Well Placement Optimization Using Differential Evolution Algorithm

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ABSTRACT: *Determining the optimal location of wells with the aid of an automated search algorithm is a significant and difficult step in the reservoir development process. It is a computationally intensive task due to the large number of simulation runs required. Therefore, the key issue to such automatic optimization is development of algorithms that can find acceptable solutions with a minimum number of function evaluations. In this study, the Differential Evolution (DE) algorithm is applied for the determination of optimal well locations. DE is a stochastic optimization algorithm that uses a population of solutions which evolve through generations to reach the global optimum. To investigate the performance of this algorithm, three example cases are considered which vary in dimension and complexity of the reservoir model. For each case, both DE algorithm and the widely used Genetic Algorithm (GA) are applied to maximize a Modified Net Present Value (MNPV) as the objective function. It is shown that DE outperforms GA in all cases considered, though the relative advantage of the DE vary from case to case. These results are very promising and demonstrate the applicability of DE for this challenging problem.*

KEY WORDS: *Well placement, Optimization, Differential evolution algorithm, Genetic algorithm.*

INTRODUCTION

Determining the best location for new wells is a complex problem that depends on reservoir and fluid properties, well and surface equipment specifications, and economic criteria. Various approaches have been proposed for this problem. One way of solving this problem is direct optimization with a numerical simulator as the function evaluation tool. The computational demand for this problem is substantial, as many function evaluations are required and each entails a full reservoir simulation. It is therefore essential that the underlying optimization algorithm be efficient and robust. There have been many previous studies addressing the optimization of well locations using different stochastic optimization algorithms. A detailed review of these

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studies is given in [1]. *Beckner & Song* applied Simulated Annealing (SA) algorithm on a well placement problem to propose the placement of a sequence of production wells [2]. *Bittencourt & Horne* investigated optimization of well placement using a hybridized GA, coupled with polytope algorithm and a tabu search method [3]. *Güyagüler et al.* also applied a Hybrid Genetic Algorithm (HGA) to find optimal location and rate for vertical wells in a water flooding project in Gulf of Mexico [4]. *Yeten et al.* designed a well placement optimization framework coupling GA with hill climbing search algorithm and an Artificial Neural Network (ANN) proxy [5]. In 2006, *Bangerth et al.* compared the performance of Simultaneous Perturbation Stochastic Approximation (SPSA), Finite Difference Gradient (FDG), Simulated Annealing (SA), Genetic Algorithm (GA) and Nelder-Mead Simplex methods during optimization of well placement in two synthetic two-dimensional reservoirs [6]. Recently, gradient based optimization techniques have been used by some researchers in different approaches for well placement optimization [7-9]. In a recent work, Particle Swarm Optimization (PSO) algorithm was applied to several well placement problems involving vertical and deviated wells by Onwunalu and Durlofsky. They found that PSO algorithm provided better results compared to the binary GA [10]. In 2011, *Afshari et al.* applied an Improved Harmony Search (IHS) algorithm to solve the problem of well placement optimization in oil reservoirs. They compared the performance of this algorithm to that of Classical Harmony Search, Particle Swarm Optimization, Simulated Annealing and Genetic Algorithm through several case studies and demonstrated that the IHS algorithm provided comparable or better results than the other methods [1]. In this work, we apply the Differential Evolution (DE) algorithm for the optimization of well locations. This algorithm has been used previously in many other application areas [11-17], but it does not appear to have been applied for the optimization of well placement problem. We demonstrate that the DE method outperforms an existing Genetic Algorithm for this application. This is potentially a very useful finding, as GAs are widely used for this and other reservoir-management related applications. The rest of the paper is organized as follows: We first describe, in next section, the components that were used for solution of the well placement problem: 1) DE algorithm and 2) Objective function. Then, in section "Results & Discussion", well placement optimization problems of three example are solved using DE and GA; the comparison of the results demonstrate the superior performance of DE for the cases considered. Finally, we conclude with a brief summary in "Conclusion" section.

THEORETICAL SECTION

Differential Evolution Algorithm

Differential Evolution (DE) was first suggested by *Storn & Price* in 1995 as a search technique for solving optimization problems [18]. DE has proven to be a promising candidate for optimizing real valued multi-modal objective functions. Besides its good convergence properties, DE is very simple to understand and to implement. DE is also particularly easy to work with, having only a few control variables which remain fixed throughout the entire optimization procedure. It uses the same operators like mutation, crossover and selection as that of GA but manipulates them in a manner different to that of GA. A brief description of DE algorithm is given here. It starts with a population of NP candidate solutions:

$$X_i^G = \left\{x_{1,i}^G, x_{2,i}^G, ..., x_{D,i}^G\right\} \qquad i = 1, 2, ..., NP \tag{1}$$

Where, the index i denotes the i-th individual of the population, D is the number of optimization parameters, and G denotes the generation to which the population belongs. The three main operators of DE are mutation, crossover and selection [19].

Mutation: The mutation operation of DE applies the vector differentials between the existing population members for determining both the degree and direction of perturbation applied to the individual subject of the mutation operation. The i-th perturbed individual, V_i^{G+1}, is generated as follows:

$$V_i^{G+1} = X_{r1}^G + F \times \left(X_{r2}^G - X_{r3}^G\right), \tag{2}$$
$$\{r1, r2, r3\} \in (1, 2, ..., NP)$$

Where X_{r1}^G, X_{r2}^G and X_{r3}^G are randomly selected three individuals in the current population set of NP individuals such that $r1 \neq r2 \neq r3 \neq i$. F is a DE control parameter which controls the amplification of differential variations and lies in the range of [0, 2].

Crossover

The perturbed individual, $V_i^{G+1}=\left(v_{1,i}^{G+1},v_{2,i}^{G+1},...,v_{D,i}^{G+1}\right)$, and the current population member, $X_i^G=\left(x_{1,i}^G,x_{2,i}^G,...,x_{D,i}^G\right)$, are then subject to the crossover operation, that finally generates the population of candidates ,or trial vectors, $U_i^{G+1}=\left(u_{1,i}^{G+1},u_{2,i}^{G+1},...,u_{D,i}^{G+1}\right)$, as follows:

$$u_{j,i}^{G+1}=\begin{cases} v_{j,i}^{G+1} & \text{if } (\text{rand}_j \le CR) \vee j=k \\ x_{j,i}^G & \text{otherwise} \end{cases} \quad (3)$$

$$j=1,2,...,D$$

Where $k\in\{1,2,...,D\}$ is a random number parameter's index, chosen once for each i, and rand_j is a random number between 0 and 1. CR is the crossover rate that controls the probability of replacing the current value of X with a new one and is in the range of [0, 1].

Selection

The selection scheme of DE also differs from that of GA. To decide whether or not it should become a member of generation, each trial vector, U_i^{G+1}, is compared to its corresponding individual in the current generation in terms of objective function. The population for the next generation is selected according to the following formula:

$$X_i^{G+1}=\begin{cases} U_i^{G+1} & \text{if } f\left(U_i^{G+1}\right)\le f\left(X_i^G\right) \\ X_i^G & \text{otherwise} \end{cases} \quad (4)$$

Thus, each individual of the temporary (trial) population is compared with its counterpart in the current population. The one with the higher objective function value (for a maximization problem) will be included in the next generation.

Objective Function

In the well placement problems, the objective function is usually the project's Net Present Value (NPV) [1]. In all problems considered in this study, we used a modified NPV as the objective function. To accommodate the constraint of minimum distance between the wells, the following expression has been used as a modification of NPV [1]:

$$MNPV=\begin{cases} 0 & \text{if } D_{i,j}\le 200 \\ \sum_{t=1}^{T}\dfrac{C_t}{(1+r)^t}-C_0 & \text{Otherwise} \end{cases} \quad (5)$$

$D_{i,j}$ is the distance between well i and well j (i,j $\in$\{1, 2, total number of wells\} and i≠j\}. In this manner, each well is constrained to have a minimum distance of 200 ft from the adjacent wells. t is the time step in years, T is the total production time in years, C_t is cash flow after time t in \$, r is annual or periodic discount rate in fraction, and C_0 is initial investment. The annual discount rate is set to 0.1 (10%). C_t and C_0 can be computed as follows:

$$C_t=R_t-E_t \quad (6)$$

$$C_0=N_{well}\times C_{well}+C_{capex} \quad (7)$$

Where R_t and E_t represent the revenue (\$) and operating expenses (\$) in time t, respectively. N_{well} is the number of wells, C_{well} is the drilling cost of each well in \$/well and C_{capex} is the capital expenditure. R_t and E_t depend on the fluid production volumes at time t:

$$R_t=P_{oil}^t\times \$_{oil}^t \quad (8)$$

$$E_t=P_{water}^t\times Cost_{P_{water}}^t+I_{water}^t\times Cost_{I_{water}}^t \quad (9)$$

Where P_{oil}^t and P_{water}^t are total oil and water production at time t in STB, I_{water}^t is total water injection at time t in STB, $\$_{oil}^t$ is oil price at time *t* in \$/STB, $Cost_{P_{water}}^t$ is produced water disposal cost at time t in \$/STB, and $Cost_{I_{water}}^t$ is water injection cost at time t in \$/STB. Table 1 shows the values of objective function parameters that will be used to compute the MNPV.

RESULTS AND DISCUSSION

DE algorithm has been applied to different case studies. These cases vary in terms of dimensions, number of well considered and petrophysical properties. Therefore, the size of the search space and consequently, the amount of computation required for function evaluation will be different for each case. All examples use a modified net present value as the objective function. In each case, we will compare the performance of the DE algorithm to that of GA.

Table 1: The value of parameters used in computing MNPV.

Objective Function (MNPV) Parameters	Parameter Value
Oil Price ($\$^t_{oil}$)	100 $/STB
Produced Water Disposal Cost ($Cost^t_{P_{water}}$)	5 $/STB
Water Injection Cost ($Cost^t_{I_{water}}$)	10 $/STB
Capital Expenditure (C_{capex})	2×10^7 $
Well Cost (C_{well})	4×10^6 $

Table 2: Tuned parameters of the algorithms used in solving well placement problem of case 1.

GA	NP = 50 , CR = 0.90
DE	NP = 10 , F = 1 , CR = 0.50

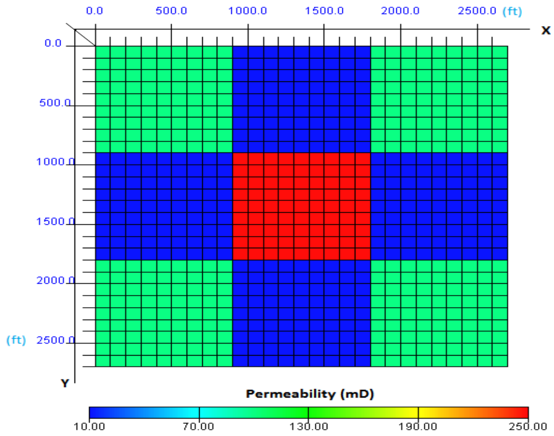

Fig. 1: Permeability distribution for reservoir model of Case 1.

DE and GA have some internal parameters that must be tuned for each problem separately in order to improve the performance them. Therefore, there was an attempt to tune these parameters for all of the cases studied here. The procedure of tuning was same as the one used by [1].

Since both GA and DE are stochastic algorithms and random in nature it is not sufficient to compare their performances based on a single run, so we need multiple number of runs. Thus, for each example case, 10 simulation runs were carried out and the average of MNPVs achieved by each optimization algorithm has been calculated over these multiple runs. These average values then were used as the basis of comparison.

Case 1

In this example, we determined the optimum location of four injection wells (I1, I2, I3, I4), in a simple 2-D synthetic reservoir model which has a single production well (P1) in the center (in block (13 , 13)). The reservoir contains 27×27 grid blocks, with each block of dimensions 100×100×50 ft. Porosity is taken to be constant and equal to 0.25 while permeability distribution is heterogeneous but very simple which are displayed in Fig. 1. The system, initially, contains oil and connate water, and the boundaries are considered to be no flow. The oil viscosity and formation volume factor are 1.24 cp and 1.05 rb/STB, respectively. Also, water viscosity and formation volume factor are considered to be 0.5 cp and 1 rb/STB, respectively. Rock compressibility is 3.26×10^{-6} psi^{-1} and solution gas oil ratio is 0.1 MSCF/STB. The system is initially at the pressure of 4500 psi. Initial water saturation and residual oil saturation are 0.25 and 0.30 respectively. The production well operates under a BHP constraint of 500 psi and injection rate for each injector is 250 STBD. Total production time is 3650 days. Regarding the simple geometry and permeability distribution of the model, one can determine the optimum locations of four injectors trivially. The optimum locations are these blocks: (0 , 13), (27 , 13), (13 , 0) and (13 , 27); and the corresponding MNPV for this well configuration is 2.6098×10^8 (global optimum). An investigation similar to the one used by [1] has been conducted to tune the internal parameters of the optimization algorithms and the results have been summarized in Table 2.

Then, we run the optimization program up to 10 times using DE and GA and compare the performance of them in solving the well placement of case 1 based on the average MNPVs. We run program until 1000 function evaluations for each case. Fig. 2 illustrates the progress of optimization process for DE solutions and GA solutions. The dashed line corresponds to the global optimum.

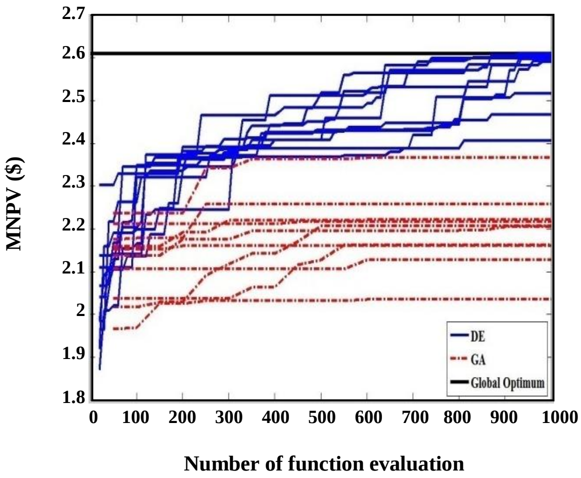

Fig. 2: The progress of optimization process for Case 1.

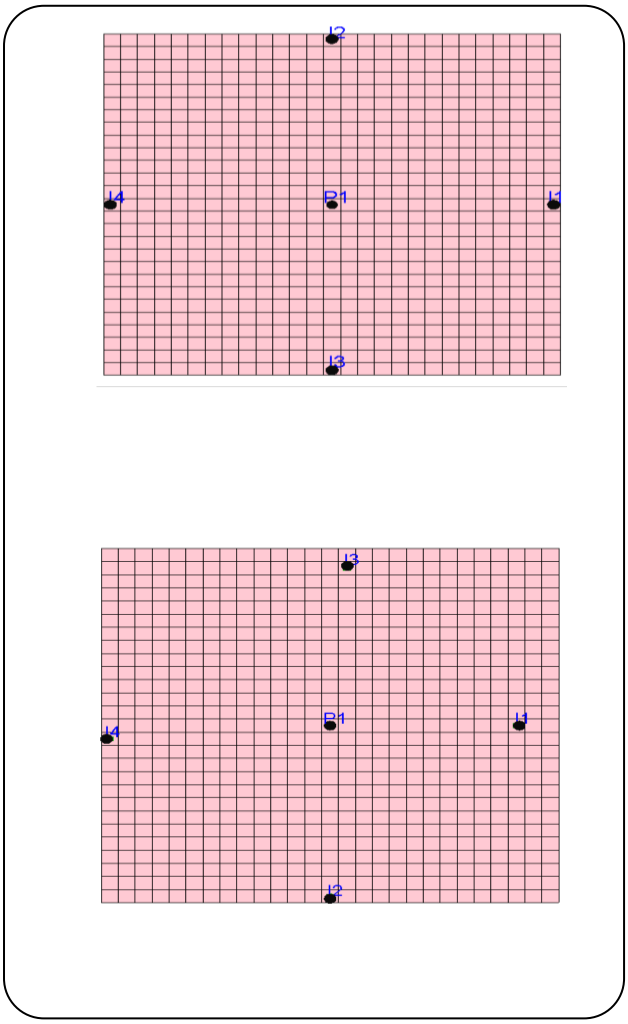

Fig. 3: Optimum location of injection wells achieved in the 5th run by (a) DE, and (b) GA for Case1.

It is evident that DE was able to found the global optimum in some runs, while GA was unable to find this solution within 1000 function evaluations. The corresponding average objective function of GA solutions is 2.1954×10^8 which is 14.3% lower than that of DE (2.5616×10^8). Fig. 3 shows the optimum solution found by these algorithms in the 5th run.

Case 2

In this case, we maximize MNPV by optimizing the location of one production well (P1) in a 2-D synthetic reservoir model. The reservoir has already four injection wells (I1, I2, I3, I4) which placed in the four corner of the reservoir (in grid blocks: (0 , 0), (220 , 0), (0 , 60), (220 , 60)). The reservoir grid contains 220×60 grid blocks, each of a size of 20×50×50 ft. Fluid properties are the same as the case 1. Porosity and permeability distribution of the model are shown in Fig. 4 and Fig. 5, respectively. The injection rate for each injector is considered to be 250 STBD and minimum BHP constraint of production well is 500 psi. Total production time is 5475 days. A procedure similar to that of case 1 has been used to tune the internal parameters of the algorithms and the results have been summarized in Table 3.

After tuning, the program is run for this case until 250 function evaluations; DE and SA are used as the optimization algorithms and their performances are compared to each other based on the average MNPVs achieved after 10 runs for each algorithm. Fig. 6 shows the optimal MNPVs for each algorithm (DE and GA solutions) versus the number of function evaluations.. The DE algorithm gives an average MNPV of 3.7412×10^8 which compared to 3.7225×10^8 of GA, shows 0.6% improvement. Fig. 7 shows the optimum location of the production well found by DE and GA in the 5th run. Comparing these locations to the porosity and permeability maps of the reservoir (Figs. 4 and 5), it is obvious that the well is not located in low permeability regions in both cases, which is acceptable by a reservoir engineer.

Case 3

This example entails the determination of the optimal

Table 3: Tuned parameters of the algorithms used in solving well placement problem of case 2.

GA	NP = 25 , CR = 0.80
DE	NP = 5 , F = 1 , CR = 0.50

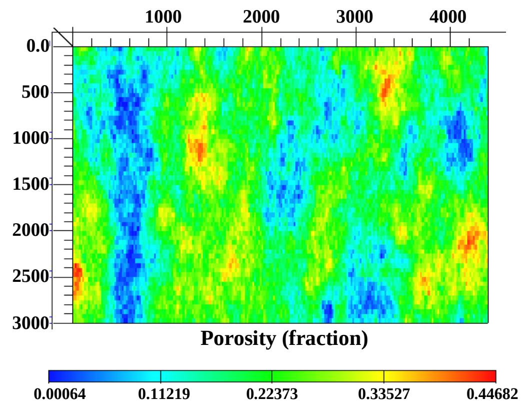

Fig. 4: Porosity distribution for reservoir models of Cases 2 & 3.

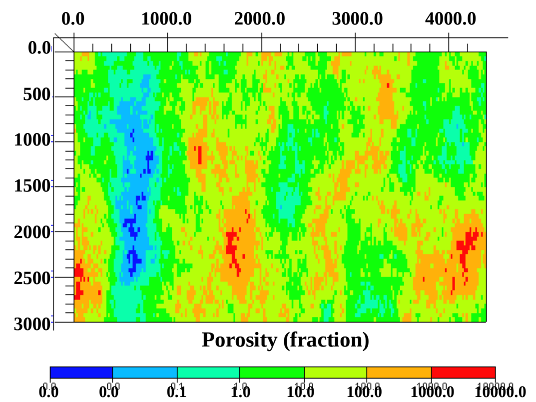

Fig. 5: Permeability distribution for reservoir models of Cases 2 & 3.

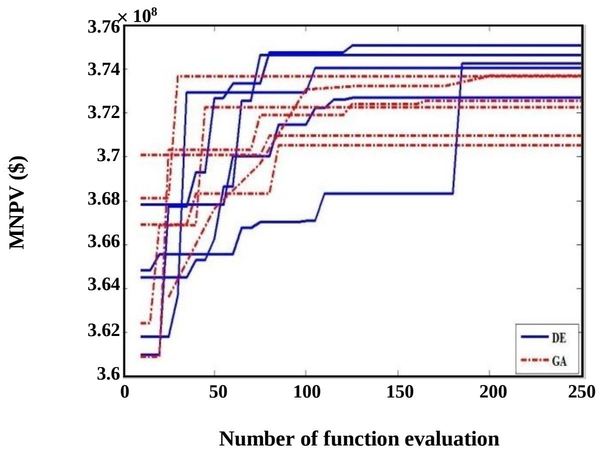

Fig. 6: The progress of optimization process for Case 2.

Table 4: Tuned parameters of the algorithms used in solving well placement problem of case 3.

GA	NP = 100 , CR = 0.80
DE	NP = 10 , F = 1 , CR = 0.50

location of four injection wells (I1, I2, I3, I4) in a reservoir which has already four fixed production wells (P1, P2, P3, P4) within itself. The reservoir model and its rock and fluid properties are the same as the case 2. Porosity and permeability distribution of the model are the same as Figs. 4 and 5, respectively. The reservoir has already four production wells in grid blocks: (68, 9), (the reservoir with minimum BHP constraint of 500 psi. Injection rate for each injector is considered to be 250 STBD and the total production time is 3650 days. A procedure similar to those of cases 1&2 has been used to tune the internal parameters of the algorithms and the results have been summarized in Table 4.

Then, we run the program for this problem using DE and GA as the optimization algorithms and compare their performances over 10 runs for each algorithm. We run the program for each case until 1000 function evaluations. Fig. 8 shows the optimization process and optimal MNVPs achieved in each case. The average MNPVs for DE and GA solutions are 2.5902×10^8 and 2.5575×10^8, respectively. Thus, DE solution is about 1.3% higher than that of GA. Optimum well locations determined in the 5th run are shown in Fig. 9. In both of them, the well locations are acceptable from the standpoint of reservoir engineering: the injection wells are located in proper distances from each other and, also, from production wells; and their locations are in the regions of reservoir which have relatively higher permeability than the other regions.

CONCLUSIONS

In this paper, we applied the DE algorithm for the problem of well placement optimization in an oil reservoir. The performance of this algorithm has been compared to GA, which is the most common optimization algorithm in the context of well placement optimization, through three case studies with the maximization of MNPV as the objective function. In all of these cases, we demonstrated that the DE algorithm provided better performance than the GA. Thus DE represents a viable alternative to GA, with a very simple structure than GA.

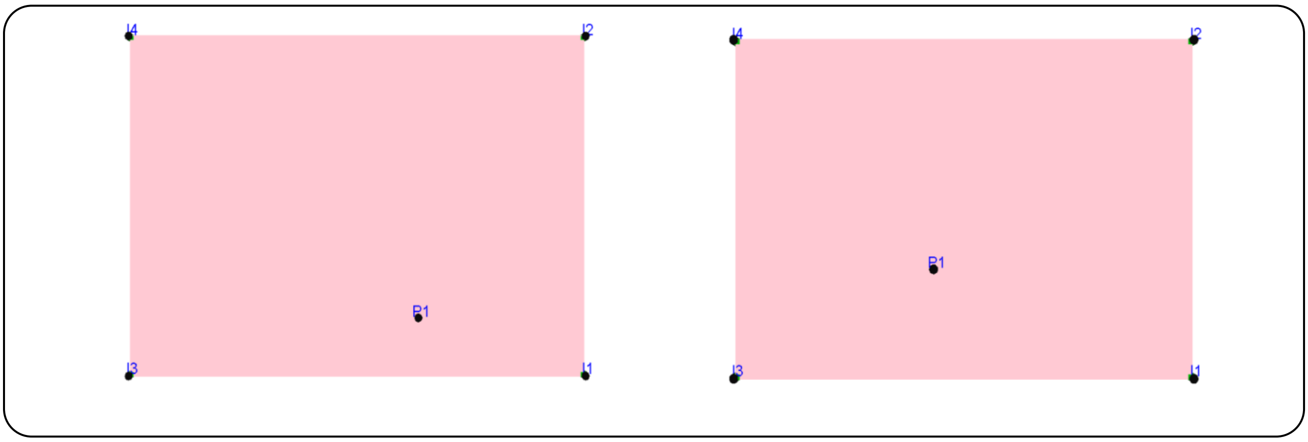

Fig. 7: Optimum location of production well achieved in the 5th run by (a) DE, and (b) GA for Case 2.

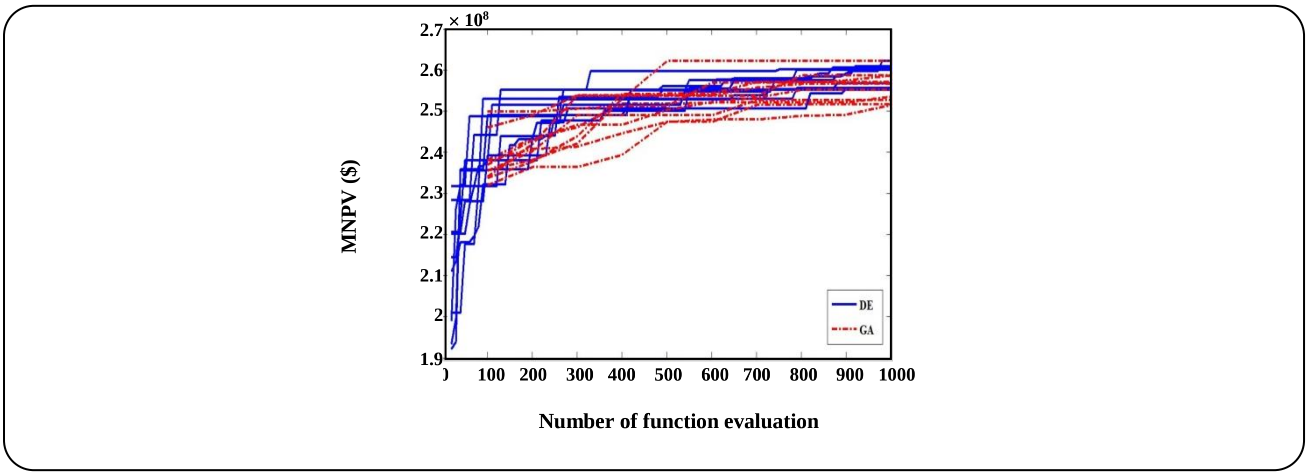

Fig. 8: The progress of optimization process for Case 3.

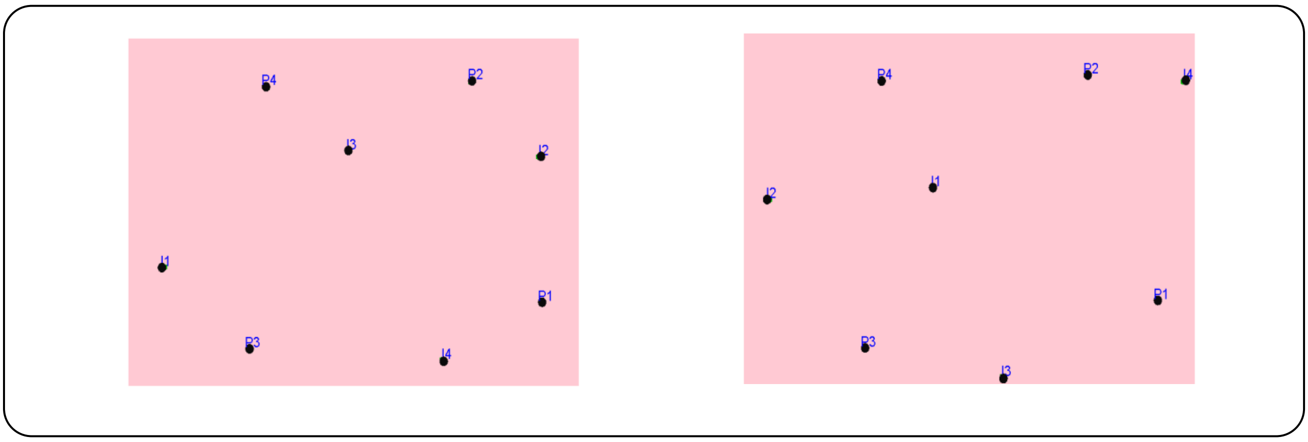

Fig. 9: Optimum location of injection wells achieved in the 5th run by (a) DE, and (b) GA for Case 3.

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