Nanofluid Condensation and MHD Flow Modeling over Rotating Plates Using Least Square Method (LSM)

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ABSTRACT: *In this study, nanofluid condensation and MHD flow analysis over an inclined and rotating plate are investigated respectively using Least Square Method (LSM) and numerical method. After presenting the governing equations and solving them by LSM, the accuracy of results is examined by the fourth order Runge-Kutta numerical method. For condensation, modeling results show that the condensate film thickness is reduced and in turn, the rate of heat transfer is enhanced by the addition of nanoparticles to the regular fluid. Effect of normalized thickness on velocity and temperature profiles reveals that increasing normalized thickness leads to an increase in f, f' and a decrease in g, θ. Effect of normalized thickness on k and s are similar to those of f' and g, respectively.*

KEYWORDS: *Nanofluid; LSM; Condensation; Nusselt number; Heat transfer.*

INTRODUCTION

Usually, scientific problems and phenomena in our world are essentially nonlinear and modeled by the nonlinear differential equations. Most of them do not have an exact analytical solution. So, numerical and approximate methods are used by researchers to solve such equations. The numerical methods are often costly and time consuming to get a complete form of results, because it gives the solution at the discrete points. Furthermore, in the numerical solution the stability and convergence should be considered to avoid divergence or inappropriate results. The first work on the flow under lubrication approximation was reported by *Stefan* [1]. After that, *Mahmood et al.* [2] investigated the heat transfer characteristics in the squeezed flow over a porous surface. *Aziz* [3] considered the outcome of time-dependent chemical reaction on the flow of a viscous

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1021-9986/2018/4/193-203 11/$/6.01

fluid past an unsteady stretching sheet. Also, Magnetohydrodynamic squeezing flow of a viscous fluid between parallel disks was analyzed by *Domairry* and *Aziz* [4]. *Mustafa et al.* [5] solved the problem of the fluid flow between parallel plates by Homotopy Analysis Method (HAM). *Turkyilmazoglu* [6] solved momentum and energy equations of nanofluids analytically to deduce the flow and heat transport phenomena in two theoretical cases, single phase and multi-phase. When the nanoparticles are uniformly distributed across the condensate boundary layer called it single phase and when the concentration of nanoparticles through the film is allowed to vary from the wall to the outer edge of the condensate film in the light of modified Buongiorno's nanofluid model named multi-phase. The solution of the particle's motion in different fluids media has been considered by the authors widely [7-10].

Comparison of the single and two-phase modeling for the nanofluids has been considered by the researchers. For instance, *Haghshenas Fard et al.* [11] compared the results of the single phase and two-phase numerical methods for nanofluids in a circular tube. They reported that for Cu-water the average relative error between experimental data and CFD results based on single-phase model was 16% while for the two-phase model was 8%. In another numerical study, *Göktepe et al.* [12] compared these two models for nanofluid convection at the entrance of a uniformly heated tube which found the same results and confirmed the accuracy of two-phase modeling. *Mohyud-Din et al.* [13] in an analytical study, considered the three dimensional heat and mass transfer with magnetic effects for the flow of a nanofluid between two parallel plates in a rotating system. As one of their main outcomes, thermophoresis, and Brownian motion parameters are directly related to heat transfer but are inversely related to the concentration profile. Also, they found that higher Coriolis forces decrease the temperature boundary layer thickness. The three-dimensional flow of nanofluids under the radiation (due to solar or etc.) has been analyzed by *Hayat et al.* [14] and *Khan et al.* [15]. They also computed and examined the effects of different parameters on the velocity, temperature, skin friction coefficient and Nusselt number of the nanofluid flow. Other works of nanofluids flow and heat transfer analysis can be found in [16-22].

There are some simple and accurate approximation techniques for solving nonlinear differential equations called the Weighted Residuals Methods (WRMs). Collocation, Galerkin and Least Square Method (LSM) are examples of the WRMs which are introduced by *Ozisik* [23] for use in heat transfer problem. *Stern* and *Rasmussen* [24] used collocation method for solving a third order linear differential equation. *Vaferi et al.* [25] have studied the feasibility of applying of Orthogonal Collocation method to solve diffusivity equation in the radial transient flow system. Recently *Hatami* and *Ganji* [26] used LSM for heat transfer study through porous fins also, they used this accurate method for fully wet circular porous fin [27], semi-spherical porous fins [28] and straight solid and porous fins [29].

Shaoqin and *Huoyuan* [30] developed and analyzed least-squares approximations for the incompressible magneto-hydrodynamic equations. *Ghasemi et al.* [31] found that LSM is more appropriate than other analytical methods for solving the nonlinear heat transfer equations.

Based on the above short review, two-phase nanofluids flow can be extended by numerical [32] or optimization [33] methods. In the present study, the authors aim to investigate the heat transfer and condensation mechanism for nanofluid flow over an inclined rotating disk under the gravity effect and MHD flow between rotating plates, respectively. The innovative points of the present study are introducing an exact and simple analytical method called LSM which does not need to any linearization or perturbation, also the effect of constant parameters appeared in the mathematical section on velocity, temperature and nanoparticles concentration are investigated.

PROBLEM DESCRIPTION

Condensation of nanofluids

Fig. 1 shows a disk rotating in its own plane with angular velocity Ω. The angle between the horizontal axis and the disk is β. A nanfluid film of thickness h is formed by spraying, with the W velocity. We assume the disk radius is large compared to the film thickness such that the end effects can be ignored. Vapor shear effects at the interface of vapor and fluid are usually also unimportant. The gravitational acceleration, $\overline{g}$, acts in the downward direction.

The temperature on the disk is T_w and the temperature on the film surface is T_0. Besides, the ambient pressure on the film surface is constant at P_0 and we can safely say the pressure is a function of z only.

Table 1: Thermal properties of base fluid (water) and nanoparticles.

Properties	Unit	Water	Al_2O_3	Cu	TiO_2	CuO	Ag
Heat capacitance	J/kg·K	4179	765	385	686.2	531.8	235
Density	kg/m^3	997.1	3970	8933	4250	6320	10500
Thermal conductivity	W/m·K	0.613	40	401	8.95	76.5	429
Thermal expansion coefficient	K^{-1}	2.1×10^{-4}	0.85×10^{-5}	1.67×10^{-5}	0.9×10^{-5}	1.8×10^{-5}	1.89×10^{-5}
Dynamic viscosity	Ns/m^2	0.001003	-	-	-		-

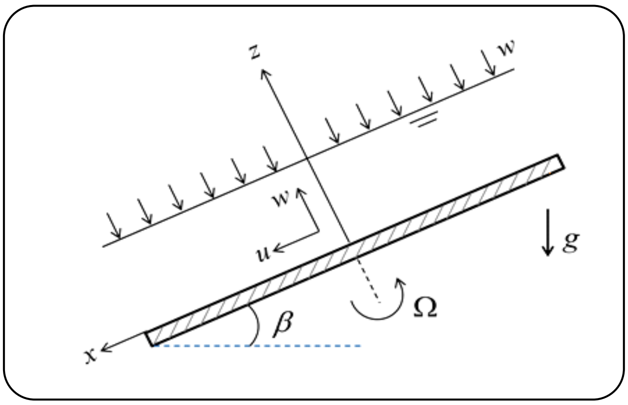

Fig. 1: Schematic diagram of the problem of nanofluid condensation.

The nanofluid is a two component mixture with the following assumptions: Incompressible; No-chemical reaction; Negligible radiative heat transfer; Nano-solid-particles and the base fluid are in thermal equilibrium and no slip occurs between them. The thermo physical properties of the nanofluid are given in Table 1.

Neglecting viscous dissipation, the continuity, momentum and energy equations for the steady state are given in the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\bar{g}\sin\beta \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}= \tag{4}$$

$$\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\bar{g}\cos\beta-\frac{p_z}{\rho_{nf}}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

In the above equations, *u, v,* and *w* indicate the velocity components in the *x, y*, and z directions, respectively.

The effective density *(ρ_{nf})*, the effective heat capacity *$(\rho C_p)_{nf}$* of the nanofluid and the effective heat capacity *$(\rho C_p)_{nf}$* of the nanofluid are defined as [21]:

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\quad \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}, \tag{6}$$

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s$$

The effective thermal conductivity of the nanofluid can be approximated by the Maxwell–Garnetts (MG) model as [21]:

$$\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)} \tag{7}$$

Supposing zero slip on the disk and zero shear stress on the film surface, the boundary conditions are:

$$u=-\Omega y,\quad v=\Omega x,\ w=0,\ T=T_w \quad \text{at } z=0 \tag{8}$$

$$u_z=0,\ v_z=0,\ w=-W,\ T=T_0,\ p=p_0,\ \text{at } z=h$$

For the mentioned problem, *Wang* introduced the following transform [12]:

$$u=-\Omega y\, g(\eta)+\Omega x f'(\eta)+\bar{g}k(\eta)\sin\frac{\beta}{\Omega'} \tag{9}$$

$$v=\Omega x\, g(\eta)+\Omega y f'(\eta)+\bar{g}s(\eta)\sin\frac{\beta}{\Omega'}$$

$$w=-2\sqrt{\Omega\upsilon_{nf}}\, f(\eta)$$

$$T=(T_0-T_w)\,\theta(\eta)+T_w$$

Where η was introduced as follows:

$$\eta = z\sqrt{\frac{\Omega}{\upsilon_{nf}}} \quad (10)$$

Continuity (1) automatically is satisfied. (2) and (3) can be written as follows:

$$f''' - (f')^2 + g^2 + 2ff'' = 0 \quad (11)$$

$$g'' - 2gf' + 2fg' = 0 \quad (12)$$

$$k'' - kf' + sg + 2fk' + 1 = 0 \quad (13)$$

$$s'' - kg - sf' + 2fs' = 0 \quad (14)$$

If the temperature is a function of the distance *z* only, (5) becomes

$$\theta'' + 2\Pr\frac{A_2 A_3}{A_1 A_4} f\theta' = 0, \quad (15)$$

$$A_1 = \frac{\rho_{nf}}{\rho_f}, A_2 = \frac{\mu_{nf}}{\mu_f}, A_3 = \frac{(\rho C_p)_{nf}}{(\rho C_p)_f}, A_4 = \frac{k_{nf}}{k_f}$$

Where $\Pr = \frac{\upsilon_f}{\alpha_f}$ is the Prandtl number of the base fluid. The boundary conditions for (11-15) are:

$$\begin{aligned} &f(0) = 0,\ f'(0) = 0, \quad f''(\delta) = 0, \quad (16)\\ &g(0) = 1, \quad g'(\delta) = 0, \quad k(0) = 0, \quad k'(\delta) = 0,\\ &s(0) = 0, \quad s'(\delta) = 0, \quad \theta(0) = 0, \quad \theta(\delta) = 1 \end{aligned}$$

and δ is the constant normalized thickness as;

$$\delta = h\sqrt{\frac{\Omega}{\upsilon_{nf}}} \quad (17)$$

which is known through the condensation or spraying velocity by:

$$f(\delta) = \frac{W}{2\sqrt{\Omega\upsilon_{nf}}} = \alpha \quad (18)$$

In this case the non-dimensional Nusselt number is obtained as:

$$Nu = \frac{k_{nf}}{k_f}\frac{\left(\frac{\partial T}{\partial z}\right)_w}{T_0 - T_w} = A_4\delta\theta'(0) \quad (19)$$

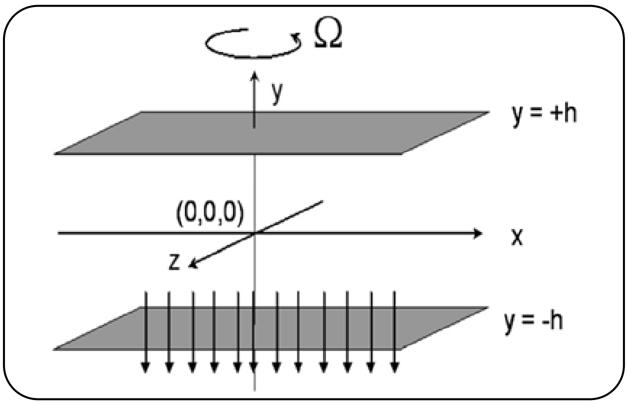

Fig. 2: The Schematic diagram of the problem.

MHD flow over the porous medium

Consider the three-dimensional, incompressible and steady flow of an electrically conducting viscous liquid between two parallel plates at y = ±h (See Fig. 2). Both the plates and the fluid rotate in unison with a constant angular velocity in direction *y*. The upper plate at y = +h is stationary and the flow is caused due to shrinking the lower porous plate at y = -h. The governing equations of the problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (20)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + 2\Omega w = \quad (21)$$

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] - \frac{\sigma B_0^2}{\rho}u$$

$$u\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] \quad (22)$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} - 2\Omega u = \nu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] - \frac{\sigma B_0^2}{\rho}w \quad (23)$$

Subject to the boundary conditions

$$\begin{aligned} &u = -ax, \quad v = -V, \quad w = 0 \quad \text{at } y = -h, \quad (24)\\ &u = 0, \quad v = 0, \quad w = 0 \quad \text{at } y = +h \end{aligned}$$

where *u*, *v* and *w* are the velocity components in directions *x*, *y* and *z*, respectively, p^* is the pressure term, B_0 is the magnetic induction, ρ is the density, σ is the electrical conductivity, ν is the kinematic viscosity and *V* is the suction velocity.

By using the following parameters

$$\eta=\frac{y}{h},\quad u=-axf'(\eta),\quad v=ahf(\eta),\quad w=axg(\eta) \tag{25}$$

The conservation law (Eq. (20)) automatically is satisfied. The Eqs. (21) to (24) can be written as follow

$$f^{IV}-M^2f''-2K_p^2g'-\mathrm{Re}\left(f'f''-ff'''\right)=0 \tag{26}$$

$$g''-M^2g+2K_p^2f'-\mathrm{Re}\left(f'g-fg'\right)=0 \tag{27}$$

where primes express the differentiation with respect to η and the following parameters are used for simplifying

$$\lambda=-\frac{V}{ah},\quad \mathrm{Re}=\frac{ah^2}{\nu},\quad M^2=\frac{\sigma B_0^2h^2}{\rho\nu},\quad K^2=\frac{\Omega h^2}{\nu} \tag{28}$$

And the transformed boundary conditions are

$$\begin{aligned}&f(-1)=\lambda,\quad f'(-1)=-1,\quad g(-1)=0\\ &f(1)=0,\quad f'(1)=0,\quad g(1)=0\end{aligned} \tag{29}$$

NUMERICAL AND ANALYTICAL APPLIED METHODS

Least Square Method (LSM)

The least square method is one of the approximation techniques for solving differential equations called the Weighted Residual Methods (WRMs) which firstly introduced by *Ozisik* [23] for heat transfer problems. For conception the main idea of this method supposes a differential operator *D* is acted on a function *u* to produce a function *p* [31]:

$$D(u(x))=p(x) \tag{30}$$

It is considered that *u* is approximated by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is,

$$u\cong\tilde{u}=\sum_{i=1}^{n}c_i\varphi_i \tag{31}$$

Now, when substituted into the differential operator, D, the result of the operations generally isn't *p(x)*. Hence an error or residual will exist:

$$R(x)=D(\tilde{u}(x))-p(x)\neq 0 \tag{32}$$

The notion in WRMs is to force the residual to zero in some average sense over the domain. That is:

$$\int_X R(x)\,W_i(x)=0\qquad i=1,2,...,n \tag{33}$$

Where the number of weight functions W_i is exactly equal the number of unknown constants c_i in $\tilde{u}$. If the continuous summation of all the squared residuals is minimized, the rationale behind the name can be seen. In other words, a minimum of

$$S=\int_X R(x)R(x)dx=\int_X R^2(x)dx \tag{34}$$

In order to achieve a minimum of this scalar function, the derivatives of *S* with respect to all the unknown parameters must be zero. That is,

$$\frac{\partial S}{\partial c_i}=2\int_X R(x)\frac{\partial R}{\partial c_i}dx=0 \tag{35}$$

Comparing with Eq. (39), the weight functions are seen to be

$$W_i=2\frac{\partial R}{\partial c_i} \tag{36}$$

However, the "2" coefficient can be dropped, since it cancels out in the equation. Therefore the weight functions for the Least Squares Method are just the derivatives of the residual with respect to the unknown constants [29],

$$W_i=\frac{\partial R}{\partial c_i} \tag{37}$$

Numerical Approach (NUM)

As mentioned before, the types of the current problems are the Boundary Value Problem (BVP) and the appropriate method which must be selected. The numerical solution is performed using the algebra package Maple 15.0, to solve the present problem. The package uses a fourth–fifth order Runge-Kutta–Fehlberg procedure for solving nonlinear boundary value problem. The algorithm is proved to be precise and accurate in solving a wide range of mathematical and engineering problems especially heat transfer cases [32].

RESULTS AND DISCUSSIONS

Many advantages of LSM compared to other analytical and numerical methods make it more valuable and motivate researchers to use it for solving heat transfer problems. For example, it solves the equations directly and no simplifications, perturbation, linearization or small parameter need [31]. Although *Ozisik* [23] and *Aziz* and *Bouaziz* [29] introduced LSM for solving a single nonlinear heat transfer equation, we successfully applied it for solving the coupled nonlinear equations in a nanofluid flow problem [27]. In this study due to nonlinearity and coupled equations, LSM is applied for the solution.

Condensation of nanofluids

Now, we want to apply this method to the present problem. Because trial functions must satisfy the boundary conditions in Eq. (16), so they will be considered as,

$$\begin{cases} f(\eta)=c_1\eta^2(\eta-\delta)^3+c_2\left(\dfrac{\eta^3}{6}-\dfrac{\delta\eta^2}{2}\right)+... \\ g(\eta)=1+c_3\eta(\eta-\delta)^2+c_4\left(\dfrac{\eta^2}{2}-\delta\eta\right)+... \\ k(\eta)=c_5\eta(\eta-\delta)^2+c_6\left(\dfrac{\eta^2}{2}-\delta\eta\right)+... \\ s(\eta)=c_7\eta(\eta-\delta)^2+c_8\left(\dfrac{\eta^2}{2}-\delta\eta\right)+... \\ \theta(\eta)=\dfrac{\eta}{\delta}+c_9\eta(\eta-\delta)+c_{10}\eta(\eta-\delta)^2+... \end{cases} \tag{38}$$

It's necessary to inform that every trial function that satisfies the boundary condition of the problem can be used and its accuracy can be improved by the number of its terms. In this problem, we have five coupled equations (Eqs. (11) - (15)) so, five residual functions will appear. Also, in this paper we considered two unknown coefficients for each trial function, so 10 unknown coefficients will be appeared (c_1-c_{10}). By substituting the residual functions, $R_1(c_1\text{-}c_{10},\eta)$, $R_2(c_1\text{-}c_{10},\eta)$, $R_3(c_1\text{-}c_{10},\eta)$, $R_4(c_1\text{-}c_{10},\eta)$ *and* $R_5(c_1\text{-}c_{10},\eta)$, into Eq. (35), a set of equations with ten equations will appear and by solving this system of equations, coefficients c_1-c_{10} will be determined. For example, using the Least Square Method for Cu-water nanofluid with ϕ=0.04, Pr=6.2 and δ=0.5 following equations will be obtained

$$\begin{cases} f(\eta)=-0.008811728449\eta^2(\eta-0.5)^3- \\ \qquad 0.9607426021\left(\dfrac{\eta^3}{6}-\dfrac{\eta^2}{4}\right) \\ g(\eta)=1+0.07794439347\eta(\eta-0.5)^2+ \\ \qquad 0.1969431465\left(\dfrac{\eta^2}{2}-\dfrac{\eta}{2}\right) \\ k(\eta)=0.008808870176\eta(\eta-0.5)^2- \\ \qquad 0.9838801251\left(\dfrac{\eta^2}{2}-\dfrac{\eta}{2}\right) \\ s(\eta)=0.03931075727\eta(\eta-0.5)^2+ \\ \qquad 0.09985540140\left(\dfrac{\eta^2}{2}-\dfrac{\eta}{2}\right) \\ \theta(\eta)=2\eta-0.1982408923\eta(\eta-0.5)- \\ \qquad 0.2492463454\eta(\eta-0.5)^2 \end{cases} \tag{39}$$

As the same manner for Cu-Water nanofluid when δ=1it can be

$$\begin{cases} f(\eta)=-0.01914194896\eta^2(\eta-1)^3- \\ \qquad 0.6564549829\left(\dfrac{\eta^3}{6}-\dfrac{\eta^2}{2}\right) \\ g(\eta)=1+0.08162945698\eta(\eta-\delta)^2+ \\ \qquad 0.4634992471\left(\dfrac{\eta^2}{2}-\eta\right) \\ k(\eta)=0.03926545013\eta(\eta-1)^2- \\ \qquad 0.8513453189\left(\dfrac{\eta^2}{2}-\eta\right) \\ s(\eta)=0.04388591688\eta(\eta-1)^2+ \\ \qquad 0.2695506788\left(\dfrac{\eta^2}{2}-\eta\right) \\ \theta(\eta)=\eta-0.4379485922\eta(\eta-1)- \\ \qquad 0.2403810627\eta(\eta-1)^2 \end{cases} \tag{40}$$

The objective of the present study was to apply the least square method to obtain an explicit analytic solution of the three-dimensional problem of condensation nanofluid film on an inclined rotating disk (Fig. 1).

The comparison between the obtained results with that of obtained with numerical results is shown in Fig. 3.

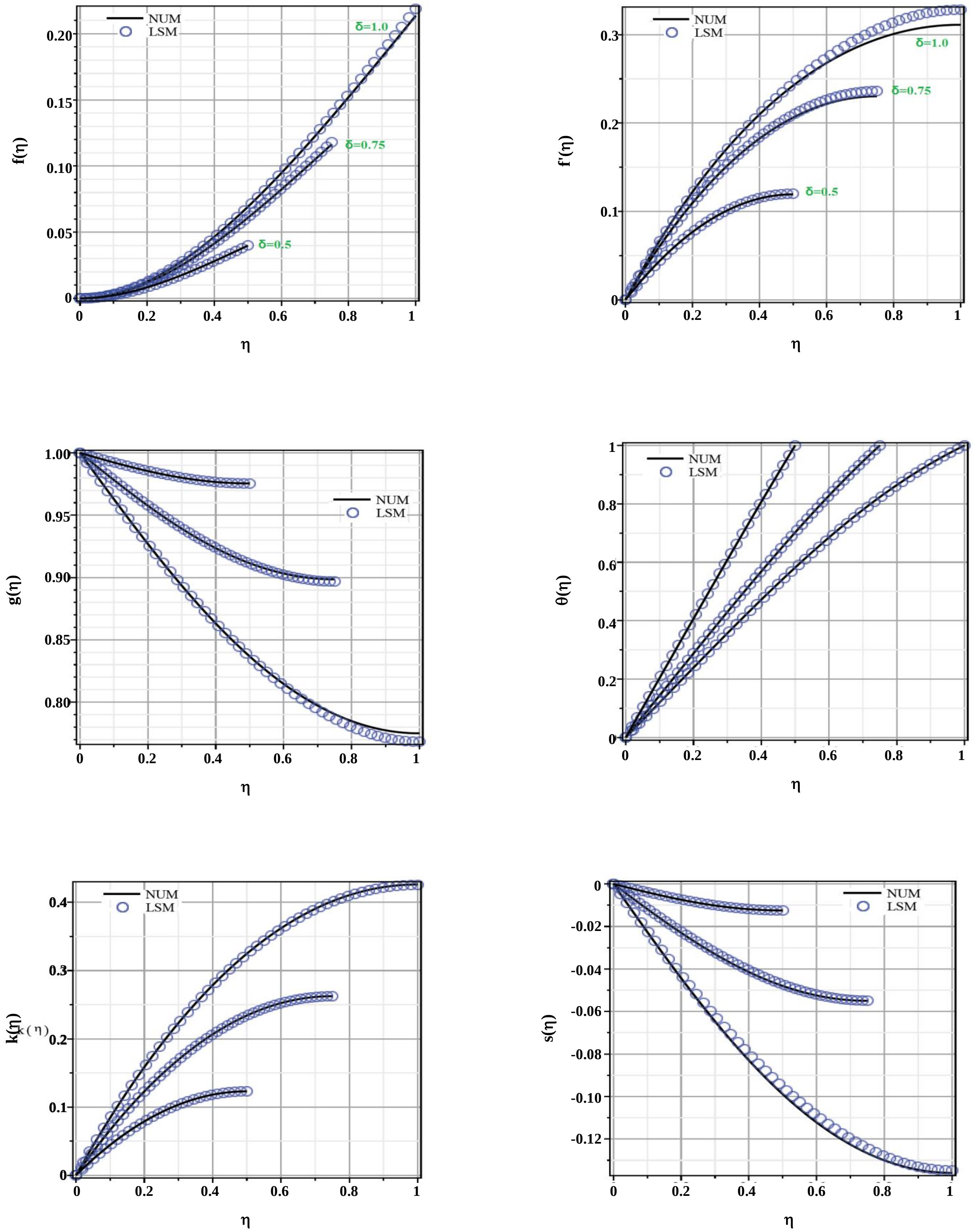

Fig. 3: Comparison of LSM and NUM for Cu-water nanofluid with ϕ=0.04 and δ=0.5, 0.75, 1.

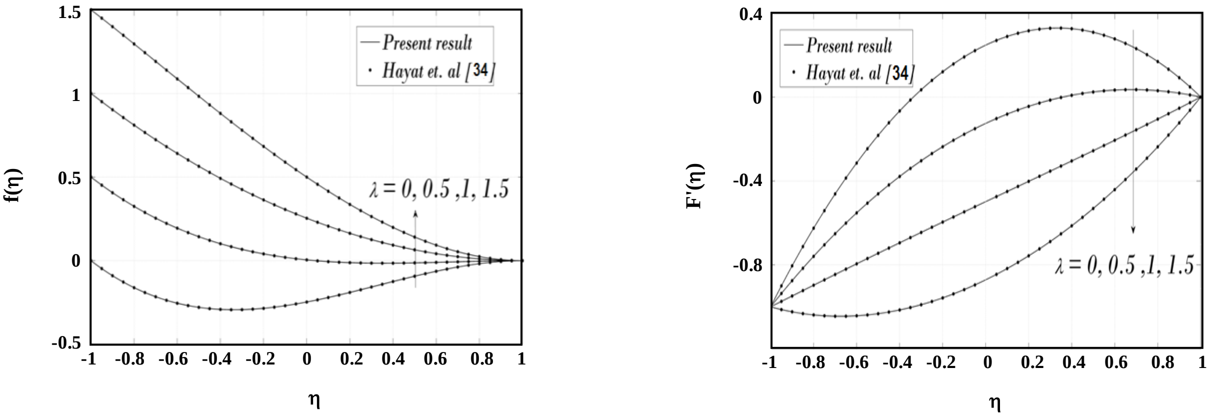

Fig. 4 Comparison of the results of the proposed hybrid method and HAM [57] when M= 0.5, K_p = 0.5 and Re=0.2.

This accuracy gives high confidence to us about the validity of this problem and reveals an excellent agreement of engineering accuracy. This investigation is completed by depicting the effects of some important parameters to evaluate how these parameters influence on this fluid. Effect of normalized thickness on velocity and temperature profiles is shown in Fig. 3. Increasing normalized thickness leads to an increase in *f*, *f'* and a decrease in g, θ. Effect of normalized thickness on k and *s* are similar to those of *f'* and g, respectively.

MHD flow between parallel plates

In the literature and introduced papers in the introduction, the detailed steps of applying LSM and its accuracy on the problems are discussed, so in the current study, we pointed briefly its application for this problem. For LSM, because base function must satisfy the boundary conditions, so it will be assumed as:

$$f(x)=\lambda-\frac{\lambda}{2}(x+1)+\left(\frac{1}{2}-\frac{\lambda}{4}\right)\left(x^2-1\right)+ \left(\frac{\lambda-1}{4}\right)\left(x^2-1\right)(x+1)+c_1\left(x^2-1\right)^2+ c_2\left(x^2-1\right)^2(x+1)+c_3\left(x^2-1\right)^3 \quad (41)$$

$$g(x)=\left(x^2-1\right)+c_4\left(x^2-1\right)(x+1)+ c_5\left(x^2-1\right)^2+c_6\left(x^2-1\right)^2(x+1) \quad (42)$$

After applying this trial function to Eqs. (26) and (27) and obtaining the residual function, the unknown parameter of the above equation will be determined. For example, when the parameters are:

$$M=0.5, K_p=0.5, Re=0.2, \lambda=1 \quad (43)$$

The solution of LSM makes the following equations:

$$f(x)=\frac{1}{4}-\frac{1}{2}x+\frac{1}{4}x^2+0.0024\left(x^2-1\right)^2+ 0.0054\left(x^2-1\right)^2(x+1)-0.0009\left(x^2-1\right)^3 \quad (44)$$

$$g(x)=x^2-1-0.2694\left(x^2-1\right)(x+1)+ \quad (45)$$

$$0.0298\left(x^2-1\right)^2-0.0052\left(x^2-1\right)^2(x+1) \quad (45)$$

Fig. 4 shows the comparison of the results of the present analytical solution and HAM in [34] to show the efficiency and accuracy of the method. Figs. 5 and 6 demonstrate the graphical representation of the results for different values of the parameters. From the physical point of view, it can be seen that by increasing the magnetic number (M), f(η) and g(η) function of flows increased while by increasing the Reynolds number, just f(η) function increases and g(η) will decrease.

CONCLUSIONS

In this paper, Least Square Method (LSM) with fourth order Runge-Kutta numerical method have been successfully applied to find the solution of nanofluid condensation and heat transfer over rotating plates. Two problems are considered, nanofluid film over an inclined plate and MHD flow between rotating palates under the magnetic field effect. The results show that LSM results are in excellent agreement with those of numerical solution. For the MHD flow between parallel plates, From the physical point of view, it was observed that

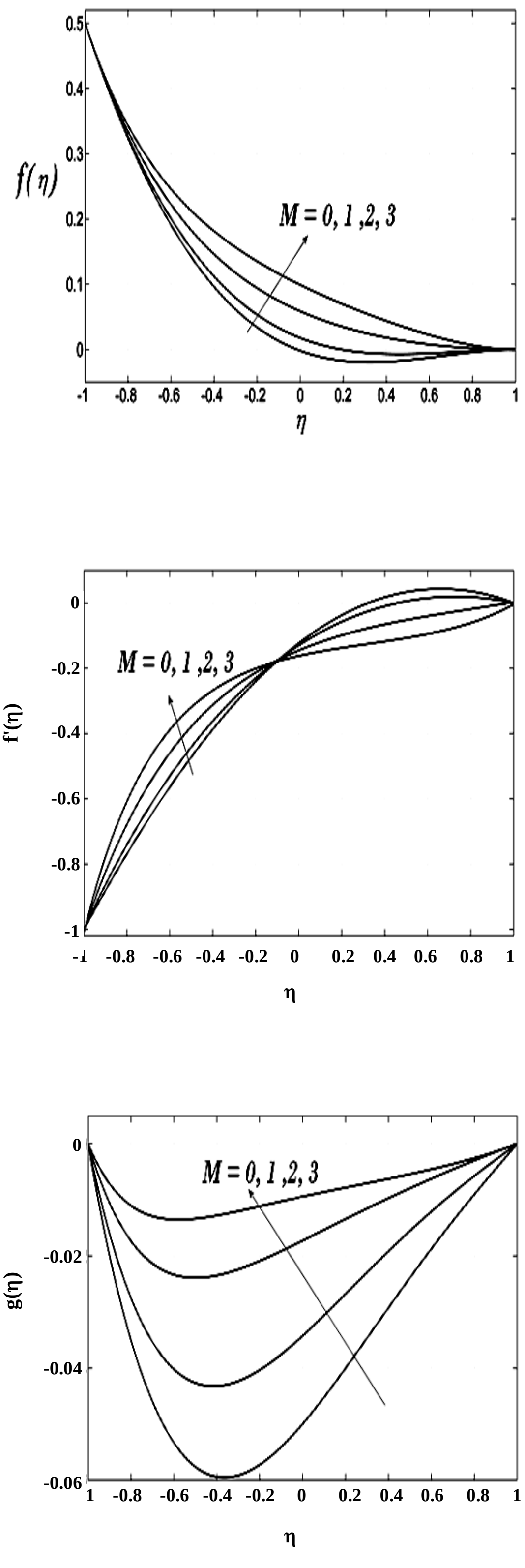

Fig. 5: The profiles $f(\eta)$, $f'(\eta)$ and $g(\eta)$ when $\lambda = 0.5$, $K_p = 0.5$ and Re=0.5.

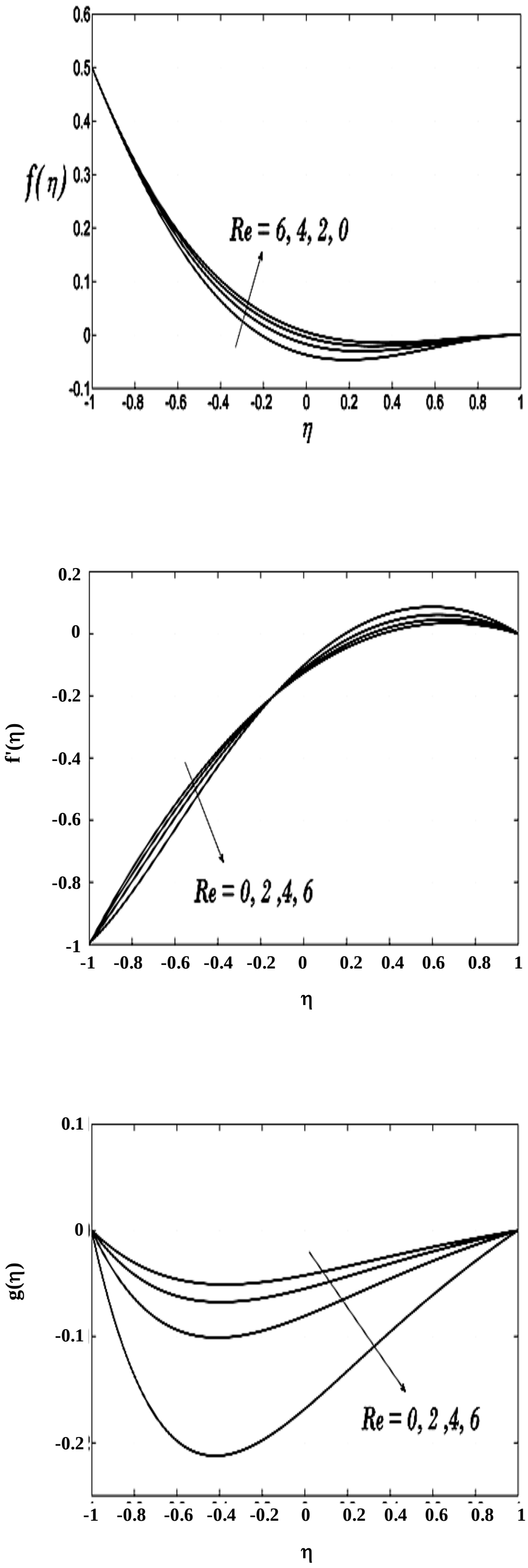

Fig. 6: The profiles $f(\eta)$, $f'(\eta)$ and $g(\eta)$ when $\lambda = 0.5$, $K_p = 0.5$ and M=0.5.

by increasing the magnetic number (M), f(η) and g(η) function of flows increased while by increasing the Reynolds number, just f(η) function increases and g(η) will decrease.

Acknowledgments

The authors gratefully acknowledge the financial support of the National Natural Science Foundation of China (No. 51422604, 21276206) and the National 863 Program of China (No. 2013AA050402). This work was also supported by the China Fundamental Research Funds for the Central Universities.

Received : May 21, 2016 ; Accepted : Feb. 5, 2018

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