Transport Property Estimation of Non-Uniform Porous Media

Ghazanfari, Mohammad Hossein; Rashtchian, Davood*+

Department of Chemical and Petroleum Engineering, Sharif University of Technology, Tehran, I.R. IRAN

Kharrat, Riyaz

Petroleum Research Center, Petroleum University of Technology, Tehran, I.R. IRAN

Vossoughi, Shapour

Department of Chemical and Petroleum Engineering, Kansas University, U.S.A.

ABSTRACT: *In this work a glass micromodel which its grains and pores are non-uniform in size, shape and distribution is considered as porous medium. A two-dimensional random network model of micromodel with non-uniform pores has been constructed. The non-uniformity of porous model is achieved by assigning parametric distribution functions to pores throat and pores length, which was measured using image analysis technique. Statically derived expressions have been used for prediction of macroscopic properties of porous model including: dispersion coefficients, permeability-porosity ratio and capillary pressure. The results confirmed that predicted transport properties are in good agreement with the available experimental data.*

KEY WORDS: *Pore size characteristics, Distribution function, Micromodel, Dispersion, Capillary pressure, Permeability, Random particle method.*

INTRODUCTION

Transport properties of porous media are commonly determined by experimental methods which are most often time consuming and expensive. The geometry and topology of the microscopic pores control the fluid content and transport properties of porous medium, e.g., dispersion, capillary pressure and permeability. On the other hand, the exact solutions of fluid flow and convection-diffusion equations at the pore scale are extremely difficult to obtain, due to the complexity of the boundary conditions at the irregular pore/grain interface [1]. Therefore, it is important to have a reliable tool that can provide plausible estimates of macroscopic properties. Instead of searching for exact solution, research efforts have focused on ways to simplify the irregular pore systems. Such simplified equivalent versions of porous medium are called 'network model', which can be the only possible means of understanding the flow through porous media from a microscopic standpoint [2].

Network models can be applied either as stochastic models or not, depending on whether the pores which constitute the connections in the network are taken as having other than a probability distribution of sizes.

* To whom correspondence should be addressed.
+ E-mail: rashtchian@sharif.edu
1021-9986/09/2/29 14/$/3.40

In a sense, the randomly oriented capillary models are simply more general network models in which the connecting links in the networks are permitted a distribution of orientation, radius and length rather than being fixed as, for example, the edges of regular polyhedral.

The general model of randomly oriented pores is exemplified by the work of *Scheidegger* [3] which employed the movement of a random particle, under laminar flow conditions, through a homogeneous isotropic porous medium comprising system of capillaries with identical macroscopic characteristics. *de Josselin de Jong* [4] visualizes a porous medium as a randomly interconnected straight channels of equal sizes, orientated at random, uniformly distributed in all directions, in which average uniform flow takes place. *Saffman* [5] studied a similar geometric porous structure in which identical capillaries are orientated randomly. The pressure gradient in the medium was taken as linear with distance with an imposed fluctuation described by a Gaussian and isotropic probability density function. The last presented models have been extended by *Greenkorn* and *Kessler* [6] to the case of non-uniform media by use of a two parametric density function, beta distribution, both for the radii and for the length distribution individually. However, in this work a four-parametric probability density function has been used to express the pores throat and pores length distributions of the micromodel which were measured using image analysis technique.

It is well known that network modeling with disparate representations of the pore space have been also extensively employed in the simulation studies of multiphase flows in porous media. The various pore network models have been reviewed by, e.g., *Sahimi* [7]. Most models are based on a regular topology that does not reflect the random nature of real porous rock. In contrast, recent advances in rock imaging techniques, micro focused computed tomography [8], capture wealth of information about the microstructure of real porous medium, and provide a quantitative jump in the pore network model capabilities [9,10]. However, in this work an economical and easy tool, image analysis technique, was employed for generating the network model of porous micromodel.

Micromodel is small-scale artificial two-dimensional porous model which is known as a novel approach that can simulate natural porous media up to a certain degree [11]. The types of micromodel studies reported in the literature vary widely in their methods and applications [11]. Most of micromodel researches reported in the literature have been observational, and only a few of these studies attempted to quantify their observations. Also, very little attention has been paid on estimation of transport properties of micromodel which being based on statically measurements of pore size characteristics.

Dispersion in a porous medium arises from spatial fluctuations of the velocity field, which, in turn, are governed by the chaotic nature of the pore space morphology [12]. The numerical calculation of dispersion coefficients in porous media has been based on pore network simulations [13], Lattice-Boltzman algorithms [14], volume averaging [15], and method of moments [16].

A common problem in simulating solute transport in porous medium is numerical dispersion, which artificially smears concentration fronts in simulation results [17]. Varieties of approaches have been developed to overcome numerical dispersion. One effective approach is the random particle method [18, 19]. In this method, solute mass is represented by a large number of particles, and advection and dispersion are determined by the movement of particles. Unlike other commonly used methods based on finite element or finite difference schemes, random particle method does not require numerical solutions of partial differential equations [2, 19]. In this work statistical approach of random particle method was applied to a simple non-uniform model, and the longitude and transverse dispersion of micromodel were estimated.

The historical development of the capillary pressure phenomenon and measurement methods such as the centrifuge, mercury injection and porous plate have been discussed and documented by various researchers [2]. On the other hand, a few of researchers attempted to derive mathematical model of capillary pressure function, e.g., an analytical derivation of a parametric capillary pressure function has been demonstrated by *Ghazanfari et al*. [20]. However, in this work the capillary pressure curves are determined by a model which is in integral from.

Absolute permeability of a porous medium is depending on pore morphology and porosity [21], and is determined by measuring the pressure drop/flow rate response and using Darcy's Law [2]. In this study the

absolute permeability-porosity ratio of porous model is calculated as a function of average pore diameter.

In this work a glass type micromodel which its grains and pores are non-uniform in size, shape and distribution was used as porous medium. The network of pores in the porous model is regarded as a network of randomly capillaries with diameters and lengths governed by probability distribution functions. The model can be made anisotropic by distributing the orientation angle. The macroscopic properties of flow are related to the pore structure of micromodel. In order to model and then to estimate the transport properties of flow through porous medium, the pore size and pore length distributions of micromodel were measured using image analysis technique. A four parameter probability distribution function was used to express the measured distributions. A capillary pressure saturation mathematical model was developed and used for prediction of measured capillary pressure data. The statistical approach of random particle method was applied for derivation of dispersion coefficient models. As a related development an estimation of the absolute permeability-porosity ratio is also presented.

STATISTICAL DISTRIBUTIONS

In general, the pore size distribution can be described in terms of cumulative distribution function, which is equal to probability that pore diameter is equal to or less than D, or in terms of probability density function, which is equal to derivative of cumulative distribution with respect to pore size [22]. In practice, one can measure pore size distribution function giving the fraction of pore space which has a pore diameter greater than a given value. The basic empirical statistical model, f(d), chosen for this study is based on parametric probability density function proposed by *Ghazanfari et al.* [20] as:

$$f(D)=\frac{n\alpha(D-D_{min})^{n-1}}{\varepsilon^{n}}e^{-\left(\frac{D-D_{min}}{\varepsilon}\right)^{n}} \qquad (1)$$

$$D_{min}\le D\le D_{max}$$

Where α, ε and n are adjustable parameters, and they are related to physical properties of porous model. More details about the physical significance of model parameters and the behavior of proposed model by changing the parameters are given elsewhere [23].

D_{min} and D_{max} is minimum and maximum pore throat or pore body size dependent to applied case, respectively. If D_{min} goes to infinity parameter α is equal to one. Therefore, α=1 is corresponding to threshold capillary pressure which equals zero [23]. Assuming that pore length, l, distribution is also obeys from probability distribution function of equation (1) as follow:

$$f(l)=\frac{n\alpha(l-l_{min})^{n-1}}{\varepsilon^{n}}e^{-\left(\frac{l-l_{min}}{\varepsilon}\right)^{n}} \qquad (2)$$

$$l_{min}\le l\le l_{max}$$

Where l_{min} and l_{max} are minimum and maximum pore lengths, respectively. The parameters of equations (1) and (2) are calculated by fitting the statistical model to the measured data of pores throat, pores body and pores length distributions of porous model. Most of probability distribution function models are two parametric. Lack of flexibility to simulate natural petrography phenomena and particularly suffering from the shortcoming of infinite range is contrary to the finite size of pore diameter and pore length of equations (1) and (2), respectively.

EXPERIMENTAL DESCRIPTIONS

Micromodel

The glass micromodels, a synthetic porous medium that lends itself well to achieving the experimental requirements of this work, are mostly fabricated by etching the desired pore network pattern on two plates of mirror glass which are then fused together. Using this method, highly intricate and detailed patterns can be etched with the dimensions of pores and throats as low as a few microns. Details of the model production procedure are given elsewhere [23, 24].

The micromodel used in this study included grains and pores which are non-uniform in size, shape and distribution. Fig. 1 shows the micromodel which is saturated with the colored water. the black and white prints the color of water and grains are gray and white, respectively.

Setup

The setup is combined an etched glass micromodel, a camera, a precise pressure transducer and a precise low rate pump which is used to control the flow rate of fluids through micromodel. The micromodel was connected

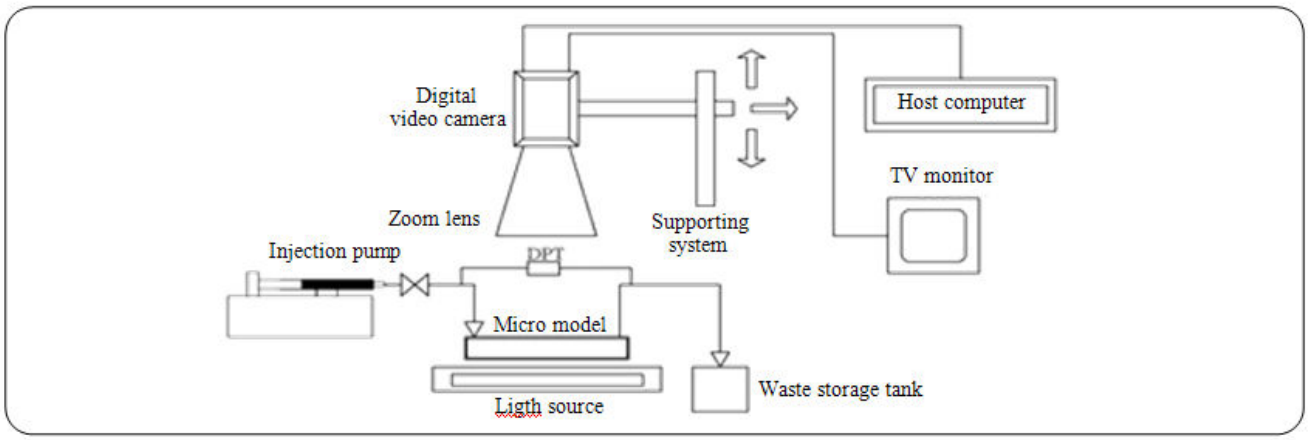

Fig. 2: Schematic of micromodel setup.

to the flow lines by specially designed clamping system. The pump is used to inject working fluid depending on the request at a fixed rate from a minimum of 1.5e-5 to a maximum 10 cc/min into the micromodel. A high resolution color TV camera with long distance focusing lens attachment mounted on a racking system was used to observe and record on video the events in micromodel. Experiments were conducted at room temperature and using a horizontal mounting. The Schematic of experimental setup is shown in Fig. 2. Detailed descriptions about the experimental setup are given elsewhere [23].

Micromodel properties

Absolute permeability of micromodel was determined using Darcy's Law by measuring the pressure drop/flow rate response. Distilled water at room temperature was injected into the micromodel at desired injection flow rate, and the pressure drop between the injection port and the exit was measured. Four different pressure drop and flow rate data sets were obtained and the best fitting straight lines passed the origin.

The porosity of micromodel was measured using image analysis technique. Fig. 1 shows the micromodel which it has been fully saturated with colored water. The etched depths of the pores in the micromodel are relatively uniform, so the ratio of colored area to the total area of micromodel, areal porosity, is equal to porosity. Due to increase the clearance, the color of water and grains has been changed to gray and white, respectively. A generated image analysis computer code and conventional enhanced image software are used to process the image.

During the injection of colored water into the micromodel with known flow rate, for a desired incremental time, the incremental area occupied by the colored water was measured using image analysis technique. Incremental time multiply by injection flow rate divided by incremental area is equal to average etched height on that region. The pore volume is equal to areal porosity multiply by average etched height. The physical and hydraulic properties of micromodel are shown in table 1.

Table 1: Physical and hydraulic properties of micromodel.

Length (cm)	7.5	Width(cm)	1.2
Absolute permeability (D)	13.3	Average areal porosity	0.485
Average depth (micron)	32	Pore volume(cm^3)	0.014

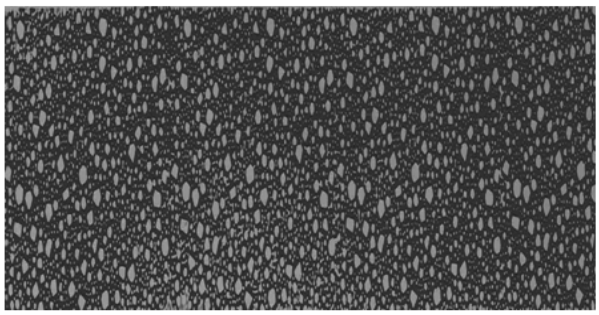

Fig. 1: Micromodel pattern which is fully saturated with colored blue water.

Pore size distribution

The pore size distribution analysis was performed on micromodel shown in Fig. 1 which is saturated with the colored water. The grains and pores of micromodel shown in this figure are non-uniform in size, shape and

distribution. This non-uniformity of geometry and topology of the microscopic pores control the fluid content and transport properties of porous medium, e.g., dispersion, capillary pressure and permeability. Therefore, it is important to measure the pore size characteristics of porous model.

A pore can be described in term of its diameter, length and orientation in a flow field. In systems of interconnected pores this implies a partitioning of pore space into pore body, pore throat and pore length. In fact, pore structure of porous medium is described by the pore throat, pore body and pore length distribution and a pore size distribution curve represents the cumulative fraction of total pore volume within porous media sample made up by particular ranges of pore sizes.

Dimensions of pores throat, pores body and pores length of porous model are computed from data obtained by two dimensional image analysis as follow: the area of pore body and the width of pore throat within a representative region of the micromodel were measured by the image analysis of porous model is shown in Fig. 1. An important condition for successful application of image analysis in the statistical analysis of the porous model is a sufficient contrast between pores and grains. Employing micromodel average etched depth, the total volume of the pore body was calculated by multiplying the planar pore area with the height. An average diameter was determined using an equivalent sphere whose volume was equal to the volume of the pore body.

To determine the pore throat size, the width of pore throat was measured and the cross-sectional area was obtained by multiplying by the height. An average pore throat diameter was determine from the area of the circle equal to the cross sectional area of a pore throat. The pore length is the distance between two connected pore bodies along the flow field. To present the exact values of measured pores size characteristics, the frequency distribution diagrams of pore throat and pore body diameters in the form of f(D) versus $D-D_{min}$ and pore lengths in the form of f(l) versus $l-l_{min}$ are plotted on Fig. 3a-c. These figures provide the minimum, maximum and average values of pores throat diameter, pores body diameter and pores length, also. The parameters α, n, of pore throat and pore body size distribution models could be estimated by fitting the statistical model f(D) of equation (1) to the measured data of pore throat and pore body size distributions on the Figs. 3a and 3b, respectively.

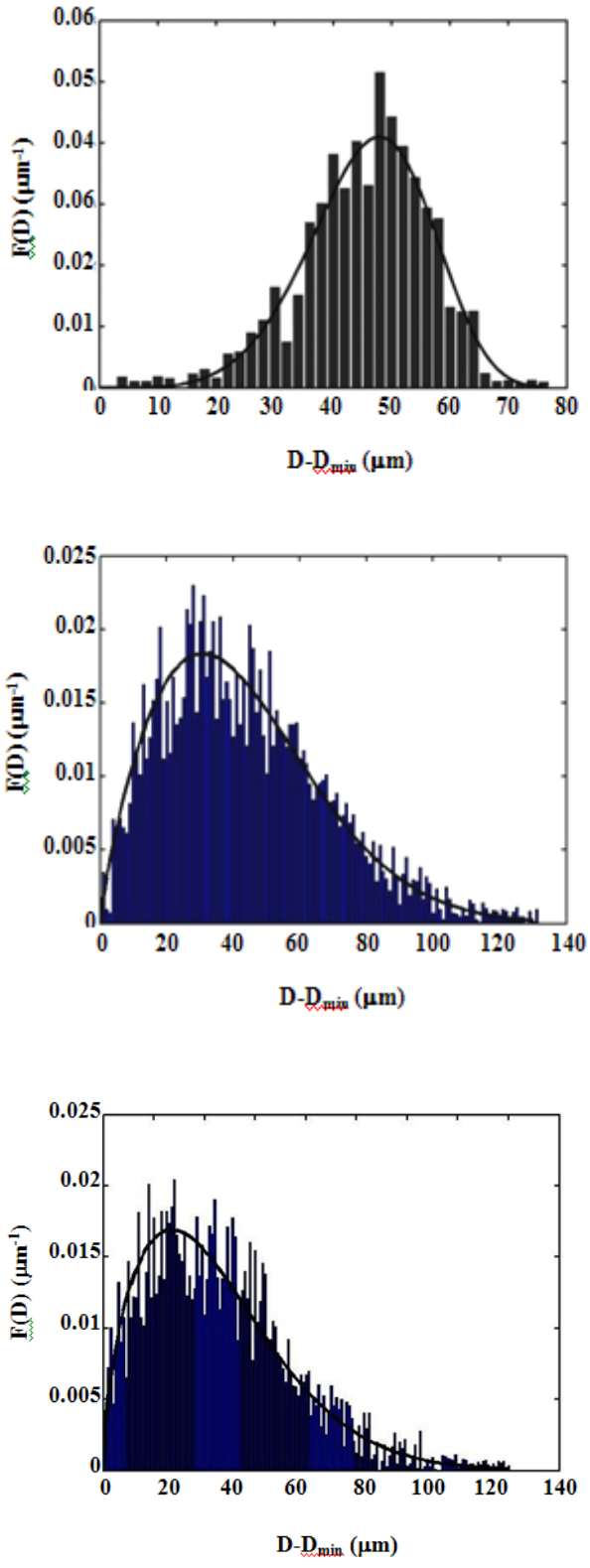

Fig. 3: (a) Frequency distribution diagrams of porous model pore throat size (b) pore body size (c) pore length. Solid lines are the fitted statistical model behavior.

Table 2: Fitted parameters of pore throats diameter, pore bodies diameter and pore lengths distribution models, and the measured minimum, maximum and average values.

	N	ε	α	Min. (μm).	Max. (μm)	Ave. (μm)
Pore throat	5.1	55	1.07	4	80	41.0
Pore body	1.8	48	1.09	4	134	44.5
Pore length	1.6	50	1.11	5	165	49.4

The parameters α, n, ε of pore length distribution model could be estimated by fitting the statistical model f(l) of equation (2) to the measured data of pore length distribution on the Fig. 3c. The measured minimum, maximum and average values of pores throat and pores body diameter, pores length and the fitted parameters of distribution models are given in table 2.

The plotted solid line in the Fig. 3a-c is the behavior of the statistical distribution functions of equations (1) and (2) with the models parameters of the table 2.

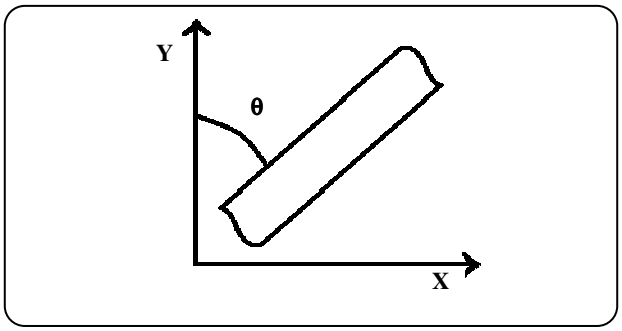

Fig. 4: Pore element description.

MATHEMATICAL MODELING

One of the difficulties in calculating the transport properties of flow through porous media is that the detailed geometry of medium is not usually known and the flow field can not be computed in detail. Therefore, theoretical method is at the present confined to the investigation of models which can be handled mathematically.

Absolute permeability

The pore space model is approximated by a random, statistically anisotropic, network of straight cylindrical capillaries of length l and diameter D, several capillaries starting and finishing at each junction. The porous model is two dimensional. The pore element in this network which is presented in Fig. 4 is oriented by an angle θ from the y axis. It is clear that $0 \le \theta \le \pi$ for the case of flow in the x direction.

Assume the size, length and orientation of pore elements are independent then the distinct probability distribution functions of pores size, f(D), and pores length, f(l), are as equations (1) and (2), respectively. The model is conceived as large number of randomly intersecting pore element. The probability of a given pore exists with size in the range l to l+dl, D to D+dD, and θ to θ + dθ is given by the product of the independent probabilities such as:

$$dP = \frac{1}{2} f(l) dl . f(D) dD . \sin\theta \, d\theta \qquad (3)$$

Where the coefficient of 1/2 is normalization factor, and is equal to $1/2\pi$ for the case of 3D pore element. The permeability of the model is found by relating the average velocity in a pore element to the average velocity in the ensemble. Consider the bulk flow through the model is only in the x direction, and consider the Reynold number of flow through capillaries is small, then according to the Hagen-Poiseuille equation:

$$v = -\frac{D^2}{32\mu} \frac{\partial p}{\partial x} \sin\theta \qquad (4)$$

The components of this velocity are:

$$v_x = v\sin\theta \quad ; \quad v_y = v\cos\theta \qquad (5)$$

The average velocities in each direction for the ensemble of pores are found by the integrating of the velocity components in equation (5) over the entire range of pores size and orientation such as:

$$\bar{v}_x = \int_P v\sin\theta \, dP \quad ; \quad \bar{v}_y = \int_P v\cos\theta \, dP \qquad (6)$$

Where dP is given by equation (3) and integrating limits are, the range of D, l and θ. The integrating of the average velocity in the y direction is $\bar{v}_y = 0$. The average velocity in the x direction is:

$$\bar{v}_z = -\frac{\langle D\rangle^2}{48\mu}\frac{\partial p}{\partial x} \tag{7}$$

Where $\langle D\rangle$ refers to average value of pores throat diameter. The average pore velocity for a fluid flowing in porous medium is given by Darcy law as:

$$V = -\frac{k}{\mu\phi}\frac{\partial p}{\partial x} \tag{8}$$

Where k, μ, ϕ, V, p are absolute permeability, viscosity, porosity, Darcy velocity and pressure, respectively. Since $V = \bar{v}_x$, combination of equation (7) and equation (8) yield:

$$\frac{k}{\phi} = \frac{\langle D\rangle^2}{48} \tag{9}$$

It might be mentioned that without considering the statistical concepts the right hand side of equation (9) the factor 1/48 become 1/32 which is a result of direct combination of Darcy's law and Hagen-Poiseuille equation [2].

Capillary pressure

Capillary pressure in porous media is simply defined as the pressure difference existing between two immiscible fluids, one of which wets the surfaces of the rock in the presence of the other. When viscous and body forces are negligible, the configuration of two fluids in porous media is governed by the Young-Laplace equation which relates capillary pressure P_c to the curvature of the interface between two fluid phases:

$$P_c = C\sigma \tag{10}$$

Where P_c is applied pressure, σ is the interfacial tension between them and C is the sum of two principal curvatures, or twice the mean curvature of the interface. Suppose that the pore throat have cylindrical configuration. Then the radiuses of curvature are equal, so C=4/D. In spite of the fact that equation (10) describes a static configuration, it is commonly applied to the displacement of one immiscible phase by another, when that displacement occurs sufficiently slowly.

The modeling to be undertaken refers to the capillary pressure de-saturation named it drainage process where a non wetting fluid is forced into pore space originally occupied by a wetting fluid. Such displacement and replacement will occur as long as a pressure difference between the non wetting and wetting fluids is greater than that to be associated with the capillary pressure that balances the forces so that the fluid-fluid interfaces remain stationary in the pore throat [25].

Since the capillary pressure of pore depends on its diameter, in the analysis presented here the assumption is made that the maximum equivalent pore throat diameter D_{max} corresponds to threshold capillary pressure P_{thr} at the 100 % water saturation S_w, and the minimum equivalent pore throat diameter D_{min} corresponds to maximum capillary pressure P_{cmax} at the residual water saturation S_{wr}. For a porous medium initially completely filled with the water, define V_P as the total pore volume occupied by water. During drainage process, the change of volume of water into the pore space of the porous model decreased by a differential saturation dS_w is $V_P dS_w$. This saturation change results from invading fluid entering all pores in the model of diameter D. So one can easily write:

$$V_P dS_w = -N\langle l\rangle(\pi D^2/4)f(D)dD \tag{11}$$

Where N is the number of pores and $\langle l\rangle$ is average of pore lengths. Equation (11) is equivalent to:

$$dS_w = -N\langle l\rangle\pi D^2 f(D)dD/4V_P \tag{12}$$

Since pores diameter and pores length are independent, the term V_P in the right hand side of equation (12) is equal to $N\langle l\rangle\pi\langle D\rangle^2/4$. So:

$$dS_w = -D^2 f(D)dD/\langle D\rangle^2 \tag{13}$$

The capillary pressure in a partially saturated pore is calculated by equation (10). Based on this equation, as the diameter of pore become smaller the capillary pressure will increase while the water saturation will decrease. Therefore, as the pore diameter varies from D_{min} to D_{max}, water saturation varies from S_{wr} to 1. Integrating equation (13) after substituting the pore diameter from equation (10) to equation (1), and omitting the term $4\sigma/P_{cmax}$ result in:

$$S_{wr} - S_w = \tag{14}$$
$$\int_0^{4\sigma/P_c} \frac{1}{\langle D\rangle^2}\left(\frac{4\sigma}{P_c}\right)^2\left(\frac{\alpha(4\sigma/P_c)^{n-1}}{\varepsilon^n}e^{(4\sigma/P_c\varepsilon)^n}\right)d\left(\frac{4\sigma}{P_c}\right)$$

At the residual wetting saturation S_{Wr} the corresponding capillary pressure P_{cmax} is assumed to be

much greater than the measured capillary pressure, Therefore the omitted term in equation (14), $4\sigma/P_{cmax}$, is negligible. Equation (14) is the capillary pressure water saturation model. The model parameters can be calculated by fitting the prediction of the model to available data. Generally, the term of capillary pressure used in this study should be considered as being the macroscopic capillary pressure for both static and dynamic fluid flow conditions [26].

Dispersion

Hydrodynamic dispersion in a porous medium occurs as a consequence of two different processes: (i) molecular diffusion, which originates from the random molecular motion of solute molecules, and (ii) mechanical dispersion, which is caused by non-uniform velocities and flow path distribution. Molecular diffusion and mechanical dispersion cannot be separated in a flow regime. The transport process of a non-reacting solute in saturated porous media is described by the macroscopic convection-diffusion equation, mass conservation law, as follow [27]:

$$D_L \frac{\partial^2 C}{\partial x^2} + D_T \frac{\partial^2 C}{\partial y^2} - v_x \frac{\partial C}{\partial x} - v_y \frac{\partial C}{\partial y} = \frac{\partial C}{\partial t} \qquad (15)$$

Where C is the solute concentration, t is time, v_x and v_y are the components of flow velocity, D_L and D_T are the hydrodynamic longitude and transverse dispersion coefficients, respectively.

The solution of the convection-diffusion equation, Equation (15), is difficult due to the co-existence of spatial first and second order terms. The first order term, which describes the advective motion, introduces a hyperbolic character, whereas the second-order term, which describes the diffusive/dispersive motion, introduces a parabolic character. Up to date, there exists no technique which yields satisfactory results under general hydrogeologic conditions and for practically feasible discretization. Numerical solution techniques can be classified as Eulerian, Lagrangian and mixed Eulerian-Lagrangian. In the well-established Eulerian approach, the equation (15) is solved on a stationary grid. Common solution techniques are finite difference, finite element or finite volume methods. These kinds of methods are well suited for solving parabolic equations like the flow equation or the diffusion equation. However, for advection-dominated problems grid-based methods suffer from numerical dispersion and numerical oscillations. To avoid these problems, severe stability constraints restricting the grid spacing and the size of the time step have to be met. One of the effective approaches to overcome this problem is the random particle method. The adaptivity of particle methods and the straightforward physical interpretation of their results make them a promising alternative to established grid-based methods in the field of solute transport dynamics in groundwater. The interest in particle methods is big due to the fact that accurate solutions not suffering from numerical dispersion and artificial oscillations can be obtained at a competitive computational cost.

The random particle is a marked fluid particle as it wanders through the porous medium selecting elemental pores for each step according to the prescribed probability function. Each passage of a particle through an individual pore is a step in random walk. The properties derived here will be fulfilled after a particle has completed a very large number, n, of statistically independent steps.

At each junction the probability of path choice must be related to the probability of existence of a pore, equation (3), and to the fluid velocity at a junction. It is assumed that a marked particle takes a random walk through the model selecting pores proportional to the to the velocity,v, given by equation (4) as:

$$dE = \frac{v}{M} dP \qquad (16)$$

where, M is the normalization constant. Substitution equations (3) and (4) into equation (16), after applying the normalization, since the integral over dE must equal unity, one can get:

$$dE = \frac{2}{\pi \langle D^2 \rangle} . f(l) d(l) . D^2 f(D) dD . \sin^2 \theta d\theta \qquad (17)$$

Then the displacement of a marked particle after n steps is a random variable with components parallel to the axes:

$$\overline{X}_n = n \int_E l \sin\theta dE \quad ; \quad \overline{Y}_n = n \int_E l \cos\theta dE \qquad (18)$$

Substituting equation (17) into equation (18), and integrating result in, $\overline{X}_n = 8n\langle l \rangle / 3\pi$ and $\overline{Y}_n = 0$.

And the time for n steps in random variable:

$$T = \overline{T}_n = n\int_E (l/v)\,dE \tag{19}$$

By substituting equations (4) and (17) into equation (19) and performing integration, one gets,

$$\overline{T}_n = 8n\langle l\rangle / 3\pi\,\overline{v}_x \tag{20}$$

Which leads to the result that a particle following the most probable path is transported through the porous medium with a velocity, $(\overline{X}_n/\overline{T}_n) = \overline{v}_x$, equal to the Darcy velocity. For randomly orientated uniform and isotropic network model the factor $8/3\pi$ in equation (20) changes to 2/3 [4, 5]. For the sake of clarification, it should be mentioned that for a randomly oriented uniform and isotropic network model the pores have equal diameter and length; hence, the time for n steps reduces to $\overline{T}_n = 2nl/3\overline{v}_x$. However, in non-uniform and anisotropic network model the pores diameter and length are distributed.

The longitude and transverse dispersion coefficients are define in terms of variance of the longitude and transverse displacements and the time required for such displacements as:

$$D_L = \frac{\overline{(X - vT)^2}}{2T} \tag{21}$$

$$D_T = \frac{\overline{Y^2}}{2T} \tag{22}$$

where, $\overline{(X - \overline{V}T)^2}$ and $\overline{Y}^2$ are the variance of the average displacements in the x and y directions, respectively.

Since the variance of Y is the sum of the variances of the transverse displacement of the individual steps and $\overline{Y}_n = 0$; hence,

$$\overline{(Y_n - \overline{Y}_n)^2} = \overline{(Y_n)^2} = n\overline{y^2} = n\int_E y^2\,dE = \tag{23}$$

$$n\int_E (l\cos\theta)^2\,dE = n\langle l\rangle^2\sigma_y^2$$

This yields,

$$\sigma_y^2 = \langle l^2\rangle / 4\langle l\rangle^2 \tag{24}$$

Let us define the following dimensionless coordinates and time,

$$\xi = \frac{Y_n - \overline{Y}_n}{\langle l\rangle n^{1/2}} \tag{25}$$

$$\tau = \frac{T_n - \overline{T}_n}{\langle l\rangle n^{1/2}/\overline{v}_x} \tag{26}$$

It should be noticed that ξ and τ have means equal to zero and variances equal to σ_y^2 and σ_T^2, respectively. Substituting the derived value of $\overline{T}_n$ from equation (20) into equation (26) we get,

$$\tau = \frac{T}{\langle l\rangle n^{1/2}/\overline{v}_x} - \frac{8n^{1/2}}{3\pi} \tag{27}$$

The subscript n in T_n is dropped for the sake of simplicity. The value of n satisfied equation (27) is:

$$n = \frac{3\pi}{8}\frac{\overline{v}_x T}{\langle l\rangle} + \frac{1}{2}\left(\frac{3\pi}{8}\right)^2\tau^2 - \tag{28}$$

$$\frac{1}{2}\tau\left(\frac{3\pi}{8}\right)^2\left(\tau^2 + 4\frac{8}{3\pi}\frac{\overline{v}_x T}{\langle l\rangle}\right)^{1/2}$$

Neglecting the terms τ^2 or $O(\sigma_T^2)$, equation (28) may be written as:

$$n = \frac{3\pi}{8}\frac{\overline{v}_x T}{\langle l\rangle} - \frac{1}{2}\left(\frac{3\pi}{8}\right)^2\tau\left(4\frac{8}{3\pi}\frac{\overline{v}_x T}{\langle l\rangle}\right)^{1/2} \tag{29}$$

Rearranging equation (29) and applying square rule, then neglecting the terms τ^2 or $O(\sigma_T^2)$,

$$n^{1/2} = \left(\frac{3\pi}{8}\frac{\overline{v}_x T}{\langle l\rangle}\right)^{1/2} - \frac{1}{2}\left(\frac{3\pi}{8}\right)\tau \tag{30}$$

Substituting the equation (30) into the equation (25), result in:

$$Y = \left(\frac{3\pi}{8}\overline{v}_x\langle l\rangle T\right)^{1/2}\xi - \tag{31}$$

$$\frac{1}{2}\left(\frac{3\pi}{8}\right)\langle l\rangle\xi\tau \approx \left(\frac{3\pi}{8}\overline{v}_x\langle l\rangle T\right)^{1/2}\xi$$

Then:

$$\frac{\overline{Y^2}}{T} = \frac{3\pi}{8}\overline{v}_x\langle l\rangle\overline{\xi^2} \tag{32}$$

The expression for the transverse dispersion coefficient is obtained from substituting the value of σ_y^2 (equation (24)), variance of ξ, into equation (32). We can immediately get:

$$D_T = \frac{3\pi}{64}\frac{\langle l^2\rangle}{\langle l\rangle}\bar{v}_x \qquad (33)$$

For the sake of clarification, it should be mentioned that for an ensemble of randomly oriented uniform and isotropic network model the pores have equal diameter and length; and the transverse dispersion expression, can be derived as $D_T = 3\bar{v}_x\, l/16$ [5, 6].

If one assumed that a marked particle takes a random walk through the model selecting pores proportional to the volumetric flow rate, the resulted equations (16), (17), (20) and (27) to (33) will be different, and the transverse dispersion expression, equation (33), reduces to $D_T = 3\pi\langle D^4\rangle\bar{v}_x\langle l^2\rangle / 64\langle D^2\rangle^2\langle l\rangle$ [23,27]. Details of derivation method of longitudinal dispersion expression, which is much longer than the transverse dispersion, for both cases in which a random walk through the model selecting pores proportional to either the volumetric flow rate [27] or to the velocity [23,27] are given elsewhere.

One condition which must be satisfied in order that this model should apply to the actual dispersion of a material quantity is that the amount of dispersion that takes place being large in comparison with the dispersion due to molecular diffusion acting alone under static conditions.

RESULTS

Absolute permeability

The absolute permeability-porosity ratio is a function of the average of pore diameter, and so depends on the pore size distribution function. Thus, the permeability-porosity ratio causes dissipation due to entrance-exit effect. Since the flow in porous media is controlled by pore throat size, substituting the average of pore throat diameter of porous model, $41\,\mu m$, which is measured experimentally, into equation (9) results in the absolute permeability-porosity ratio equal to 35.02 D. It is in good agreement with the experimentally measured value, 27.42 D, and the error is 27.7 %. The calculations without considering the statistical concepts gives the absolute permeability porosity ratio equal to 52.53 D which it has a amount of error 91.5 % with the measured data. The foregoing discussion leads to an interesting possibility to determine the average pore throat size of porous model using permeability and porosity measurements with equation (9).

Capillary pressure

For a long time capillary pressure verses saturation data have been employed by researchers interested to the pore size characteristics. Since the capillary pressure of a pore depends on its diameter, the capillary pressure of a porous model depends on distribution of pore throat diameter. So the location and shape of capillary pressure curve can be defined by the parameters of pore throat size distribution function. The location of the capillary pressure versus saturation curve is defined by the position of its two asymptotes. At infinite pressure the capillary pressure touches the vertical asymptote, which indicates the bulk volume occupied at the infinite pressure or residual water saturation or may be total interconnected pore volume.

When the bulk volume occupied by injected fluid is zero, the curve touches the horizontal asymptote which indicates the extrapolated displacement pressure named it threshold pressure. Capillary pressure and relative permeability both depend on the same fluid-fluid and rock-fluid interaction energies. Relative permeability is used to describe quantitatively simultaneous transport of two or more immiscible phases through a sample porous medium. If the capillary pressure vanishes, the residual saturations approach zero and the relative permeability of a phase becomes equal to its saturation. Relative permeability models in the literature are therefore often inferred from a capillary pressure correlation coupled with pore network models.

The parameters of capillary pressure model, α, n, n, ε in equation (14), were estimated by fitting the capillary model to the measured capillary pressure data [28] of drainage process in micromodel at different of capillary numbers. The circles in Fig. 5(a) to 5(d) are shown that the model predicted capillary pressure values in which matched well with the experimental data. The parameters of fitted model are given in table 3. The correlation coefficients R_{xy} are all above 0.985.

Table 3: Fitted parameters of statistical function at different capillary numbers.

Ca	S_{wr}	n	ε	α	R_{xy}
1.6e-7	0.22	8.5	120	1.04	0.997
8e-7	0.16	7.1	96	1.05	0.995
1.6e-6	0.13	5.2	72	1.08	0.992
8e-6	0.11	4.1	62	1.08	0.989

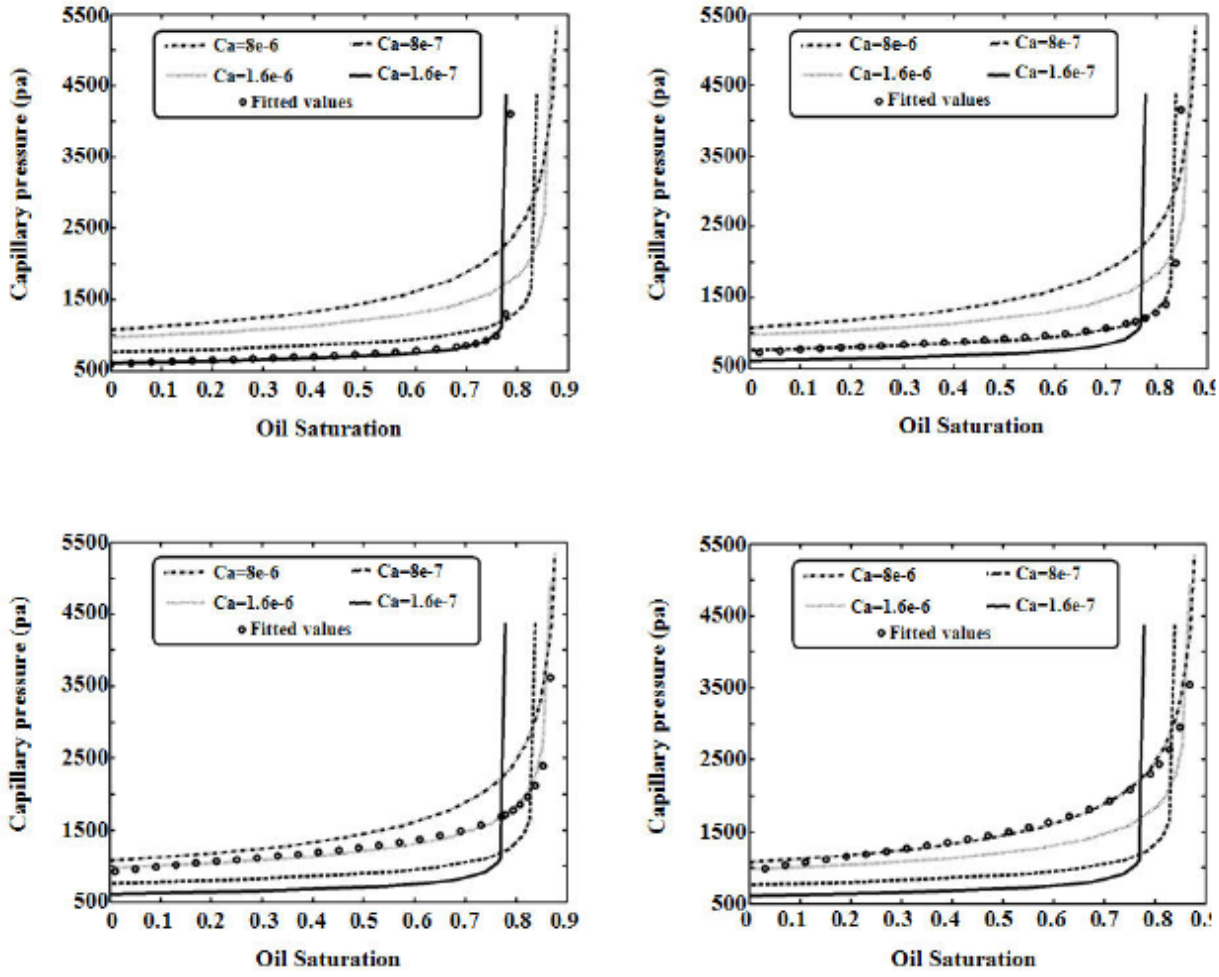

Fig. 5: Fitted capillary pressure model and experimentally measured capillary pressure data of drainage process at four level of capillary number.

Dispersion

The convection-diffusion equation is valid only if the solute particles have sufficient time to be distributed by molecular diffusion between all the streamlines of a representative elementary volume. The characteristic time required for the distribution of the solute particles between the available streamlines is of the order $\langle 1 \rangle^2/D_m$, where D_m is the molecular diffusion. This time must be at least equal to the characteristic time $\langle 1 \rangle/V$ required for the solute to move from the injection point, x=0, to the observation point x. Very often, under experimental conditions, and in heterogeneous media like stratified systems this criterion breaks down and in this case the equation of permanent dispersion is no longer valid.

Therefore, the question is how to describe this transient dispersion process mathematically, which in not the purpose of this study.

Molecular diffusion affects the material quantity moving through a pore in two ways. First, the material quantity diffuses sideways across the pore so that an element of material quantity does not stay on a streamline with constant radius but spreads out over the neighboring streamlines. The second is transport of material quantity by diffusion along the pore. Here, the value of the ratio of longitude diffusivity characteristic time to radial diffusivity characteristic time, $(\langle 1\rangle^2/2D_m)/(\langle D\rangle^2/32D_m)\approx 100$, so the fast radial diffusivity assumption is reasonable.

The model results for transverse dispersion as a function of fluid velocity are plotted in Fig. 6 together with experimental data of micromodel [23]. It is obvious that the predictions of transverse dispersion coefficient using equations (33) are in close agreement with the measured data. For the experimental data presented here, molecular diffusion becomes controlling mechanism of transverse dispersion for the fluid velocities lower than 0.01 cm/s. That is why no data were compared with the model prediction for the fluid velocities below this level. As mentioned earlier, the model assumes that the amount of dispersion that takes place being large in comparison with the dispersion due to molecular diffusion acting alone.

The close agreement observed between prediction of transverse dispersion coefficient and experimental data shown in Fig. 6, when dispersion is large in comparison with the molecular diffusion, confirms that the assumptions of theory and statistical network model, even though is simple, are not unreasonable. When molecular diffusion being negligible, empirical equations for transverse dispersion coefficient shows that this coefficient is proportional to the average velocity and pore space characteristics. This is what essentially shown by equation (33).

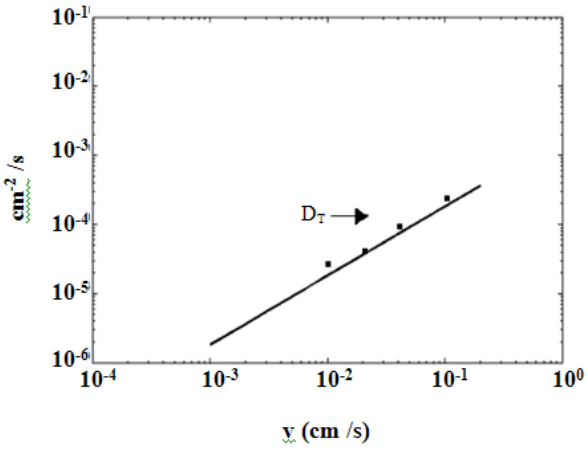

Fig. 6: Comparison of predicted transverse dispersion coefficient (solid lines) with the experimental data of the micromodel.

DISCUSSIONS

Actual porous media even though seemingly homogenous and isotropic are most often non-uniform. This fact may affect the transport properties of porous media such as dispersion, permeability and capillary pressure which is a result of the tortuous and circuitous nature of the flow paths in medium, and are commonly obtained from experimental measurements. Any theoretical or numerical approach to solve the governing equations of flow through porous media not only needs a detailed understanding of the phase displacement mechanisms at the pore level but also an accurate and realistic characterization of the structure of the porous medium. An attractive alternative approach is an ensemble of randomly oriented and distributed non-uniform pores which the pores are supposed to be connected with each other at the ends and several may start or finish at this end-points.

In the study presented here, a glass micromodel which its grains and pores are non-uniform in size, shape and distribution was used as porous medium. Transparent nature of interconnected pores and throats of the micromodel makes it very attractive to enhance understanding of various aspects of transport phenomena in porous medium. In order to relate the macroscopic properties to the structure of porous model, the pores throat, pores body and pores length distributions of micromodel were measured by applying the image analysis technique. The measured pore size characteristics expressed by a four-parametric probability density function in which the parameters have physical significances. A simple random model of micromodel pore structure, which is a non-uniform pores network, the connecting links in the networks are permitted a distribution of orientation, radius and length, was used for prediction of transport properties of micromodel. A capillary pressure saturation mathematical model was

developed which is in integral form. The model parameters are estimated by fitting the statistical model to the measured capillary pressure data at different of capillary numbers. The obtained capillary pressure results matched quite well with the measured values. The random particle method in combination with the distribution functions of pores size and pores length, permitted a new derivation of transverse dispersion expression. Predictions from the derived model of transverse dispersion coefficient are in close agreement with the experimental data under the condition that the molecular diffusion is negligible. This confirms that the assumptions of theory and statistical network model, even though is simple, are not unreasonable. As a related development based on measured pore size distribution the absolute permeability-porosity ratio as a function of average pore diameter is calculated.

Despite the simplicity of the proposed network model, it seems that it is a strong tool for investigation of the processes which occur in micromodel. In particular, it allows a priori predictions of macroscopic behavior. This capability is the most important aspect of the approach. Because there are no prescribed parameters, verifying the models predictions via experimental data provides physical insight. If the predictions are not consistent with the data, it can be concluded that the model does not account for some essential part of the physical situation, and then seek a more realistic model. On the other hand, a successful prediction allows some confidence that the model can be taken as a reasonable approximation of the actual situation and, thus, has a predictive capability and may be used to examine different porous medium.

CONCLUSIONS

The most important conclusions of the work are outlined below:

- A simple non-uniform network model has been proposed and successfully used for modeling of transport properties in micromodel.

- The image analysis technique was successfully applied for measuring of pores throat, pores body and pores length distributions of micromodel.

- A parametric capillary pressure saturation mathematical model which is in the integral form was obtained. The predicated capillary pressure results matched quite well with the measured data at different of capillary numbers.

- The estimated absolute permeability-porosity ratio based on statistical model of pore size distribution is in agreement with the experimentally measured value.

- The absolute permeability-porosity ratio is a function of average pore diameter and the foregoing discussion leads to an interesting possibility to determine average pore throat size of porous model using permeability and porosity measurements.

- The statistical approach of random particle method was applied and a new model of transverse dispersion has been derived. The model results match well with the experimental data.

- The close agreement observed between predictions of transport properties models and experimental data confirms that the assumptions of theory and statistical network model, even though is simple, are not unreasonable.

Received : 2nd October 2007 ; Accepted : 22nd July 2008

REFERENCES

[1] Man, H. N., Jing, X. D., Pore Network Modeling of Electrical Resistivity and Capillary Pressure Characteristics, *Transp. Porous Media*, **41**, 263, (2000).

[2] Dullien, F.A.L., "Porous Media: Fluid Transport and Pore Structure", 2nd Edition, Academic Press, New York, (1992).

[3] Scheidegger, A.E., Statistical Hydrodynamics in Porous Media, *J. Appl. Phys*., **25**(8), 994, (1954).

[4] De Josselin de Jong, G., Longitudinal and Transverse Diffusion in Granular Deposits, *Trans. American Geophys. Union,* **39**, 67, (1958).

[5] Saffman, P.G., A Theory of Dispersion in a Porous Medium, *J. Fluid Mech.,* **6**(3), 321, (1959).

[6] Greenkorn, R.A., Kessler, D.P., Dispersion in Heterogeneous, Non-Uniform Anisotropic Porous Media, in "Flow through Porous Media", American Chem. Soc., Washigton, D.C., (1970).

[7] Sahimi, M., Flow Phenomena in Rocks: From Continuum Models to Fractals, Percolation, Cellular Automata, and Simulating Annealing, *Rev. Mod. Phys*., **65**, 1393, (1993).

[8] Holt, R.M., Fjaer, E., Torsaeter, O., Bakke, S., Petrophysical Laboratory Measurements for Basin and Reservoir Evaluation, *Mar. Pet. Geol.*, **13**(4), 383, (1996).

[9] Patzek, T. W., Verification of a Complete Pore Network Simulator of Drainage and Imbibition, *Soc. Petrol. Eng. J.*, **6**, 144, (2001).

[10] Piri, M., Blunt, M. J., Three-Dimensional Mixed-Wet Random Pore-Scale Network Modeling of Two- and Three-Phase Flow in Porous Media, I: Model Description, *Phys. Rev. E*, **71**, 026301(2005).

[11] Buckley, J., Multiphase Displacement in Micro-models, in "Interfacial Phenomena in Petroleum Technology", Edited by N. Morrow, Marcel Decker, New York, (1991).

[12] Sahimi, M., Hughes, B.D., Scriven, L.E., Davis, H.T., Dispersion in Flow through Porous Media, I: One-Phase Flow, *Chem. Eng. Sci*. 41, 2103, (1986).

[13] Bruderer, C., Bernabe, Y., Network Modeling of Dispersion: Transition from Taylor Dispersion in Homogeneous Networks to Mechanical Dispersion in Very Heterogeneous Ones, *Water Resour. Res.,* **37**, 897, (2001).

[14] Lowe, C. P., Frenkel, D., Do Hydrodynamic Dispersion Coefficients Exist?, *Phys. Rev. Lett.,* **77**, 4552, (1996).

[15] Souto, H. P. A., Moyne, C., Dispersion in Two-Dimensional Periodic Porous Media, Part II:Dispersion Tensor, *Phys. Fluids* 9, 2253, (1997).

[16] Huseby, O., Thovert, J.F., Adler, P.M., Dispersion in Three-Dimensional Fracture Networks, *Phys. Fluids,* **13**, 594, (2001).

[17] Wolfsberg, A. V., Freyberg, D. L., Efficient Simulation of Single Species and Multispecies Transport in Groundwater with Local Adaptive Grid Refinement, *Water Resour. Res*., **30** (11), 2979, (1994).

[18] Tompson, A. F. B., Gelhar, L. W., Numerical Simulation of Solute Transport in 3D, Randomly Heterogeneous Porous Media, *Water Resour. Res*., **26**(10), 2541, (1990).

[19] LaBolle, E. M., Fogg, G. E., Tompson, A. F. B., Random-Walk Simulation of Transport in Heterogeneous Porous Media: Local Mass Conservation Problem and Implementation Methods, *Water Resour. Res*., **32**(3), 583, (1996).

[20] Ghazanfari, M. H., Rashtchian, D., Kharrat, R., Voussughi, S., Capillary Pressure Estimation of Porous Media Using Statistical Pore Size Function, *Chem. Eng. Tech*., **30**, 862, (2007).

[21] Marle, C.M., "Multiphase Flow in Porous Media":, Gulf Publishing, Houston, (1981).

[22] Andrew, L.Z., Aimar, P., Meireles, M., Pimbely, J.M., Belfort, G., Use of the Log-Normal Density Function to Analyze Membrane Pore Size Distribution: Functional Forms and Discrepancies, *J. Mem. Sci.*, **91**, 293, (1994)

[23] Ghazanfari, M.H., Prediction of Multiphase Flow Properties in Porous Media Using Micromodel Experiments and Pore-Scale Modeling, PhD. Thesis, Sharif University of Technology, Tehran (2008).

[24] McKellar, M.., Wardlaw , N., A Method of Making Two-Dimensional Glass Micromodels of Pore Systems, *J. Can. Pet. Technol.*, **21**, 39, (1982).

[25] Bear, J, Bachmat, Y., "Introduction to Modeling of Transport Phenomena in Porous Media", Kluwer Academic Publishers, Dordrecht, (1990).

[26] Bear, J., "Dynamics of Fluids in Porous Media", Elsevier, New York , (1975).

[27] Ghazanfari M.H., Kharrat, R., Rachtchian, D. and Vossoughi S., Statistical Model of Dispersion in 2-D Glass Micromodel, *SPE 113343, IOR2008*, Tusla, (2008).

[28] Ghazanfari, M. H., Rashtchian, D., Kharrat, R., Vossoughi, S., Khodabakhsh, M., Unsteady State Relative Permeability and Capillary Pressure Estimation of Porous Media, XVI International CMWR Conf., Denmark, (2006).