Effect of Bed Diameter on the Hydrodynamics of Gas-Solid Fluidized Beds

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ABSTRACT: *Effect of scale on the hydrodynamics of gas-solid fluidized beds was investigated in two fluidized beds of 152 mm and 78 mm in diameter. Air at room temperature was used as the fluidizing gas in the bed of sand particles. The Radioactive Particle Tracking (RPT) technique was employed to obtain the instantaneous positions of the particles at every 20 ms of the experiments. These data were used to calculate hydrodynamic parameters, such as mean velocity of upward and downward-moving particles, jump frequency, cycle frequency and axial and radial diffusivities, which are representative of solid mixing and diffusion of particles in the bed. These hydrodynamic parameters were compared in both scales in order to determine the scale effect on the hydrodynamics of the gas-solid fluidized bed. In all cases, it was shown that solid mixing and diffusivity of particles increase by increasing column diameter. The results of this study would help to understand solid mixing which might be critical in industrial fluidized bed reactors.*

KEY WORDS: *Scale effects, Fluidized bed, Radioactive Particle Tracking (RPT).*

INTRODUCTION

Fluidized bed reactors have been used in many catalytic and chemical processes. In general, these reactors are scaled-dependent and their scale-up has been a challenge since the advent of using such reactors. In general, information obtained from small or pilot-scale units must be used for designing commercial units.

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One discouraging problem would be the decrease in reactor performance when a pilot plant is scaled up to a larger commercial one. This can be related to undesirable gas flow patterns and poor solid mixing [1].

The legendary scale-up problem was that of unsuccessful scale-up of the first bubbling-bed reactor in 1950 in Brownsville, Texas. Two 5m diameter reactors were constructed based on the results of small (0.3 m dia) pilot plant tests with a Geldart Group B iron catalyst. The catalyst slugged in the pilot plant reactors, but when added to the large commercial reactors, the void (bubble) size increased significantly and bubble velocity increased proportionately with bubble size. This caused a corresponding decrease in gas residence time in the commercial reactor and result in production of plant that was lower than the laboratory and pilot-scale [2].

The major difficulties in the scale-up are that the hydrodynamic behavior of the fluidized bed varies with the scale. If mixing rates and gas-solid contact efficiencies are kept constant between beds of different size, then thermal characteristics and chemical reaction rates should be similar. However, in general, the bed hydrodynamics would not remain similar. In other words, the flow regime may be different in small and large beds using the same particles, superficial gas velocity and solids circulation rate. The issue of scale-up involves an understanding of these hydrodynamic changes and how they, in turn, influence chemical and thermal conditions by variations in gas-solid contact, residence time, solid circulation, solid mixing and gas distribution.

In order to avoid large upset in product yields as well as delays in the commissioning of commercial reactors, it is essential to detect and identify the hydrodynamic parameters that vary at different scales [2, 3, 4]. In the previous study, it was shown that Radioactive Particle Tracking (RPT) data could be used to measure hydrodynamic parameters [4, 5].

The good solid mixing, high mass and heat transfer rate and also isothermal condition, caused by rapid mixing of solids and solid diffusion are some of the significant advantages of fluidized beds in comparison to fixed-beds. Therefore, in this work the time-position data obtained by the RPT technique were used at two different scales to investigate the effect of scale on hydrodynamic parameters related to solid mixing and diffusivity through the bed.

Table 1: Properties of solids used in experiments.

Solid	ρ_s (kg/m^3)	d_s (μm)	U_c (m/s)	U_{mf} (m/s)
Sand	2650	385	1.5	0.24
Sand	2650	250	1.0	0.076

EXPERIMENTAL SECTION

The experiments were carried out in gas-solid fluidized beds which were made of Plexiglas pipe with 152 mm and 78 mm inside diameter and 1500 mm and 750 mm height, respectively. The properties of solids used in the experiments are shown in Table 1. Air at room temperature was injected into the bed and its flow rate was measured by an orifice plate connected to a water manometer. A cyclone placed at the air outlet of the column returns the entrained solids back to the freeboard.

Two types of tracers were used in the experiments. In the experiments in the 78 mm column, the tracer was made of scandium oxide and in the experiments in the 152 mm column the tracer was a mixture of gold powder and epoxy resin with a density and size close to those of the bed material. They were being activated in the SLOWPOKE nuclear reactor of École Polytechnique up to about 3.67 MBq (100 μCi*)* prior to each experiment. The produced isotope ^{198}Au emits *γ*-rays which were counted by 16 cylindrical NaI(Tl) scintillation detectors on sliding rails. Two personal computers simultaneously registered the number of *γ*-rays detected by each detector in every sampling period of 20 ms. These number of counts were used later to calculate the coordinates of the tracer. Details of the system calibration and the inverse reconstruction strategy for tracer position rendition are described by *Larachi et al.* [6, 7]. Fig. 1 shows a schematic presentation of reactor and detectors around the bed.

In each experiment a single tracer was placed into the bed to move freely with the other particles. Movement of the tracer was then monitored for about 3.5 h during which the count rates of the tracer at some 615000 points were acquired. The bed was then made empty and the tracer was recovered from the solids in order to be used in the next experiment. A typical run of experiments involved: (a) preparation of the tracer; (b) start-up of the fluidized bed system; (c) start-up of the reactor and measurement system; (d) introduction of the tracer into the bed and collecting the γ-ray counts by each detector; and (e) reconstructing the tracer position.

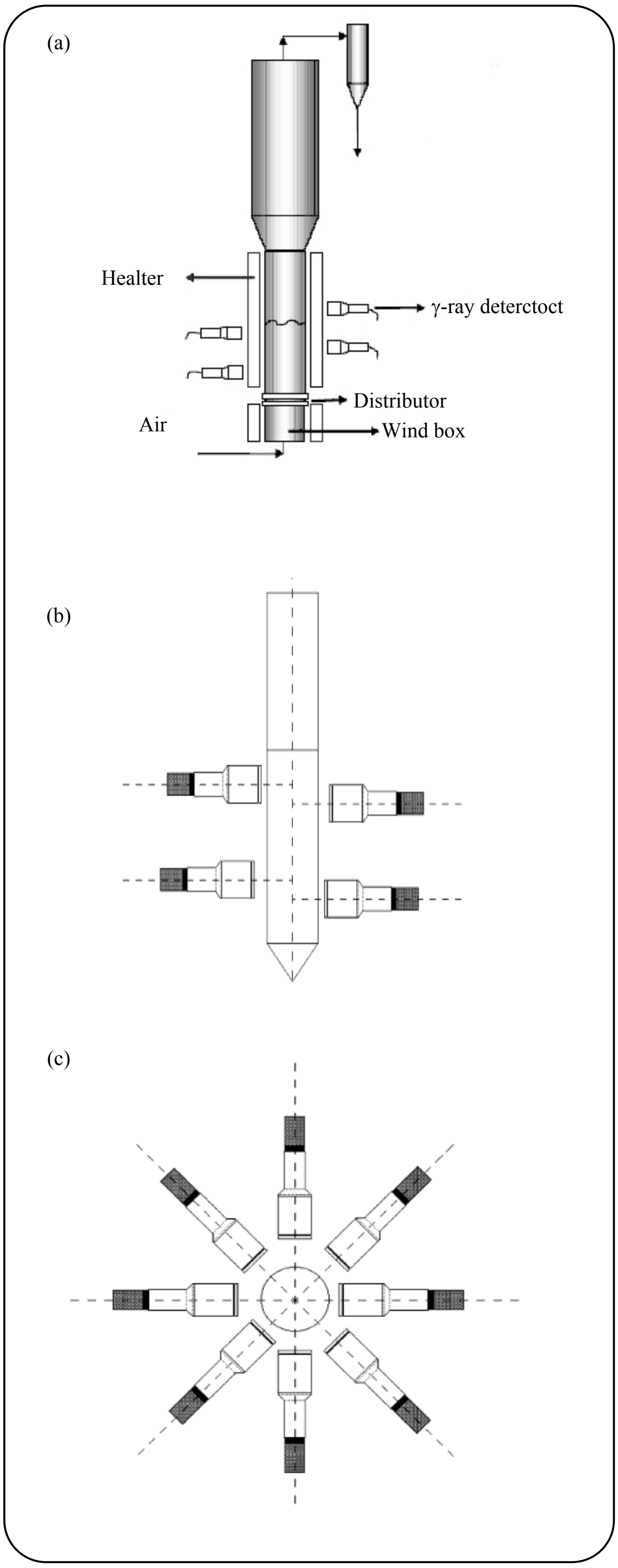

Fig. 1: (a) Schematic illustration of fluidized bed reactor (b) Top view of typical configuration of the detectors around the bed (c) Side view of typical configuration of the detectors around the bed.

TREATMENT OF DATA

The obtained time-position data from RPT is used to determine some hydrodynamic parameters which are representative to solid mixing and diffusion of particles through the bed as followings.

Mean Velocity of Upward and Downward-Moving Particles

The particles in the gas–solid fluidized bed usually do not move as single and isolated particles [8,9] but they do as clusters. They may move partly in emulsion, ascending cluster, descending cluster, bubble wake and the bubble induced drift [5, 10]. The particles in the emulsion phase can form clusters, moving either upward or downward. In a bubbling fluidized bed, the bubbles of constant size rise at constant velocity. Therefore, the tracer particle rising inside the bubble wake or cloud would exhibit a constant velocity as the bubble and represents the bubble rise velocity. Moreover, upward movement of bubbles induces a net upward movement of emulsion in the region surrounding the bubble. Consequently, the particles located in the vicinity of rising bubbles can also show persistent upward movement. Therefore, the particles in the bed can be categorized into two main groups including upward-moving (bubble wake and ascending clusters) and downward-moving (descending clusters) particles. The above idea forms the basis of the algorithm for recognizing the bubbles and clusters among the trajectories in the RPT experiments, as fully reported by *Mostoufi & Chaouki* [5]. The bubbles and clusters detected in this way, have reached either their equilibrium size or the change in their size was negligible in the period of observation. A computer program was developed based on their algorithm. This program finds the linear segments of the trajectory and calculates the velocity from their slope and finally the velocity distribution of these movements is obtained.

As an example, Fig. 2 shows velocity distribution of upward and downward particles for the superficial gas velocity of 0.5 m/s and in the bed of 78 mm in diameter. As can be seen in this figure, there are two distinguishable segments in this distribution, corresponding to velocities of downward and upward-moving particles. Evidently, the left peak with negative velocity corresponds to the downward-moving clusters and the right peak with positive velocity belongs to the upward-moving particles. Therefore, ascending bubbles/clusters and descending clusters in the fluidized beds could be recognized by plotting the

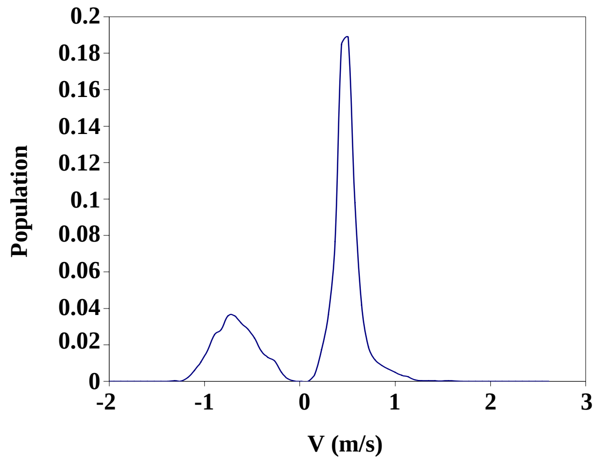

Fig. 2: Velocity distribution of ascending and descending particles at U_0 = 0.5 m/s and ID = 78 mm.

velocity distribution of these species. At the end, based on velocity distributions, mean velocities of upward and downward-moving particles were calculated.

Jump Frequency

Jump time, as proposed by *Stein et al.* [8], is the time that a particle spends in the wake of a bubble. According to *Stein et al.* [8], a jump is defined as a vertical upward movement of a particle with a vertical velocity superior than 0.1 mm/s and a magnitude greater than 24mm. Since jump is a short upward motion of solids in bed, the jump time is a characteristic of the local solid mixing in the fluidized bed. A computer program was developed based on the above described conditions. This program finds the segments of the trajectory which have upward movement with the velocity superior that 0.1mm/s and displacement greater than 24 mm. Since jump is considered as a short upward movement, the minimum number of points in each set has been set to 2. It means that if the height of second point is 24 mm or much greater than first one and slope of line which connect these two points is greater than 0.1mm/s, this would be a jump motion. In such a case, subsequent data points were added one by one to the set of two points in order to check if these subsequent points also belong to the same conditions of segment of trajectory. This addition of subsequent points to the segment continues until addition of another point would lower the slope of the line to less than 0.1 mm/s or displacement lower than 24mm. When the selected two data points don't have any of three mentioned conditions, the program would then continue the search by performing the same calculation for the next subsequent set of 2 data points**.** Based on what was explained**,** the number of jumps and their duration can be evaluated.

Cycle Frequency

The circulation rate and consequently, mixing of solids are caused by the upward motion of bubbles in fluidized beds. The upward movement of solid is compensated by the downward movement in emulsion within the region close to the wall. The cycle time is thus a characteristic of axial solid mixing in the fluidized bed which has been defined by *Stein et al.* [11] as the circulation of the tracer from below 30% to higher than 70% of the bed height and back. In other words, the tracer was pursued until it reached the bottom 30% level of the bed, followed afterwards till it reached the top 70% level of the bed and then returned to the bottom. The schematic of such motion is shown in Fig. 3. Based on this definition, a computer program was developed to detect the cycles through the bed. This program finds the first data point with the height lower than 30% of dense bed height. After finding the first point with this condition, subsequent data points are added one by one till reaching the point with the height higher than 70% of dense bed height. By reaching to the point, adding of subsequent data points is continued till achieving the point which has the height near the start point of this detected cycle (below 30% of dense bed height). The program would then continue the search by performing the same calculation for the next data points. As a sample, Fig. 4 shows a detected cycle from trajectory data points.

Solid Diffusivity

Solid diffusivity can be evaluated from the particle trajectory in the Lagrangian coordinates. Details of this method have been described *by Mostoufi & Chaouki* [8]. In order to obtain axial and radial diffusivity of the solids as a function of axial and radial positions, a large number of solid tracers should be injected in a small cell in the bed, first. However, there was only one tracer in the RPT experiments. Therefore, it is essential to employ the principle of ergodicity for processing the data obtained in the RPT experiments, as described by *Mostoufi & Chouki* [8]. Based on the principle of ergodicity, the time averaged motion of particles is the same for all initial points or, as statistical point of view, the system that evolves for a long time forgets its initial state. Using their method, a sample reconstruction of self-diffusion of 1000 solid particles vs. time is shown in Fig. 5. The mean-square axial and radial displacements of the points shown in Fig. 5 are plotted in Figs. 6 a-b, respectively. It can be seen from these figures that at the beginning, the mean-square displacement is increased linearly, from slope of which the diffusivity could be evaluated [8]. However, after a long-enough time, the tracers are completely mixed in the bed forming a uniform mixture. The tracers are in the dynamic equilibrium at this time and no more diffusion takes place after that due to the fact that the concentration gradient has vanished.

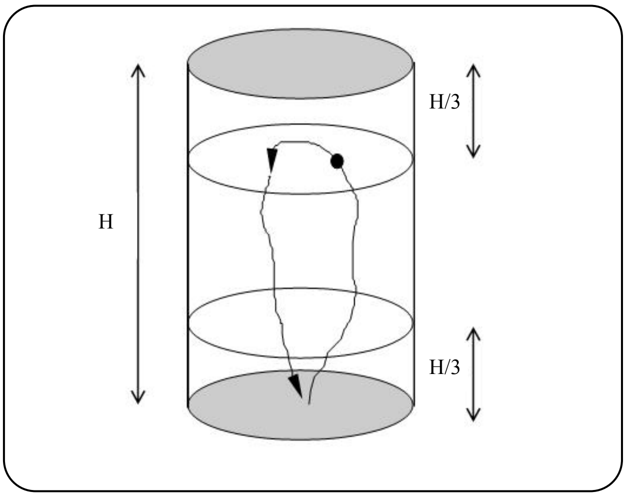

Fig. 3: Schematics of a cycle motion in the bed.

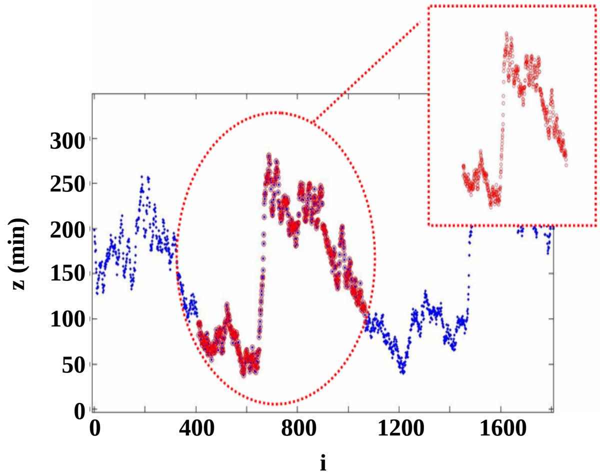

Fig. 4: Sample of a detected cycle through the bed.

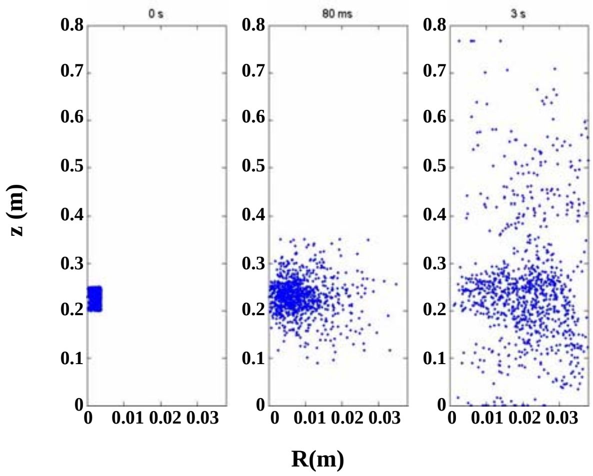

Fig.5: Self-diffusion of 1000 tracers virtually injected in the imaginary compartment (U_0=76 mm/s, ID=78mm).

RESULTS AND DISCUSSION

The effect of scale and superficial gas velocity on hydrodynamic parameters is presented and discussed in the followings.

Mean velocity of Upward and Downward-moving Particles

The velocities of all upward or downward-moving clusters and bubbles were found by the above-described algorithm for each experiment. The velocity distribution functions for all experiments were then determined. The effect of scale on the hydrodynamics of fluidized beds was investigated based on the mean velocity of bubbles, upward and downward moving clusters. Figs. 7 a-b illustrates these velocities as a function of dimensionless superficial velocity in both scales. As can be seen in these figures, the mean velocity of upward and downward-moving particles in the larger bed is higher than that of the smaller bed. The bubble and cluster velocities are also increased by an increase in the gas velocity. In fact, by increasing both the bed diameter and gas velocity, larger bubbles form in the bed, thus the velocity of upward-moving particles are increased. On the other hand, the same flow rate of solids should move back down the bed in order to keep the net flux of solids circulation equal to zero. Therefore, increasing the upward velocity of solids corresponds to the same trend in downward movement of solids.

Bubbles in a smaller diameter column tend to rise slower than bubbles in a larger diameter column due to the restraining effects of the column walls. Such wall effects can be expected to diminish with increasing column diameter. The influence of the column diameter

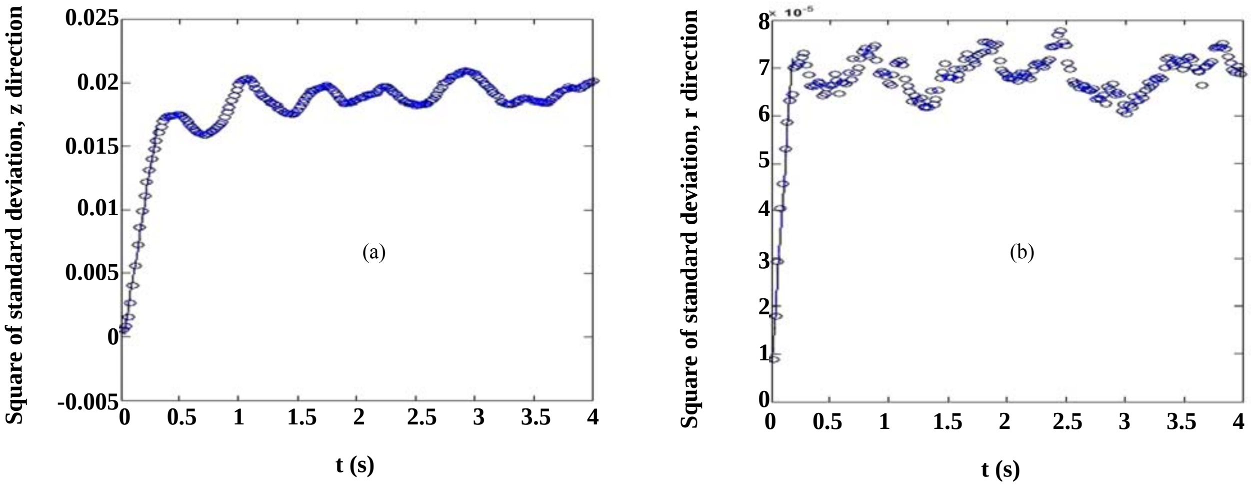

Fig. 6: Mean square displacement of 1000 tracers virtually injected in the imaginary compartment (a) Axial displacement (b) Radial displacement.

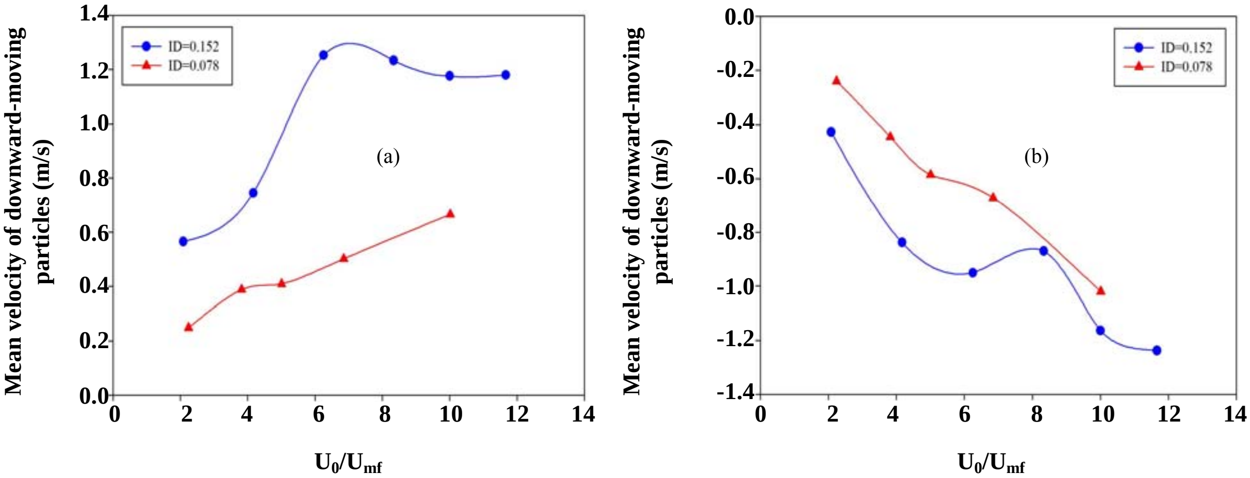

Fig. 7: Mean velocity of particles vs. dimensionless velocity: (a) upward particles (b) downward particles.

on the rise velocity of bubble in liquid has been taken into account by introducing a scale factor correction into the *Davies-Taylor* relation [7]:

$$V_b^0 = 0.71\sqrt{gd_b}\,(SF) \tag{1}$$

where the superscript 0 is used to emphasize that the rise velocity refers to that of a single, isolated bubble. *Collins* [7] determined the scale correction factor:

$$SF = \begin{cases} 1 & \dfrac{d_b}{D_T} < 0.125 \\ 1.13\exp\left(-\dfrac{d_b}{D_T}\right) & 0.125 < \dfrac{d_b}{D_T} < 0.6 \\ 0.496\sqrt{\dfrac{D_T}{d_b}} & \dfrac{d_b}{D_T} < 0.6 \end{cases} \tag{2}$$

Although Eqs. (1) and (2) were proposed for the rise of a single gas bubble in a liquid in an inviscid flow [12-17], they are also valid for rising of a single bubble in gas-solid fluidized bed [12, 14, 16, 18].

The rise velocity in a swarm of bubbles is higher than that of a single bubble due to wake interactions [15]. A bubble which enters into the wake of a preceding bubble gets accelerated. Such acceleration effects could be observed in both gas-solid and gas-liquid systems [19-22]. To consider such wake interaction effects, *Krishna et al.* [23] introduced a multiplying factor, AF, which is in fact the wake acceleration factor:

$$V_b = V_b^0(AF) \tag{3}$$

The acceleration factor AF can be expected to depend on the average distance of separation between the bubbles;

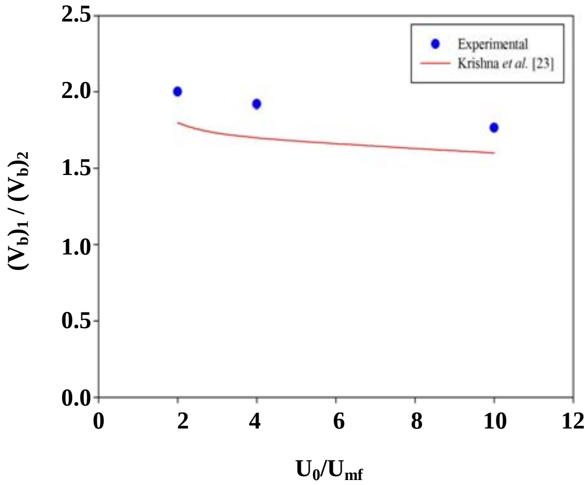

Fig. 8: Ratio of bubble velocities in two scales.

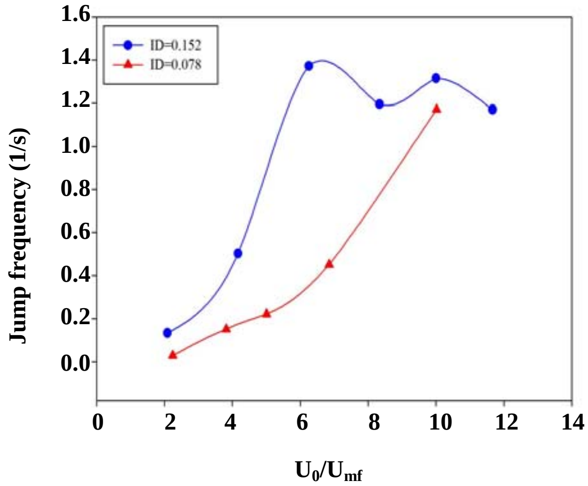

Fig. 9: Jump frequency vs. dimensionless velocity.

the smaller separation distance has greater acceleration effect. Above correlation show that the particles associated with bubble have more velocity in larger diameter. To compare the results with the correlation of *Krishna et al.* [23], the ratio of mean bubble velocity at two scales, is plotted against dimensionless velocity in Fig. 8. As can be seen in this figure, the experimental results are in close agreement with the correlation of *Krishna et al.* [23]. Based on this figure, it can be concluded that the correlation of *Krishna et al.* can predict the effect of bed diameter on bubble velocity with just 10% underestimation.

Jump Frequency

To study the effect of scale on the jump motion in bed, jump frequencies were evaluated at each scale. The jump frequency was obtained from dividing the number of jumps by the time in which these jumps were detected. In other words,

$$\text{Jump Frequancy} = \frac{N_{Jump}}{(N_{Points} - 1)\Delta t} \tag{4}$$

in which N_{jumps} is the number of jumps detected in each experiment. Estimated jump frequencies as a function of dimensionless velocity for both scales are shown in Fig. 9. This figure demonstrates that the jump frequency is increased by increasing both bed diameter and gas velocity. In fact, by increasing the upward mean velocities, particles would have more chance to participate in a jump. Another important phenomena which could be seen in Fig. 9 is that in the larger bed, jump frequency is increased up to $U_0/U_{mf} = 6.2$, after which the frequency remains approximately constant. This point can be attributed to the transition point from bubbling to turbulent regime of fluidization. It means that in the turbulent regime of fluidization, increasing the superficial gas velocity has no significant effect on the jump frequency. It was also shown that velocity and size of the bubbles remain constant or even reduce with increasing superficial gas velocity in the turbulent regime of fluidization [24-26]. Therefore, in turbulent regime, number of particles associated to the jump motion reduces due to reduction of bubble and void size and velocity.

Number of jumps as a function of jump duration is shown in Figs. 10 a-b for 152 mm and 78 mm diameter columns, respectively. These figures illustrate that the peak is increased considerably with increasing the gas velocity which indicates that at the same jump duration, number of jumps is increased by increasing the gas velocity. In fact, increasing the bubble size with increasing the superficial gas velocity results in increasing the number of particles affected by bubbles, thus, the number of particles participated in a jump increases accordingly.

Cycle Frequency

Similar to the jump frequency, the cycle frequency was obtained from dividing the number of cycles by the time in which these cycles were detected:

$$\text{Cycle Frequency} = \frac{N_{cycle}}{(N_{points} - 1)\Delta t} \tag{5}$$

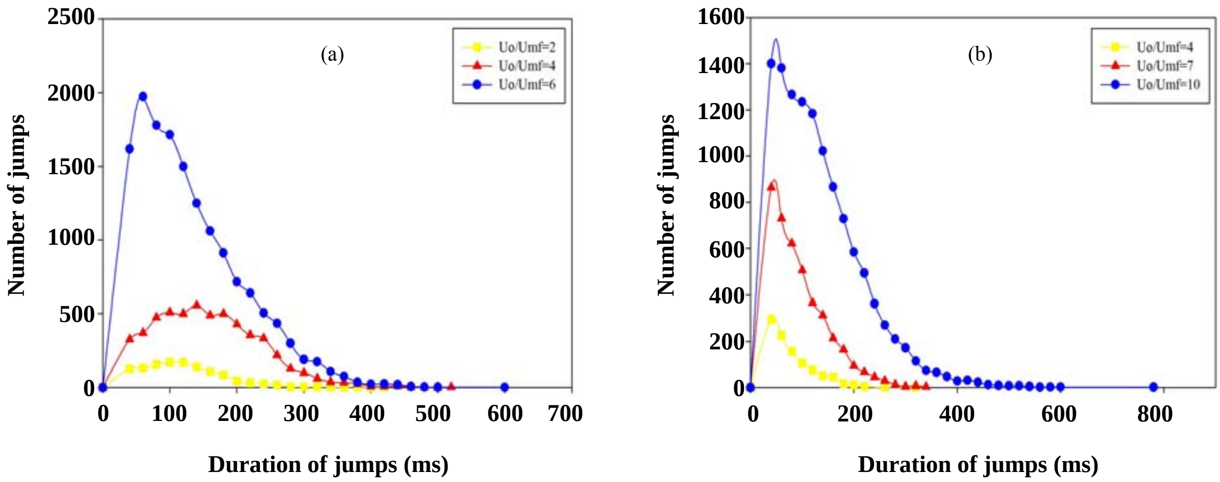

Fig. 10: Number of jump vs. duration of jump (a) ID=152mm (b) ID=78mm.

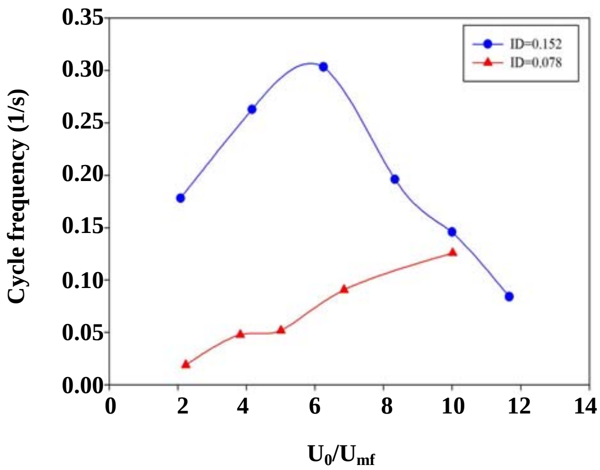

Fig. 11: Cycle frequency vs. dimensionless velocity.

Fig. 11 shows the average cycle frequency as a function of dimensionless velocity in both scales. As can be seen in this figure, the cycle frequency, which is representative of solids mixing, is increased by increasing the gas velocity up to the transition to turbulent regime. The reason for such a behavior is that higher gas velocity increases turbulency and mixing in the bed. In fact, the cycle frequency is increased due to an increase in the velocity of upward and downward-moving clusters and bubbles caused by increasing both superficial gas velocity and bed diameter. Fig. 11 also shows that for the larger column the trend of changes in the cycle frequency vs. velocity in the turbulent regime of fluidization is different from what was described in the bubbling regime. This may be attributed to the regime changes where the particles have less chance to take place from below 30% to higher than 70% of the bed height and back; consequently the cycle frequency is decreased with increasing the superficial gas velocity to the turbulent regime. Another possible reason for such a trend is that the velocity and size of bubbles are reduced in the turbulent regime of fluidization by increasing superficial gas velocity, as discussed earlier.

Solid Diffusivity

In this work, axially averaged diffusivities have been predicted and the results are presented as function of radial position only. The predicted axial and radial diffusivities in the center of the bed are shown in Figs. 12 a-b, respectively. It is apparent that diffusivities in both directions (axial and radial) are increased with increasing the superficial gas velocity. Bubbles and voids are the main reason of solids circulation in the fluidized beds. Shape, size and frequency of the bubbles influence the diffusivity of the solids. An increase in the superficial gas velocity increases the bubble velocity and bubble frequency. Therefore, both axial and radial diffusivities are increased accordingly. In addition, diffusivities in both directions are higher in the larger column. For the smaller column, the radial diffusivity is even an order of magnitude smaller than that of the larger column which indicates that the solid distribution is almost uniform in the radial direction for small columns. Lower solids diffusivity in smaller column can be attributed to the wall effect where restraining forces caused by the wall of the bed can be considered as an obstacle for mobility of particles.

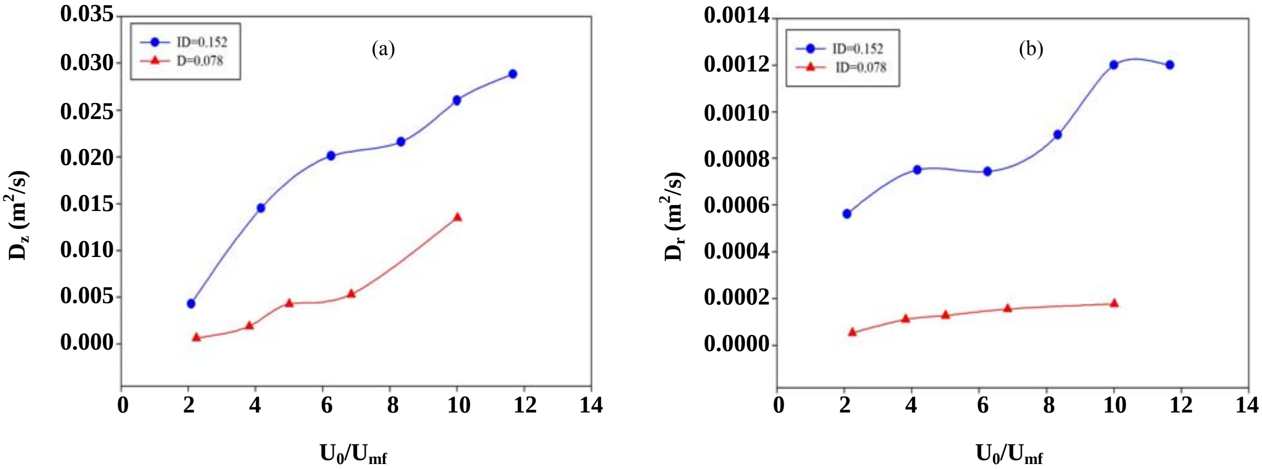

Fig. 12: Mean solid diffusivities as a function of dimensionless velocity at the center of the bed (a) axial diffusivity (b) radial diffusivity.

Therefore, in the smaller columns, in which wall effect is more significant, particles would not be able to diffuse through the bed easily.

CONCLUSIONS

In present work, the RPT technique was used to study the particle movement in fluidized beds. The time-position data of the tracer was obtained in all experiments and was used to calculate hydrodynamic parameters, such as mean velocity of upward and downward particles, jump frequency, cycle frequency and axial and radial diffusivities, which are representative of solid mixing and diffusivity through the bed. Comparison of the results obtained in different beds shows that the solid mixing and diffusion of particles are increased by increasing the column diameter. This result can be attributed to wall effect. Restraining forces caused by the bed wall, can be considered as an obstacle for mobility of particles in smaller columns. It is worth noting that those parameters that change with the scale should be subject to more investigation when designing a large scale unit. Measurements in the lab scale should be used during the scale-up with caution.

Nomenclature

d_b Bubble diameter, m
d_s Mean particle diameter, m
ID Inside diameter, m
U_0 Superficial velocity, m/s
U_{mf} Minimum fluidization velocity, m/s
U_c Superficial velocity at onset of turbulent fluidization, m/s
V Velocity, m/s
V_b Bubble velocity, m/s
V_b^0 Single bubble velocity, m/s
$(v_b)_1$ Bubble velocity in ID=0.152 m, m/s
$(v_b)_2$ Bubble velocity in ID=0.078 m, m/s
N_{jumps} Number of jumps
N_{points} Number of points
N_{cycle} Number of cycles
D_T Bed diameter, m
D_z Axial diffusivity, m^2/s
D_r Radial diffusivity, m^2/s
z Axial coordinate, m

Greek symbols

ρ_s Solid density, kg/m^3
Δt Sampling time, s

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