Pseudo Steady State Gas Flow in Tight Reservoir under Dual Mechanism Flow

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ABSTRACT: Gas reservoirs with low permeability (k<0.1 mD) are among the unconventional reservoirs and are commonly termed as "Tight Gas Reservoirs". In conventional gas reservoirs that have high permeability, the flow of gas is basically controlled by the reservoir permeability and it is calculated using the Darcy equation. In these reservoirs, gas flow due to gas diffusion is ignored compared to Darcy flow. However, diffusion phenomenon has a significant impact on the gas flow in tight gas reservoirs and the mechanism of gas diffusion can no longer be ignored in comparison to Darcy flow. In this study, a dual mechanism based on Darcy flow as well as diffusion is used for the gas flow modeling in tight gas reservoirs. The diffusivity equation is obtained using this method that it indicates the gas flow in a porous media. The conventional dry gas pseudo pressure function is not able to linearize the diffusivity equation including diffusion effect. Subsequently, a new real gas pseudo pressure function is used and a novel real gas pseudo time function is introduced. These pseudo functions consider changes in gas properties with pressure and linearize the diffusivity equation. The linear diffusivity equation is solved analytically for constant gas flow boundary condition under Pseudo Steady State (PSS) situation. Then, pseudo steady state analytical solution, based on new functions of pseudo pressure and pseudo time, is obtained. The calculation of reservoir parameters such as permeability, effective diffusion coefficient and original gas in place (OGIP) using reservoir data is the first application of analytical solution. Reservoir data is required to analysis the results of application of introduced model in low permeability gas reservoir.

KEY WORDS: Tight gas reservoir, Real gas pseudo pressure function, Pseudo time function, Effective diffusion coefficient, OGIP.

INTRODUCTION

Gas reservoirs with permeability less than 0.1 mD (9.869E-17 m²) are termed as "Tight Gas" reservoirs. These reservoirs worldwide present a considerable potential for hydrocarbon production [1-3].

Al-Hussainy et al. [4] introduced a pseudo pressure function for real gas as,

\[ m(p) = 2 \int_{p'}^{p} \frac{p}{\mu z} dp \]  \hspace{1cm} (1)

Where gas properties are considered functions of pressure. Then, he used the aforementioned relation to linearize the diffusivity equation for real gases. Also,
this function is known as conventional dry gas pseudo pressure function. Kikani & Pedrosa [5] modified real gas pseudo pressure function for situations where the reservoir permeability \( k \) is a function of pressure as,

\[
m'(p) = 2 \int \frac{p k(p)}{\rho z} dp
\]  

Agarwal [6] and Lee & Holditch [7] employed pseudo time concept to linearize the transient gas flow equation for gas reservoirs having large hydraulic fractures as shown below,

\[
t_a = \int_0^t \frac{1}{\mu(p_w)c_i(p_w)} dt
\]  

Variations of viscosity as well as compressibility were considered as a function of wellbore pressure \( p_w \). Fraim & Wattenbarger [8] proposed a normalized pseudo time function by considering the variations of gas properties as follow,

\[
t_n = (\phi \mu c_i)\int_0^t \frac{1}{\phi(p) \mu(p) c_i(p)} dt
\]  

Later, Ibrahim et al. [9] introduced a new normalized pseudo time function by considering reservoir properties to be functions of average pressure as follow,

\[
t_n = (\phi \mu c_i)\int_0^t \frac{1}{\phi(p) \mu(p) c_i(p)} dt
\]  

Ertekin et al. [10] suggested that flow mechanisms of gas in tight reservoirs are influenced by Darcy flow as well as gas molecular concentration gradient. In fact, in tight porous systems gas flow due to diffusion effects might be the dominant mechanism as discussed by Ayala et al. [11, 12]. Consequently, gas flow velocity in tight reservoirs may be formulated as below:

a. Darcy flow velocity due to pressure gradient [13],

\[
v^D = -\frac{k}{\mu} \nabla p
\]  

b. Diffusion flow velocity due to gas molecular concentration gradient,

\[
v^F = -\phi S_e \frac{D_e}{P_g} \nabla P_g = -\phi S_e D_e c_e \nabla p
\]  

Therefore, the total gas flow velocity will be,

\[
v^T = v^D + v^F
\]  

Applying some simple manipulations,

\[
v^T = -(\frac{k}{\mu} + \phi S_e D_e c_e) \nabla p
\]  

Note that, \( D_e \) represents the combined diffusion mechanisms that might happen in the porous media. In gas reservoirs with reasonable permeability, the contribution of diffusion effects to flow could be ignored and Eq. (9) reduces to Darcy flow, Eq. (6).

In the previous study [14], a dual mechanism based on Darcy flow as well as diffusion was presented for the gas flow modeling in homogeneous porous media. Then, a new pseudo pressure function was introduced and used to linearize the gas flow governing differential equation with the diffusion effect. Finally a set of analytical solutions was presented for analyzing steady-state and transient gas flow through porous media including effective diffusion.

THEORITICAL SECTION

Mathematical model

Assuming a homogeneous tight gas reservoir, the flow of gas is obtained from the continuity equation as below [14],

\[
\nabla \cdot \left( \rho v^T \right) = -\frac{\partial}{\partial t}(\rho \phi)
\]  

Substitution of total gas flow velocity, \( v^T \), from Eq. (9) and real gas equation of state as \( p = \rho M/\rho R T \) in Eq. (10) and employing some manipulation yields [14],

\[
\nabla \cdot \left( \frac{P}{\mu} \left( \frac{k}{\mu} + \phi S_e D_e c_e \right) \nabla P \right) = \phi S_e c_e \frac{\partial p}{\partial t}
\]  

The above nonlinear partial differential equation represents the flow of gas in tight reservoirs including diffusion effect. The conventional dry gas pseudo pressure function, Eq. (1) is not able to linearize Eq. (11). Therefore, the followings pseudo functions are introduced,

Pseudo pressure function

In order to linearize the Eq. (11), a new pseudo pressure function was defined as follows [14],
\[ m'(p) = \frac{\beta}{p} \int_0^P \left( \frac{k}{\mu} + \phi S_g D_c c_g \right) dp \]  
(12)

Using Eq. (12) in Eq. (11) and employing some simple mathematical manipulation yields [14],

\[ \nabla \cdot (\nabla m'(p)) = \frac{\phi c_i}{(\frac{\mu}{k} + \phi S_g D_c c_g)} \frac{\partial m'(p)}{\partial t} \]  
(13)

Now, the Left Hand Side (LHS) of Eq. (13) is linear. However, the Right Hand Side (RHS) of Eq. (13) contains strongly pressure dependent parameters and it is nonlinear; especially during pseudo steady state flow situation and buildup tests while the average reservoir pressure changes sharply.

**Pseudo time function**

In order to linearize the RHS of Eq. (13), a novel pseudo time function is introduced as below,

\[ t'_s(t) = \int_0^t \frac{\left( \frac{k}{\mu} + \phi S_g D_c c_g \right)}{\phi S_g D_c c_g} d\tau \]  
(14)

Derivative of pseudo time function with respect to time is shown below,

\[ \frac{\partial t'_s}{\partial t} = \frac{\left( \frac{k}{\mu} + \phi S_g D_c c_g \right)}{\phi S_g D_c c_g} \]  
(15)

**Gas flow equation with diffusion effect**

Substitution of Eq. (15) in Eq. (13) becomes as,

\[ \nabla \cdot (\nabla m'(p)) = \frac{\partial m'(p)}{\partial t'_s} \]  
(16)

In cylindrical coordinate system Eq. (16) becomes as,

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial m'(p)}{\partial r} \right] = \frac{\partial m'(p)}{\partial t'_s} \]  
(17)

**Analytical solutions**

The problem under consideration is that of radial flow from a finite cylindrical gas reservoir with sealed upper and lower surfaces at a constant production rate. The inner boundary is the well. The reservoir is volumetric, that is, the outer boundary is sealed. Under constant production rate boundary condition, after pressure variation reaches the boundaries of reservoir, System enters into pseudo steady state. The initial and boundary conditions for pseudo steady state flow are as follows,

a. Constant gas flow rate at well,

\[ \left( r \frac{\partial m'(p)}{\partial r} \right)_{r=r_w} = \frac{p_i q_w T_i}{\pi h T_i} \quad \text{for all } t'_s(0) \]  
(18)

Where \( q_w \) is a constant gas flow rate.

b. Initially, uniform pressure prevails throughout the system,

\[ m'(p) = m'(p_i) \quad \text{at } t'_s = 0, \quad \text{for all } r \]  
(19)

c. No flow exists at the boundaries,

\[ \frac{\partial m'(p)}{\partial r} \bigg|_{r=r_b} = 0 \quad \text{for all } t'_s \]  
(20)

In this situation using the Laplace Transformation, Van Everdingen & Hurst [15, 16] introduced the following analytical solution for gas flow equation (Eq. (17)) [17],

\[ m'(p_i) - m'(p_w) = \frac{2p_i q_w T_i}{T_i V_b} t'_s(t) + \frac{p_i q_i T_i}{\pi h T_i} \left[ \ln \frac{r_e}{r_w} - \frac{3}{4} \right] \]  
(21)

Noting that; original gas in place, \( G = V_{ps} S_{gi}/B_{gi} \) pore volume, \( V_{ps} = \phi_i V_b \) the bulk volume, \( V_b = \pi (r_i^2 - r_w^2) h \) and \( B_{gi} = T_i p_i z_i/T_i p_i \). Then Eq. (21) may be rewritten as,

\[ m'(p_i) - m'(p_w) = \frac{2 \phi_i S_{gi} p_i q_i}{z_i G} t'_s(t) + \frac{p_i q_i T_i}{\pi h T_i} \left[ \ln \frac{r_e}{r_w} - \frac{3}{4} \right] \]  
(22)

Eq. (22) suggests that a graph of \([m'(p_i) - m'(p_w)]\) versus \( t'_s(t) \) will be a straight line of slope:

\[ SL = \frac{2 \phi_i S_{gi} p_i q_i}{z_i G} \]  
(23)

And intercept,

\[ 1 = \frac{p_i q_i T_i}{\pi h T_i} \left[ \ln \frac{r_e}{r_w} - \frac{3}{4} \right] \]  
(24)
RESULTS AND DISCUSSION

It should be noted that, in the course of oncoming calculations one requires knowledge of the average reservoir pressure ($\bar{p}$), as a function of time. Since $\bar{p}$ is not known, calculations should be iterative. In the followings, the procedure to calculate $\bar{p}$ and G is explained:

Assuming $k$ and $D_e$ to be constant, the pseudo pressure function Eq. (12), can be written as bellow:

$$m'(p) = k m'_i(p) + D_e m'_2(p)$$  \hspace{1cm} (25)

Where,

$$m'_i(p) = \int_{\bar{p}}^{p} \frac{2p}{\mu z} dp$$  \hspace{1cm} (26)

And,

$$m'_2(p) = \int_{\bar{p}}^{p} \frac{2p_0 S_p c_g}{\rho'} dp$$  \hspace{1cm} (27)

Similarly, Eq. (14), representing pseudo time function can be written as bellow:

$$t'_i(t) = k t_{s1}(t) + D_e t_{s2}(t)$$  \hspace{1cm} (28)

Where,

$$t_{s1}(t) = \int_{0}^{t} \frac{d t}{\mu(\bar{p}) \rho(\bar{p}) c_1(\bar{p})}$$  \hspace{1cm} (29)

And,

$$t_{s2}(t) = \int_{0}^{t} \frac{S_p(\bar{p}) c_g(\bar{p}) d t}{c_1(\bar{p})}$$  \hspace{1cm} (30)

As mentioned earlier, we need to know $\bar{p}$ in order to calculate the pseudo time functions presented in the Eqs. (29) and (30). Since $\bar{p}$ is not known, it is calculated from the gas material balance equation, which is given as [18],

$$\left( \frac{\bar{p}}{Z} \right) = \left( \frac{p_i}{Z_i} \right) \left( 1 - \frac{G_p}{G} \right)$$  \hspace{1cm} (31)

A logical iteration technique consists of, evaluating $\bar{p}$ from an initial guess of the original gas in place (G). This initial estimate could be a value a little bit greater than the final cumulative gas production ($G_p$). The average pressure, $\bar{p}$ obtained in this way is used to calculate the pseudo time functions and then $[m'(p_i) - m'(p_w)]$ versus $t'_i(t)$ plot could be constructed. The slope of this plot is used to evaluate G, as suggested by Eq. (23). Subsequently, new value of $G$ is used for the next iteration and this process continues until convergence tolerance for G is met. An algorithm for these calculations is presented in follow.

**Determination of reservoir permeability, effective diffusion coefficient and OGIP**

In a real situation, the production data, i.e., well bore pressure versus time and flow rate versus time are known. These data is employed in the analytical solution in conjunction with an iterative technique to obtain reservoir permeability ($k$), reservoir effective diffusion coefficient ($D_e$) and original gas in place (G);

Noting that Eq. (22) represents pseudo steady state gas flow in the reservoir and assuming permeability ($k$) and effective diffusion coefficient ($D_e$) to be constant, the left hand side of the Eq. (22) can be expanded as below:

$$m'(p_i) - m'(p_w) =$$  \hspace{1cm} (32)

$$k (m'_i(p_i) - m'_i(p_w)) + D_e (m'_2(p_i) - m'_2(p_w))$$

Where

$$m'_i p_w = (m'_i (p_i) - m'_i (p_w))$$  \hspace{1cm} (33)

And

$$m'_2 p_w = (m'_2 (p_i) - m'_2 (p_w))$$  \hspace{1cm} (34)

Then

$$m'(p_i) - m'(p_w) = k m'_i p_w + D_e m'_2 p_w$$  \hspace{1cm} (35)

Therefore

$$k m'_i p_w + D_e m'_2 p_w =$$

$$\frac{2p_0 S_p p_c k (t_{s1}(t) + D_e t_{s2}(t)) + \rho_0 k T_s}{\pi h T_s} \ln\left( \frac{r_w}{r_c} \right) - \frac{3}{4}$$  \hspace{1cm} (36)

Employing well bore pressure data versus time, the values of $m'_i p_w$ and $m'_2 p_w$ can be obtained. To determine the values of $t_{s1}(t)$ and $t_{s2}(t)$ (Eq. (29) and Eq. (30)) the values of average reservoir pressure (p̄) is needed which can be obtained from Eq. (31). Using above defined parameters, Eq. (36) is simplified as,

$$k m'_i p_w + D_e m'_2 p_w = SL (k t_{s1}(t) + D_e t_{s2}(t)) + 1$$  \hspace{1cm} (37)

Eq. (37) is rearranged as Eq. (38),
\[ k (m_1 p_w - SL t_{a1}(t)) + D_e (m_2' p_w - SL t_{a2}(t)) = 1 \] (38)

External reservoir radius is determined as,

\[ r_e = \left( \frac{G p_t z_i T_i}{\pi h \phi S_g p_t T_w} + r_w^2 \right)^{1/2} \] (39)

Cumulative gas production can be determined as below,

\[ G_p = \int_0^t q dt = t q_s \] (40)

Iterative procedure to determine the average reservoir pressure \( \bar{p} \), \( k \), \( D_e \) and OGIP from PSS gas flow data

The average reservoir pressure (\( \bar{p} \)) vs. time, reservoir permeability (\( k \)), reservoir effective diffusion coefficient (\( D_e \)) and reservoir original gas in place (\( G \)) could be determined employing an iterative technique as below,

1- Cumulative gas production (\( G_p \)) with respect to time is calculated using Eq. (40).
2- Make an initial guess for \( G \) which should be grater than final \( G_p \).
3- Using \( G \) and Eq. (31) determine \( \bar{p} \) versus time.
4- Using \( \bar{p} \) and Eqs. (29) and (30) determine \( t_{a1}(t) \) and \( t_{a2}(t) \).
5- Using Eqs. (23), (39) and (24) determine values of \( SL \) and \( I \).
6- Using wellbore pressure data and Eqs. (33) and (34) calculate the values of \( m_1' p_w \) and \( m_2' p_w \).
7- Using Eq. (38), and values of \( SL \) and \( I \) determine \( k \) and \( D_e \) employing least square technique.
8- From Eq. (37), determine \( G \) from the slope of \( (k m_1' p_w + D_e m_2' p_w) \) curve vs. \( (k t_{a1}(t) + D_e t_{a2}(t)) \).
9- Go to step 1 and iterate until the value of \( G \) converges to a constant value.

CONCLUSIONS

Flow of gas in tight gas reservoirs was modeled using dual flow mechanism. Linearization of the equation was made by introducing new pseudo pressure and pseudo time functions. The equation was then solved analytically. An algorithm to evaluate reservoir parameters such as permeability (\( k \)), effective diffusion coefficient (\( D_e \)) and OGIP (\( G \)) was introduced.