the time of capillary rise. Lucas [4], not being aware of Washburn’s work, had derived the same equation independently.

The main shortcomings of the combined Lucas-Washburn's approach are due to the following presumptions and approximations:

- Porous media is assumed to be a bundle of single, straight line capillaries of radius $r$.
- Influence of gravity is not taken into considerations.
- Laminar flow of the liquid is assumed.
- Evaporation from porous media is not taken into account.
- In contrast with Lucas-Washburn’s approach, the contact angle apparently varies as a function of the liquid used [5-9].

- Adsorption of liquid by the solid surface is ignored.
- Effect of inertia on imbibition, relating to the mass of liquid under motion, is neglected.

- For large times, the liquid rise can not fully be described by the Lucas-Washburn’s equation [10-13]. The change in the behavior of the height of capillary rise for large times was suggested [11] to be due to a change in the mechanism of meniscus’s motion from piston-like to a more slower process. The change in the mechanism of meniscus’s motion should be reflected in a change in the macroscopic capillary pressure, however, since Lucas-Washburn’s equation, was obtained under the assumption that capillary pressure was constant, then this will no longer be valid [11]. A further reason for this phenomenon could be attributed to the neglecting of the dynamic saturation gradients implicit to the formulation of the Lucas-Washburn’s equation [10].

The above assumptions have caused non-conformity of the experimental data to the Lucas-Washburn’s equation [5-15]. Therefore, many modifications of the Lucas-Washburn’s equation have been suggested [9,11-26] in order to attain closer correlations to experimental data. The main modifications of this equation are as follows:

1- In order to consider the tortuous nature of the flow path in porous media, the following modifications were suggested [14-20]:

i) If the porous media is considered to be a distribution of cylindrical capillaries having various radii, then Darcy’s length represents the length of the wetting front based on the geometric mean of these capillary radii [15]. The Darcy’s length, $x_{Darcy}$, is defined as:

$$x_{Darcy} = \frac{V(t)}{A\phi} \quad (3)$$

where $\phi$ is the porosity of the porous media, $V(t)$ is the volume of liquid absorbed by the porous media at a given time $t$, and $A$ is the mean cross-sectional area of such porous media. If an equivalent hydraulic capillary (ehc) based on Darcy’s length is derived, similar to what was defined by Lucas-Washburn, it would then follow that:

$$r_{ehcDarcy} = \left( \frac{V(t)}{A\phi} \right)^2 \frac{2\eta}{\gamma_{lv} \cos \theta} \quad (4)$$

The most important drawback using $r_{ehcDarcy}$, (which includes an assumed porosity term), is, that it presupposes that at each stage, the porous media is completely filled by the liquid front [15].
ii) A variable type of porous media under investigation is introduced into the Lucas-Washburn’s equation by the aid of an empirical value $c_r$. This empirical value consists of the average pore radius $r$ and a so-called tortuosity term $c$, to take into account all deviations of the real pore geometry from the assumed cylindrically straight, linear capillary model [14,16-18]. The modified version of eq. (2) then becomes:

$$t_2 \cos r c_r v^2 \eta \theta \gamma = (5)$$

iii) For a given porous media, the effective pore size for penetration can be related to the volume fraction of solids within the porous media ($\phi$). It has been suggested [19] that the Lucas-Washburn’s equation can be augmented to explicitly show the dependency on the porosity of the porous media as:

$$t_1 \frac{K}{\eta} 2 \frac{\epsilon^2}{1-\epsilon} \eta = (6)$$

where $\epsilon(=1-\phi)$ is the porosity of the porous media and $K$ is a constant.

iv) To account for an unusual and complicated cross section, a hydraulic radius is introduced. The hydraulic radius is defined as the ratio of the cross sectional area available for flow, to the wetted perimeter [20]. This hydraulic radius ($r_h$) replaces $r$ in the Lucas-Washburn’s equation (eq.(2)):

$$h^2 = \frac{r_h \gamma_r c \cos \theta}{\eta} t$$

(7)

It is clear that penetration of the porous media by liquids with vastly different wetting properties are by no means similar. Therefore, the assumption that the equivalent radius is constant for liquids with vastly different wetting properties is a major drawback of the modified Lucas-Washburn’s equation [20].

2 - To account for evaporation effects in the Lucas-Washburn’s equation, an evaporation rate proportional to the area of the liquid exposed to air is assumed [21]. On the other hand, the net loss of liquid mass per unit time due to evaporation is:

$$Q_e = -2 \epsilon \rho h^2$$

(8)

where $\rho$ is the liquid density and $\epsilon$ is a phenomenological evaporation rate per unit area. Thus the differential form of the Lucas-Washburn’s equation is modified to:

$$\frac{dh}{dt} = \frac{K}{\eta} \frac{(\Delta P_c)}{h} (\rho g - 2 \epsilon \rho h)$$

(9)

where $K$ is a constant and $g$ is the gravitational acceleration. This form obviously neglects changes in the concentration profile towards the surface as well as the evaporation at the surface, itself [21].

3 - To consider the dynamic advancing contact angle, in the case of a completely spreading liquid, van Remoortere et al. [9] found an empirical equation for the advancing contact angle:

$$\cos \theta = \left( \frac{\eta V}{r} \right)^{1/2}$$

(10)

4 - Only a few workers [22-26] in the field of imbibition recognized inertial effects in the interconnected void network of a real porous media. Whilst in a capillary, the inertial regime is relevant only in the initial extremely short times of absorption, in a porous media, however, there is an inertial contribution each time the liquid is accelerated. These acceleration events, sometimes observed in the most extreme cases as the Haines-jumps, are well-known and observable microscopically [15]. These workers claimed that such modified equations considering the inertial terms, gave improved results [22-26].

5 - For large times, an algebraical equation is sometimes used to reproduce an empirical behavior during capillary rise [11-13]:

$$h = h_0 + \frac{v_0 t_0}{1-b} \left( \frac{t}{t_0} \right)^{1-b}$$

with $b < 1$. Where $b$ is a constant and at time $t_0$ the front is at position $h_0$ with velocity $v_0$.

**Artificial neural network (ANN) and the analysing capillary rise in porous media**

An artificial neural network can be considered as a black box consisting of a series of complicated equations for the calculation of outputs based on a given series of input values. A typical artificial neural network system is depicted in Fig. 1. Therefore, the task of the network is to perform a set of mappings between its inputs and outputs.
Artificial neural networks consist of collections of connected processing elements or neurons. As shown in Fig. 2, the neuron consists of an input $p$, which is multiplied by a weight $w$, and then is summed by a bias $b$. The product is computed by a mathematical function $f$ which determines a neuron output $a$. The mathematical function is called the activation or the transfer function.

Artificial neural networks are currently being extensively applied in many fields of science and engineering [27-34]. The major reason for this rapid growth and diverse application of artificial neural networks is their ability to estimate virtually any function in a stable and efficient way. Hence, they create a platform on which different models can be constructed. One of the main problems with analysing capillary rise in porous media is that the application of an exact theory is not computationally practical and an approximation to it has to be employed. It is expected that an artificial neural network approach for analysing capillary rise in porous media would offer a profitable new solution to this problem. Therefore, in the present work, the performance of a designed ANN for the prediction of the time of capillary rise was put to test; in addition to comparing its performance with other known methods.

**EXPERIMENTAL**

**Materials**

Imbibition experiments were carried out with porous columns made from fifteen packed powders. Physical and chemical properties of which are given in Ref. [35] and are summarized in table 1. The suppliers of these powders are also mentioned in table 1. The powders provide a wide range of particle size (i.e. 0.012-45 µm) and bulk density (i.e. 50-1060 kg/m$^3$). Some properties of the probe liquids used are shown in table 2. Formamide was provided by the Roth Company and all the other liquids were provided by the Merck Company. The properties of these liquids were obtained from Ref. [36]. The liquids provide a wide range of surface tension (i.e. 15.45-71.99 mJ/m$^2$) and viscosity (i.e. 0.25-21 mPa.s, excluding Glycerine).

**Capillary rise experiments**

Two different approaches for capillary rise experiments have been broadly employed: In the first approach, the weight increase as a consequence of the capillary rise in the porous media is evaluated; whilst in the second approach, evaluation is made of the length advanced by the liquid through the porous media. However, Labajos-Broncano et al. [37] has proved that the measurement of mass or length in the experimental procedure give equivalent results and have no effect in
### Table 1: Some physical properties and chemical nomenclature of the probe powders.

<table>
<thead>
<tr>
<th>Powders</th>
<th>Active component</th>
<th>Particle size (µm)</th>
<th>Bulk density (kg/m³)</th>
<th>Supplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerosil 200</td>
<td>Silica</td>
<td>0.012</td>
<td>50</td>
<td>Degussa</td>
</tr>
<tr>
<td>Printex V</td>
<td>Carbon black</td>
<td>0.025</td>
<td>190</td>
<td>Degussa</td>
</tr>
<tr>
<td>Bayferrox 130</td>
<td>Iron oxide</td>
<td>0.17</td>
<td>1050</td>
<td>Bayer</td>
</tr>
<tr>
<td>Kemira 650</td>
<td>Titanium dioxide</td>
<td>0.22</td>
<td>900</td>
<td>Kemira</td>
</tr>
<tr>
<td>Ti-Pure R-900</td>
<td>Titanium dioxide</td>
<td>0.41</td>
<td>820</td>
<td>DuPont</td>
</tr>
<tr>
<td>Penta Calcium Carbonate</td>
<td>Calcium carbonate</td>
<td>1.9</td>
<td>700</td>
<td>Omya Pars</td>
</tr>
<tr>
<td>Super Talk WT-2500</td>
<td>Magnesium silicate</td>
<td>2.4</td>
<td>500</td>
<td>Sanaye Poodre Sangsar</td>
</tr>
<tr>
<td>Baryte Ba-2500</td>
<td>Barium sulfate</td>
<td>2.4</td>
<td>1060</td>
<td>Sanaye Poodre Sangsar</td>
</tr>
<tr>
<td>Kaoline KSP</td>
<td>Aluminium silicate</td>
<td>2.4</td>
<td>440</td>
<td>Khorasan Kaoline</td>
</tr>
<tr>
<td>Kaoline KCC</td>
<td>Aluminium silicate</td>
<td>3.8</td>
<td>610</td>
<td>Khorasan Kaoline</td>
</tr>
<tr>
<td>Syloid ED 30</td>
<td>Silica</td>
<td>5.5</td>
<td>130</td>
<td>Boysan</td>
</tr>
<tr>
<td>Super Talk WT-1500</td>
<td>Magnesium silicate</td>
<td>8.3</td>
<td>630</td>
<td>Sanaye Poodre Sangsar</td>
</tr>
<tr>
<td>Super Talk WT-800</td>
<td>Magnesium silicate</td>
<td>15</td>
<td>790</td>
<td>Sanaye Poodre Sangsar</td>
</tr>
<tr>
<td>Milky Calcium Carbonate-800</td>
<td>Calcium carbonate</td>
<td>15</td>
<td>820</td>
<td>Micronized Powder</td>
</tr>
<tr>
<td>Zinc Oxide Z-1</td>
<td>Zine oxide</td>
<td>45</td>
<td>620</td>
<td>Farayand Powder</td>
</tr>
</tbody>
</table>

### Table 2: Some properties of the probe liquids.

<table>
<thead>
<tr>
<th>Liquids</th>
<th>Density (kg/m³)</th>
<th>Surface tension (mJ/m²)</th>
<th>Viscosity (mPa.s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethane-1,2-diol</td>
<td>1110</td>
<td>47.99</td>
<td>21</td>
</tr>
<tr>
<td>2,6,10,15,19,23- Hexamethyltetraocosa-2,6,10,14,18,22-hexaene</td>
<td>860</td>
<td>31.85</td>
<td>15</td>
</tr>
<tr>
<td>Hexadecane</td>
<td>770</td>
<td>27.05</td>
<td>3.59</td>
</tr>
<tr>
<td>Formamide</td>
<td>1130</td>
<td>57.03</td>
<td>3.31</td>
</tr>
<tr>
<td>Diiodomethane</td>
<td>3320</td>
<td>66.18</td>
<td>2.8</td>
</tr>
<tr>
<td>Propan-2-ol</td>
<td>800</td>
<td>20.93</td>
<td>2.2</td>
</tr>
<tr>
<td>Nitrobenzene</td>
<td>1200</td>
<td>43.45</td>
<td>1.86</td>
</tr>
<tr>
<td>Distilled water</td>
<td>1000</td>
<td>71.99</td>
<td>1</td>
</tr>
<tr>
<td>Decane</td>
<td>730</td>
<td>23.54</td>
<td>0.81</td>
</tr>
<tr>
<td>Chlorobenzene</td>
<td>1110</td>
<td>32.99</td>
<td>0.8</td>
</tr>
<tr>
<td>Methanol</td>
<td>790</td>
<td>22.07</td>
<td>0.60</td>
</tr>
<tr>
<td>Ethyl acetate</td>
<td>900</td>
<td>23.39</td>
<td>0.44</td>
</tr>
<tr>
<td>Butanal</td>
<td>800</td>
<td>24.36</td>
<td>0.41</td>
</tr>
<tr>
<td>Acetone</td>
<td>790</td>
<td>23.46</td>
<td>0.32</td>
</tr>
<tr>
<td>Pentane</td>
<td>630</td>
<td>15.45</td>
<td>0.25</td>
</tr>
<tr>
<td>Glycerine *</td>
<td>1230</td>
<td>62.82</td>
<td>1412</td>
</tr>
</tbody>
</table>

*a) Glycerine was only used to determine the surface free energy of Printex V (i.e. Carbon black powder) since the times of capillary rise for the other liquids were too small.*
the final outcome. In this work, the distance-time imbibition was employed.

The apparatus, depicted in Fig. 3, was used for the capillary rise experiments and consisted of a reservoir of a certain liquid and a graduated vertical cylindrical tube supported by a clamp. The cylindrical tube was in fact a 10 -ml volumetric pipette with the bottom tip cut off. Glass wool was packed into the bottom of the tube to act as a support bed for each powder. The tube was then filled with a known mass of a given powder. The tube was then cleaned carefully with a cleansing solution followed by repeated rinsings with distilled water and finally with acetone. The tube was then thoroughly dried in an oven. All column wicking were performed at the temperature of about 25±1 °C. Each reported value is the average of three independent measurements for liquids for which small standard deviations were observed. In the case of liquids where large standard deviations were observed, the reported value is the average of five to six measurements.

**Determination of surface free energy of powders**

Since the surface free energy of powders \( \gamma_s \) plays a critical role in analysing the diffusion of liquids into porous media and its value is usually not provided by the supplier, then it had to be determined separately. This was carried out with reference to eq.(5) from which the following can be interpreted:

\[
\frac{h^2}{2\eta t} = \frac{K \gamma_{lv} \cos \theta}{2\eta t}
\]  

(12)

It is evident from eq. (12) that \( 2\eta h^2/t \) equals \( K \gamma_{lv} \cos \theta \) (where \( K = c \tilde{r} \) ), therefore a plot of \( 2\eta h^2/t \) versus the liquid surface tension \( \gamma_{lv} \) of the test liquids used, would give valuable information regarding the surface tension of the solid-vapor. In fact this plot will show a maximum which is analogous to the solid-vapor surface tension of the investigated material [38-43]. It must be noted that the penetration time for each liquid to reach a certain height for each powder was considered.

Figs. 4 to 8 show the normalized wetting rates \( 2\eta h^2/t \) plotted as a function of their corresponding liquid surface tensions. Such plots show a maximum, where the \( \gamma_{lv} \) equals the \( \gamma_s \). In these Figs., the calculated values were connected to their neighbors by smoothing spline fitting of the Curve Fitting Toolbox of MATLAB 7.0 [44]. The estimated surface free energies are also summarized in table 3.

**Time prediction of capillary rise using an artificial neural network**

The Neural Network Toolbox of MATLAB 7.0 was used for the prediction of the time of capillary rise. The

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**Fig. 3: Schematic representation of the apparatus for the capillary rise experiments.**
Fig. 4: Plots of the normalized wetting rates as a function of the liquid surface tensions for: Penta Calcium Carbonate (●); Ti-Pure R-900 (♦); Super Talk WT-800 (▲); Zinc Oxide Z-1 (▲).

Fig. 5: Plots of the normalized wetting rates as a function of the liquid surface tensions for: Milky Calcium Carbonate-800 (●); Syloid ED 30 (♦); Super Talk WT-1500 (▲); Super Talk WT-2500 (▲).

Fig. 6: Plots of the normalized wetting rates as a function of the liquid surface tensions for: Aerosil 200 (●); Kaoline KSP (♦); Kaoline KCC (▲); Kemira 650 (▲).

Fig. 7: Plots of the normalized wetting rates as a function of the liquid surface tensions for: Baryte Ba-2500 (●); Bayferrox 130 (♦).

Fig. 8: Plot of the normalized wetting rates as a function of the liquid surface tensions for: Printex V (●).

Fig. 9: Plot of MSE versus number of epochs for both set of training and testing data for the designed ANN.
input layer of the network consisted of density, surface tension, and viscosity for the liquids and particle size, bulk density, packing density, and surface free energy for the powders. The output layer of the network corresponded to the time of capillary rise in order to reach a given height (i.e. 0.036 m).

Three initial preexamination steps were undertaken prior to the prediction by ANN.

Step one was carried out in order to determine the significant effect of the powders and liquids on the time of capillary rise. A two-way ANOVA (ANalysis Of VAriance) of the Statistics Toolbox of MATLAB 7.0 was used for this purpose. For a 95 percent confidence limit, significant values of less than 0.05 for the powders and liquids showed that both influence the time of capillary rise.

In the second step, the Curve Fitting Toolbox of MATLAB 7.0 was used to correlate each variable as the network input to the time of capillary rise. The results showed that use of only one variable did not correlate well with the time of capillary rise.

In the third and final step, the principal components analysis (PCA) of the Statistics Toolbox of MATLAB 7.0, was used in order to determine the minimum number of variables required to give the maximal accurate prediction. All the possible number of variables from two to a maximum of 7 were used. In this investigation, the results showed that compression of data was not possible and only when all seven variables were used, the errors were minimized.

Since the number of inputs were determined by statistical means, the next step of the work was to determine the most appropriate architecture for the network. Several networks were created, trained, and tested. The number of layers, the optimum number of neurons per layer, and the transfer function(s) in the hidden layer(s) for each network were obtained by trial and error. Care was taken to avoid overtraining. In a sense, overtraining indicates that the ANN adapts itself too well to the training data in such a way that further improvements based on the training data would not only impair further improvements in the predictions of the testing data but it could also have adverse effects on the predictions. Therefore, the number of weights and biases in the network must not exceed the number of data in the training data set. The 154 times of capillary rise recorded were in the range 0 to 5000 s. These times were divided in two groups: a training set (136 times of capillary rise) and a testing set (18 times of capillary rise). Both training and test sets were selected so as to cover that same range. Therefore, the performance of the ANNs was found to be independent of choice of the powders or the liquids. Networks were trained using the Levenberg-Marquardt back propagation algorithm. A linear activation function was used in output layer of the networks. All the input and output data were normalized to the interval [-1 to 1] before training and testing.

The performance factor (PF/3) and the product moment correlation coefficient ($r^2$) were used in order to assess the accuracy of the actual times in comparison with the times predicted by the trained networks. The performance factor (PF/3) was derived by Guan et al. [45] and in fact combines three different statistical measures of fit (suitably weighted) into a single numerical value which is given in eq.(13). This facilitates ease of comparison by declaring the errors involved in the predictions, as follows:

$$PF/3 = \frac{100 \times \left[ \frac{\gamma + V_{AB}}{100} + CV - 1 \right]}{3}$$

where $CV$ is the coefficient of variation, $\gamma$ was proposed by Coates et al. [46], and $V_{AB}$ was derived by Schultz [47].

It must be noted that for a perfect correlation $r^2 = 1$ and PF/3 = 0. An epoch is defined as the presentation of the set of training (input and/or output) to a network and the calculation of new weights and biases. Since the initial values of weights and biases are initiated from random values and the number of epochs were prefixed at 1000, then it is obvious that each run might produce different output values. Therefore, each artificial neural network was made to run five times and the average value for $r^2$ and PF/3 were reported. Finally, the ANN with a $7 \times 12 \times 1$ architecture having a tan-sigmoid transfer function in its single hidden layer had the maximum $r^2$ value and the minimum PF/3 value, therefore this network was selected for the rest of the work. Fig. 9 shows the learning progress of the selected ANN with time.
Table 3: Estimated surface free energy of the probe powders.

<table>
<thead>
<tr>
<th>Powders</th>
<th>Surface free energy (mJ/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kemira 650</td>
<td>35.61</td>
</tr>
<tr>
<td>Super Talk WT-2500</td>
<td>28.05</td>
</tr>
<tr>
<td>Baryte Ba-2500</td>
<td>26.83</td>
</tr>
<tr>
<td>Penta Calcium Carbonate</td>
<td>26.13</td>
</tr>
<tr>
<td>Aerosil 200</td>
<td>58.34</td>
</tr>
<tr>
<td>Printex V</td>
<td>62.19</td>
</tr>
<tr>
<td>Syloid ED 30</td>
<td>27.79</td>
</tr>
<tr>
<td>Tri-Pure R-900</td>
<td>23.96</td>
</tr>
<tr>
<td>Bayferrox 130</td>
<td>26.02</td>
</tr>
<tr>
<td>Zinc Oxide Z-1</td>
<td>25.54</td>
</tr>
<tr>
<td>Kaoline KSP</td>
<td>27.38</td>
</tr>
<tr>
<td>Super Talk WT-800</td>
<td>27.94</td>
</tr>
<tr>
<td>Milky Calcium Carbonate-800</td>
<td>28.27</td>
</tr>
<tr>
<td>Kaoline KCC</td>
<td>29.17</td>
</tr>
<tr>
<td>Super Talk WT-1500</td>
<td>28.06</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION

In the present study attempts were made to compare the experimentally determined times of capillary rise with their equivalence as predicted by a proposed ANN approach and additionally as commonly calculated by the Lucas-Washburn’s equation (eq.(2)) as well as six of its modifications (see eqs. (4), (5), (6), and (7)). Some of this comparison is summarized in table 4. Also, the ranked comparison was carried out by the aid of the performance factor (PF/3) and the product moment correlation coefficient ($r^2$) as shown in table 5. The maximum ($\Delta t_{\text{max}}$) and average ($\Delta t_{\text{ave}}$) deviations of predicted or calculated times of capillary rise from those determined experimentally are also shown in table 5.

It is evident from table 5 that the proposed approach of artificial neural network was able to predict the time of capillary rise with excellent accuracy (i.e. $r^2 = 0.91$, PF/3 = 55, $\Delta t_{\text{max}} = 1802$ s, $\Delta t_{\text{ave}} = 20$ s). The Lucas-Washburn’s equation, however, gave the worst correlation (i.e. $r^2 = 0.11$, PF/3 = 1016, $\Delta t_{\text{max}} = 670851$ s, $\Delta t_{\text{ave}} = 12503$ s). Furthermore, some of the modifications of this equation as proposed by different workers did not seem to conspicuously improve the relationships giving a range of inferior correlations compared to the proposed ANN approach between the calculated and experimentally determined times of capillary rise (i.e. $r^2 = 0.24$ to 0.44, PF/3 = 129 to 293, $\Delta t_{\text{max}} = 4107$ s to 4364 s, $\Delta t_{\text{ave}} = 276$ s to 727 s). Additionally, it must be noted that the common approach of calculating the time of capillary rise by the Lucas-Washburn’s equation or its proposed modifications have the inherent weaknesses of requiring tedious experimental procedures as well as rigorous and complex mathematical treatments. Some of which are discussed below:

1- The determination of the contact angle in eqs. (2), (4), (5), and (7) is usually carried out by three methods: i) Direct measurement of the contact angle of a liquid droplet on the surface of a powdered solid. The main drawback of this method is that compressed pellets of powdered solids usually give a rough and porous surface causing the measured contact angle to be smaller than when a smooth surface of the solid would have been produced [18].

ii) Eq. (5) could be used to determine the contact angle. However, before the contact angle could be determined from such an equation, $c \Gamma$ must be estimated. This usually involves a separate calibration procedure. In one procedure [17], the wetting experiment is initially conducted with a liquid which completely wets the solid. For such a liquid the value of $\cos \theta$ is unity, thus $c \Gamma$ is directly yielded from eq. (5). The estimated value of $c \Gamma$ is now assumed to hold for all other liquids wetting the porous media. Suppose that such an assumption was valid, this would involve the onerous task of finding a liquid that completely wets all of the chemically different fractions (organic and inorganic) that are bound to be present in a heterogeneous substance [20]. In a second procedure [48], $c \Gamma$ is determined through the use of mercury. The contact angle of mercury for most solids is known to lie between 130° to 140°. Therefore a value in this range is assumed and $c \Gamma$ is calculated from mercury porosimetry data. The main drawback of this procedure is that mercury does not spontaneously wet solids for which the contact angles are greater than 90° [20]. Mercury would have to be forced in with an externally applied pressure, and the extent to which it penetrates the porous media would depend on the applied pressure. Therefore, the assumption that porosity is constant, implicit to the assumption of constant $c \Gamma$ would clearly not be valid [20].
### Table 4: Comparison of the experimentally determined times of capillary rise ($t_1$) equivalently predicted by the ANN ($t_2$) and correspondingly calculated by various equations ($t_3$ to $t_9$)

<table>
<thead>
<tr>
<th>Run no.</th>
<th>$t_1$(s)</th>
<th>$t_2$(s)</th>
<th>$t_3$(s)</th>
<th>$t_4$(s)</th>
<th>$t_5$(s)</th>
<th>$t_6$(s)</th>
<th>$t_7$(s)</th>
<th>$t_8$(s)</th>
<th>$t_9$(s)</th>
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<td>86</td>
<td>66</td>
<td>69</td>
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<td>42</td>
<td>63</td>
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<td>64</td>
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<tr>
<td>14</td>
<td>142</td>
<td>179</td>
<td>14</td>
<td>189</td>
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<td>182</td>
<td>105</td>
<td>189</td>
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<tr>
<td>24</td>
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<td>345</td>
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<td>50</td>
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<td>2402</td>
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<td>182</td>
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<td>55</td>
<td>55</td>
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<td>49</td>
<td>51</td>
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<tr>
<td>84</td>
<td>667</td>
<td>625</td>
<td>132</td>
<td>1812</td>
<td>1812</td>
<td>735</td>
<td>1323</td>
<td>2082</td>
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<td>94</td>
<td>358</td>
<td>357</td>
<td>96</td>
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<td>389</td>
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<td>295</td>
<td>479</td>
<td>389</td>
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<tr>
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<td>1411</td>
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<td>38</td>
<td>527</td>
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<td>295</td>
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<td>2929</td>
<td>2472</td>
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<td>171</td>
<td>681</td>
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<tr>
<td>154</td>
<td>3163</td>
<td>3155</td>
<td>132</td>
<td>504</td>
<td>504</td>
<td>146</td>
<td>252</td>
<td>995</td>
<td>496</td>
</tr>
</tbody>
</table>

$t_1$: Times of capillary rise, experimentally determined; $t_2$: Times of capillary rise as predicted by the ANN; $t_3$: Times of capillary rise as calculated by eq. (2); $t_4$: Times of capillary rise as calculated by eq. (4); $t_5$: Times of capillary rise as calculated by eq. (5); $t_6$: Times of capillary rise as calculated by eq. (5) taking into consideration the gravity; $t_7$: Times of capillary rise as calculated by eq. (5) taking into consideration eq. (10); $t_8$: Times of capillary rise as calculated by eq. (6); $t_9$: Times of capillary rise as calculated by eq. (7).

iii) The contact angle can also be obtained by the use of the Young's equation with a slight modification as follows [49]:

$$
\gamma_s = \frac{(\gamma_{lv} - 0.8)(\cos \theta + 1)}{2}
$$

(14)

In this investigation, the recent approach was used to determine the contact angle in eqs. (2), (4), (5), and (7).

2- The porosity and packing of the porous media, and the hydraulic radius required additional calculations.

3- For each powder, the constants in eqs. (4), (5), and (7) namely $n_{eDarcy}$, $c_T$, and $K$ were determined graphically by the use of the average slope of the straight line fit by $h^2$-$t$ diagrams.

4- Consideration of gravity and dynamic contact angle also required further calculations.

It must be noted that the phenomenological evaporation rate was not considered and since the recorded times were relatively small, eq. (11) was not considered, as per recommendations [11]. In contrast, however, the proposed artificial neural network approach has the additional advantages listed below:

i) There is no need for a mathematical model relating the time of capillary rise to any one or several parameters.

ii) Further assumptions regarding the effectiveness of additional phenomena such as inertia, evaporation of liquid, gravity, dynamic contact angle, adsorption of liquid by the solid surface, etc. need not be considered.

iii) Additional tedious experimentation need not be carried out.

iv) Rigorous mathematical treatments are not required.
Table 5: The variation in the accuracy of the estimated times of capillary rise by the ANN and the various equations in terms of $r^2$, PF/3, $\Delta t_{\text{max}}$ and $\Delta t_{\text{ave}}$.

<table>
<thead>
<tr>
<th></th>
<th>$r^2$</th>
<th>PF/3</th>
<th>$\Delta t_{\text{max}}$ (s)</th>
<th>$\Delta t_{\text{ave}}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANN</td>
<td>0.91</td>
<td>55</td>
<td>1802</td>
<td>20</td>
</tr>
<tr>
<td>Eq. (2)</td>
<td>0.11</td>
<td>1016</td>
<td>670851</td>
<td>12503</td>
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<tr>
<td>Eq. (4)</td>
<td>0.31</td>
<td>164</td>
<td>4108</td>
<td>390</td>
</tr>
<tr>
<td>Eq. (5)</td>
<td>0.31</td>
<td>164</td>
<td>4107</td>
<td>390</td>
</tr>
<tr>
<td>Eq. (5) taking into consideration the gravity</td>
<td>0.24</td>
<td>293</td>
<td>4364</td>
<td>727</td>
</tr>
<tr>
<td>Eq. (5) taking into consideration eq. (10)</td>
<td>0.24</td>
<td>216</td>
<td>4214</td>
<td>582</td>
</tr>
<tr>
<td>Eq. (6)</td>
<td>0.44</td>
<td>129</td>
<td>4244</td>
<td>276</td>
</tr>
<tr>
<td>Eq. (7)</td>
<td>0.27</td>
<td>169</td>
<td>4108</td>
<td>349</td>
</tr>
</tbody>
</table>

v) All the parameters required by the ANN are either available or easily obtainable.
vi) Further improvements to the ANN approach could be envisaged by the use of hybrid systems involving genetic algorithms and fuzzy logic.

CONCLUSIONS
The prediction of time of capillary rise in porous media is a complicated task. Previously proposed and commonly used approaches require many tedious experimentations, as well as rigorous and complex mathematical treatments. Even so these approaches leave much to be desired in terms of the accuracy of predictions. The new approach of artificial neural network proposed in the present investigation avoids the problems mentioned above. Furthermore, it provides a simple and direct means by which accurate and consistent predictions of time of capillary rise are facilitated. The proposed artificial neural network is capable of doing all this since its overall performance is independent of abnormalities in the experimental data or non-conformity to a certain mathematical model.

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