AUTOMATED ANALYSIS OF PRESSURE BUILD UP TESTS AFFECTED BY PHASE REDISTRIBUTION

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ABSTRACT: Analytical solutions and type curves for the constant rate radial flow of fluid in both conventional and naturally fractured reservoirs including the effect of wellbore phase redistribution are presented. An automated procedure for non-linear least square minimization using the analytical solutions and their derivatives with respect to the unknown parameters developed to analyze the pressure build up data affected by phase redistribution. Field examples and analysis are also presented.

KEY WORDS: Pressure build up test, Wellbore phase redistribution, Type curve, Optimization, Non-linear regression.

INTRODUCTION
Theoretically the Horner plot of a pressure build up test in a radial infinite porous media should be a straight line. Some of the variations from this behavior, such as the curved portion immediately after shut-in which results from after production and skin effects, and the flattened end portion due to boundary effects are known. The bottom hole pressure (BHP) response in a pressure build up test is controlled by many factors in both the reservoir (i.e. heterogeneities and boundaries) and the wellbore conditions such as limited entry, skin and wellbore contents effect.

The effect of the wellbore contents on a pressure build up test has been described by two phenomena: wellbore storage and phase redistribution. Wellbore storage, noted by Van Everdingen and Hurst [1], has been described in
detail by Ramey [2] and Agarwal et al [3]. When a producing oil well is shut-in at the surface, due to compressible nature of the content of the well, fluids continue to flow from the reservoir into the wellbore (after flow) for a time. If after shut-in gas and liquid are simultaneously flowing into the wellbore, gravity effects will cause segregation of phases. Since the gas cannot expand on its way to the top of the well and because of the difference between gas and oil compressibilities, it causes a net increase in the wellbore pressure.

General analysis of wellbore phase redistribution was first presented by Stegmeier and Matthews [4] and Pitzer et al [5]. Fair [6] later showed the effect of wellbore phase redistribution to be a storage effect and presented analytical solutions for the radial infinite reservoir. Witterfeld [7] examined the effect of wellbore on pressure build up tests by solving the flow equations in the wellbore/reservoir system with finite difference approximation and found that after a sufficiently long time, gas and liquid phases were completely segregated.

In this work we propose, new type curves for analyzing pressure transient data affected by phase redistribution in naturally fractured reservoirs. We have also treated Fair's conventional model differently and have obtained type curves in terms of \( P_w \) vs \( t_p / D \) similar to those presented by Gringarten et al [8] but including the effect of phase redistribution. We have considered the Laplace space solutions and their derivatives with respect to the unknown reservoir parameters with an automated procedure and have been successful in analyzing a pressure build up test conducted on a well located in southwest of Iran.

ANALYTICAL SOLUTIONS TO THE MATHEMATICAL MODELS

The dimensionless wellbore pressure in Laplace space for a well, producing with a constant rate from an infinite system including the effects of skin, wellbore storage and phase redistribution, as derived in the Appendix, is:

\[
L(P_w) = \frac{1}{s(1 + C_D s) \left( \frac{K_0(V_S(s))}{V_S(s)} \right) + S(1 + C_D s) \left( \frac{K_0(V_S(s))}{V_S(s)} \right) + S)}
\]

where \( s \) is the Laplace variable, \( f(s) = 1 \) for conventional, and for naturally fractured systems \( f(s) \) is a function of the two characteristic parameters \( \lambda \) and \( \omega \) as introduced by Warren and Root [9] for the pseudo-steady state flow between matrix and fractures as follows:

\[
f(s) = \frac{\omega (1 - \omega) s + \lambda}{(1 - \omega) s + \lambda}
\]

According to Bourdet and Gringarten [10] the function \( S + K_0(V_S(s))/V_S(s)K_0(V_S(s)) \) can always be approximated \( \ln(2/(\sqrt{\pi} e^{-2S})) \) where \( \gamma = 1.781 \) is the exponential of the Euler constant. By using this approximation and transforming the time base from \( t_p \) to \( t_p/(C_D \pi)^{1/2} \) Eqs. 1 and 2 for all practical purposes may be written as:

\[
L(P_w) = \frac{1}{s(1 + C_D s) \left( \frac{1}{s(C_D)f + m} - \frac{1}{s(C_D)f + m + \lambda A_D} \right) + \lambda A_D}
\]

and

\[
f(s) = \frac{\omega (1 - \omega) s + \lambda}{(1 - \omega) s + \lambda}
\]

At early times \( s \to \infty \) the \( f(s) \) function is equal to \( \omega \) and Eq. (3) becomes:

\[
L(P_w) = \frac{1}{s(1 + C_D s) \left( \frac{1}{s(C_D)f + m} - \frac{1}{s(C_D)f + m + \lambda A_D} \right) + \lambda A_D}
\]

or as shown in Eq. 6:

\[
L(P_w) = \frac{1}{s(1 + C_D s) \left( \frac{1}{s(C_D)f + m} - \frac{1}{s(C_D)f + m + \lambda A_D} \right) + \lambda A_D}
\]
where \((C_D)_f\) represents the dimensionless wellbore storage based on the storativity of the fracture system.

At late times \((s \to 0)\), the \(f(s)\) function is equal to unity, therefore Eq.(3) reduces to:

\[
L(P_{wD}) = \left[ 1 + \frac{C_{eD}}{C_D f + m} s^2 \left( \frac{1}{su(C_D)f + m} - \frac{1}{su(C_D)f + m + 1/C_D} \right) \right] \left[ s + (1/a) \left( \frac{2}{\gamma N + (C_D f + m) e^{2s}} \right) \right]^{-1}
\]

Eqs. 6 and 7 may be also written as follows:

\[
L(P_{wD}) = \frac{\left[ 1 + \frac{s^2 C_{eD}}{C_{wD} + 1/C_D} \right]}{\frac{1}{s} + \frac{1}{C_{wD}} \left( \frac{C_D}{C_{AD}} - 1 \right)}
\]

\[
L(P_{wD}) = \frac{s + (1/a) \left( \frac{2}{\gamma N + (C_D f + m) e^{2s}} \right)}{\frac{1}{s} + \frac{1}{C_{wD}} \left( \frac{C_D}{C_{AD}} - 1 \right)}
\]

where:

\[
C_{AD} = \frac{1}{C_{wD} + 1/C_D}
\]

Eqs.(8) and (9) are identical, except for the value of the dimensionless wellbore storage constant.

In order to obtain the real time dimensionless pressure, Eqs. 8 and 9 have been numerically inverted by using Stehfest algorithm [11] and new type curves for several values of the wellbore phase redistribution coefficient \(C_{eD}\) and \(C_D e^{2s}\) have been generated. Results are presented in Figs. 1 and 2 which indicate that under certain conditions the pressure functions may show a "hump". These type curves may be used in the same manner as described by Gringarten et al [8] to obtain permeability, skin, and wellbore storage. In addition to these parameters phase redistribution coefficient and the two parameters characteristic of dual porosity systems may be obtained as follows.

\[t_D \times \frac{\lambda}{4} = 1.0\]  

\[
\omega = \frac{(C_D e^{2s})_{f+m}}{(C_D e^{2s})_f}
\]

The Laplace transform of the dimensionless wellbore pressure for conventional reservoirs may be written as follows:

\[
L(P_{wD}) = \frac{\left[ 1 + \frac{s^2 C_{eD}}{C_{wD} + 1/C_D} \right]}{\frac{1}{s} + \frac{1}{C_{wD} + 1/C_D} \left( \frac{C_D}{C_{AD}} - 1 \right)} \left[ s + (1/a) \left( \frac{2}{\gamma N + (C_D f + m) e^{2s}} \right) \right]^{-1}
\]
This equation is also numerically inverted into real time domain for several values of the wellbore phase redistribution coefficient $C_{PD}$ and $C_{D} e^{25}$. Results are shown in Figs. 3 and 4.

\[ r_w = 0.3542 \text{ ft.} \]
\[ \phi = 0.22 \]
\[ c_i = 1.32 \times 10^{-5} \text{ psi}^{-1} \]
\[ P_{m}(\Delta t=0) = 3615.36 \text{ psia} \]

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**Field Examples**

**Example 1**

This example is a pressure build up test conducted on an oil well producing from a sandstone reservoir located in southwest of Iran. The transient analysis indicates a homogeneous reservoir with no indication of double porosity or double permeability behavior. The actual pressure build up data are shown in Table 1.

**Table 1: Pressure build up data**

<table>
<thead>
<tr>
<th>$\Delta t$ (hr)</th>
<th>$\Delta P_w$ (psi)</th>
<th>$\Delta t$ (hr)</th>
<th>$\Delta P_w$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0022</td>
<td>5.5470</td>
<td>0.0602</td>
<td>10.2660</td>
</tr>
<tr>
<td>0.0027</td>
<td>5.9670</td>
<td>0.0666</td>
<td>10.4850</td>
</tr>
<tr>
<td>0.0033</td>
<td>6.4280</td>
<td>0.0738</td>
<td>10.7120</td>
</tr>
<tr>
<td>0.0038</td>
<td>6.7540</td>
<td>0.0819</td>
<td>10.9590</td>
</tr>
<tr>
<td>0.0044</td>
<td>7.0600</td>
<td>0.0902</td>
<td>11.1920</td>
</tr>
<tr>
<td>0.0049</td>
<td>7.2530</td>
<td>0.0999</td>
<td>11.4430</td>
</tr>
<tr>
<td>0.0055</td>
<td>7.4260</td>
<td>0.1285</td>
<td>12.0640</td>
</tr>
<tr>
<td>0.0060</td>
<td>7.5910</td>
<td>0.1655</td>
<td>12.7090</td>
</tr>
<tr>
<td>0.0066</td>
<td>7.6300</td>
<td>0.2130</td>
<td>13.3590</td>
</tr>
<tr>
<td>0.0069</td>
<td>7.6040</td>
<td>0.2741</td>
<td>13.9900</td>
</tr>
<tr>
<td>0.0074</td>
<td>7.5410</td>
<td>0.3524</td>
<td>14.6190</td>
</tr>
<tr>
<td>0.0077</td>
<td>7.5290</td>
<td>0.4533</td>
<td>15.2150</td>
</tr>
<tr>
<td>0.0083</td>
<td>7.5320</td>
<td>0.5830</td>
<td>15.7670</td>
</tr>
<tr>
<td>0.0085</td>
<td>7.5260</td>
<td>0.7499</td>
<td>16.3490</td>
</tr>
<tr>
<td>0.0091</td>
<td>7.4940</td>
<td>0.9647</td>
<td>16.8320</td>
</tr>
<tr>
<td>0.0097</td>
<td>7.4710</td>
<td>1.2408</td>
<td>17.3600</td>
</tr>
<tr>
<td>0.0102</td>
<td>7.4220</td>
<td>1.7647</td>
<td>18.0460</td>
</tr>
<tr>
<td>0.0105</td>
<td>7.3970</td>
<td>2.5092</td>
<td>18.6960</td>
</tr>
<tr>
<td>0.0110</td>
<td>7.3570</td>
<td>2.6391</td>
<td>18.788</td>
</tr>
<tr>
<td>0.0119</td>
<td>7.3590</td>
<td>2.7752</td>
<td>18.878</td>
</tr>
<tr>
<td>0.0124</td>
<td>7.3950</td>
<td>2.9165</td>
<td>18.966</td>
</tr>
<tr>
<td>0.0130</td>
<td>7.4560</td>
<td>3.0691</td>
<td>19.063</td>
</tr>
<tr>
<td>0.0138</td>
<td>7.5800</td>
<td>3.2274</td>
<td>19.144</td>
</tr>
<tr>
<td>0.0144</td>
<td>7.6670</td>
<td>3.3941</td>
<td>19.243</td>
</tr>
<tr>
<td>0.0160</td>
<td>7.9530</td>
<td>3.5694</td>
<td>19.328</td>
</tr>
<tr>
<td>0.0177</td>
<td>8.1880</td>
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<td>19.429</td>
</tr>
<tr>
<td>0.0197</td>
<td>8.3410</td>
<td>3.9472</td>
<td>19.509</td>
</tr>
<tr>
<td>0.0219</td>
<td>8.4510</td>
<td>4.1508</td>
<td>19.604</td>
</tr>
<tr>
<td>0.0241</td>
<td>8.5270</td>
<td>4.3549</td>
<td>19.668</td>
</tr>
<tr>
<td>0.0269</td>
<td>8.6950</td>
<td>4.5902</td>
<td>19.761</td>
</tr>
<tr>
<td>0.0297</td>
<td>8.9000</td>
<td>4.8272</td>
<td>19.867</td>
</tr>
<tr>
<td>0.0327</td>
<td>9.0840</td>
<td>5.0763</td>
<td>19.933</td>
</tr>
<tr>
<td>0.0363</td>
<td>9.2520</td>
<td>5.3383</td>
<td>20.027</td>
</tr>
<tr>
<td>0.0402</td>
<td>9.4340</td>
<td>5.6135</td>
<td>20.108</td>
</tr>
<tr>
<td>0.0444</td>
<td>9.6380</td>
<td>5.9032</td>
<td>20.180</td>
</tr>
<tr>
<td>0.0494</td>
<td>9.8480</td>
<td>6.2080</td>
<td>20.253</td>
</tr>
<tr>
<td>0.0544</td>
<td>10.0470</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The model and field data were optimized by using non-linear least square technique [14, 15, 16, 17]. The output [18] is shown in Fig. 5 and the reservoir parameters are given in Table (2) (Appendix 3).

<table>
<thead>
<tr>
<th>Table 2: Reservoir parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_D = 112.428$, $C_{\phi D} = 148.96$, $\alpha_D = 731.73$</td>
</tr>
<tr>
<td>$K = 2458.7$ md, $S = -4.5$</td>
</tr>
</tbody>
</table>

As can be seen there is an excellent match between field data and model output (Fig. 5).

![Fig. 5: Optimization match. Example 1](image)

**Example 2**

Example 2 is an actual set of pressure build up data measured in a gas lifted oil well in southeast Louisiana [6]. Fig. 6 compares the model output with the field input data. The results of the analysis are presented in Table (3).

![Fig. 6: Optimization match. Example 2 (southeast Louisiana)](image)

Here again there is a satisfactory agreement between the two sets of data.

**Example 3**

Example 3 consists of data obtained from a well in southeast Louisiana [6] producing at low rate. The field data and model results are shown in Fig. 7. Results of analysis are given in Table 4.

![Fig. 7: Optimization match. Example 3 (southeast Louisiana)](image)

**Table 3: Results of optimization match for example 2**

<table>
<thead>
<tr>
<th>Type curve matching[6]</th>
<th>$K$ (md)</th>
<th>$S$</th>
<th>$C_D$</th>
<th>$C_{\phi D}$</th>
<th>$\alpha_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Data</td>
<td>134</td>
<td>0</td>
<td>750</td>
<td>10</td>
<td>8571.43</td>
</tr>
<tr>
<td>Model data</td>
<td>132.7</td>
<td>-0.004</td>
<td>647</td>
<td>9.94</td>
<td>8430.35</td>
</tr>
</tbody>
</table>

Fig. 7 shows good agreement between the field and model data. It also reveals that the last three data points in the example are approaching the semilog straight line. Using semilog analysis, the permeability and skin are estimated to be 13.41 md and 4.51 respectively (Fig. 8). These values are in adequate agreement with those obtained from the optimization technique.

**Table 4: Results of optimization match for example 3**

<table>
<thead>
<tr>
<th>Type curve matching[6]</th>
<th>$K$ (md)</th>
<th>$S$</th>
<th>$C_D$</th>
<th>$C_{\phi D}$</th>
<th>$\alpha_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Data</td>
<td>1.45</td>
<td>5</td>
<td>100</td>
<td>10</td>
<td>250</td>
</tr>
<tr>
<td>Semi-Log</td>
<td>13.41</td>
<td>4.51</td>
<td>51.064</td>
<td>11.34</td>
<td>202.6</td>
</tr>
<tr>
<td>Model data</td>
<td>14.016</td>
<td>4.95</td>
<td>51.064</td>
<td>11.34</td>
<td>202.6</td>
</tr>
</tbody>
</table>
Comparing the permeabilities which have been obtained by semi-Log analysis and the optimization method with that obtained from type curve matching, one can see that the permeability from type curve and reported by Fair [6] is in error.

![Graph](image)

**Fig. 8**: Semi-Log analysis (example 3)

**DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS**

Mathematical analysis of phase redistribution has been extended and a new type curve for Log-Log analysis of conventional reservoirs with phase redistribution has been constructed.

The diffusivity equation in fractured reservoirs has been solved including the phase redistribution boundary conditions and solutions in terms of dimensionless well bore pressure has been presented in Laplace space.

A new type curve for Log-Log analysis of fractured reservoirs with phase redistribution has been constructed by considering definition of dimensionless time $t_D/C_D$.

In order to use the new type curves for both conventional and fractured reservoirs, the apparent dimensionless storage coefficient ($C_{AD}$) and the dimensionless well bore storage coefficient ($C_D$) need to be calculated. It is therefore recommended that in conjunction with the pressure build up data, the fluid gradient under flowing and static conditions be measured.

From the type curve for fractured reservoirs in addition to the reservoir parameters (permeability, skin, well bore storage constant), the two parameters characteristic of the fissuration: $\omega$ the storativity ratio and $\lambda$, the interporosity flow coefficients may also be determined.

An automated procedure of non-linear regression analysis using Marquardt method has been developed for reservoir parameter estimation [18]. A computer program has been provided on the IBM-PC to estimate optimal values of permeability ($K$), skin ($S$), well bore storage ($C_D$), phase redistribution pressure parameter ($C_\phi$) and phase redistribution time parameter ($\alpha$) in conventional and fractured reservoirs. In the option of fractured reservoirs in addition to $K$, $S$, $C_D$, $C_\phi$ and $\alpha$, $\omega$, the storativity ratio and $\lambda$, the interporosity flow coefficient are also calculated.

The optimization program has been examined for two generated data, two field examples in Louisiana and a field example in southwest of Iran. Results from semi-Log analysis and the optimization program showed to be in good agreement.

**NOMENCLATURE**

**English Letter Symbols**

- $A$: constant
- $a$: array of parameters (unknown)
- $B$: constant
- $C$: well bore storage coefficient, bbl/psi
- $C_a$: apparent storage coefficient, bbl/psi
- $C_{AD}$: apparent dimensionless storage coefficient
- $C_f$: compressibility of fluid within the fissure system
- $c_m$: compressibility of fluid within the matrix
- $c_t$: total compressibility, $\text{psi}^{-1}$
- $C_D$: dimensionless well bore storage coefficient
- $C_{\phi}$: phase redistribution pressure parameter, psi
- $C_{D\phi}$: dimensionless phase redistribution pressure parameter
- $h$: reservoir thickness, ft
- $K$: reservoir permeability, md
- $L(f)$: Laplace transform of function $f$
- $M$: number of parameter (unknown)
- $md$: milli-darcy
- $N$: number of data points
- $P$: pressure, psi
\( P_{ID} \) dimensionless fissure pressure
\( P_w \) wellbore pressure, psi
\( P_{wD} \) dimensionless wellbore pressure
\( P_D \) dimensionless pressure
\( P_\phi \) phase redistribution pressure, psi
\( P_{\phi D} \) dimensionless phase redistribution pressure
\( \Delta P \) pressure difference, psi
\( q \) flow rate, B/D
\( q_{mf} \) sand-face flow rate, B/D
\( r \) radius, ft
\( RB \) reservoir barrel
\( r_w \) wellbore radius, ft
\( r_D \) dimensionless radius, \( r_D = r/r_w \)
\( s \) Laplace transform variable
\( S \) skin factor
\( t \) time, hours
\( t_D \) dimensionless time
\( \Delta t \) shut-in time
\( V_w \) wellbore volume

\( \omega \) flowing bottom hole
\( \omega_s \) shut-in bottom hole

**Mathematical Notations**

\( I_0 \) modified Bessel function of the first kind and of order zero
\( I_1 \) modified Bessel function of the first kind and of order one
\( K_0 \) modified Bessel function of the second kind and of order zero
\( K_1 \) modified Bessel function of the second kind and of order one
\( L \) Laplace transform
\( L^{-1} \) Laplace inversion operation
\( \text{Ln} \) Logarithm to the base \( e \) (natural logarithm)
\( \partial_x \) partial derivative of \( x \)
\( \Delta \) finite increment
\( \Sigma \) summation

**Greek Letter Symbols**

\( \alpha \) phase redistribution time parameter, hours
\( \alpha_D \) dimensionless phase redistribution time parameter
\( \beta \) oil formation volume factor RB/STB
\( \psi \) Euler's constant = 0.57721566
\( \lambda \) Interporosity flow parameter
\( \mu \) fluid viscosity, cp
\( \phi \) porosity (fraction)
\( \omega \) storativity ratio

**Subscript Letter Symbols**

\( b \) bulk
\( D \) dimensionless
\( f \) fissure
\( fb \) bulk fissure
\( fD \) dimensionless fissure
\( m \) matrix
\( ma \) matrix block, matrix average
\( mb \) bulk matrix
\( s \) surface
\( sf \) sandface
\( t \) total
\( w \) well

**ACKNOWLEDGMENT**

The authors wish to express their gratitude to the management of National Iranian Oil Co. for the provision of the field data.

**REFERENCES**


APPENDIX
1. Mathematical Analysis of Phase Redistribution

For a well where wellbore storage occurs, the contribution of the expansion of the wellbore liquid may be written as:

\[ q_t - q_{sf} = cV_w \frac{dP_w}{dt} \]  

where \( c \) is the compressibility of the fluid in the wellbore, \( V_w \) is the wellbore volume, \( q_t \) is the surface flow rate and \( q_{sf} \) is the sandface flow rate. Defining the wellbore storage constant \( C = cV_w \) and introducing the dimensionless parameters shown below, Eq. (1.1) may be written as:

\[ \frac{dP_{wd}}{dt_D} = \frac{1}{C_D} \left( 1 - \frac{q_{sf}}{q} \right) \]  

where \( C_D \), \( t_D \) and \( P_{wd} \) are dimensionless wellbore storage, dimensionless time and dimensionless wellbore pressure respectively.

Fair [6] modified Eq. (1.2) by adding a term describing the pressure change caused by phase redistribution as follows:

\[ \frac{dP_{wd}}{dt_D} = \frac{1}{C_D} \left( 1 - \frac{q_{sf}}{q} \right) + \frac{dP_{\phi D}}{dt_D} \]  

This equation may be rearranged to show the sandface flow rate dependency:

\[ \frac{q_{sf}}{q} = 1 - C_D \left( \frac{dP_{wd}}{dt_D} - \frac{dP_{\phi D}}{dt_D} \right) \]  

where

\[ P_{\phi D} = C_{\phi D} (1 - e^{(\alpha_D \phi_D)}) \]  

\( P_{\phi D} \) is the dimensionless wellbore phase redistribution pressure, \( C_{\phi D} \) and \( \alpha_D \) are
dimensionless phase redistribution pressure and
time parameters defined below:

\[ C_{AD} = \frac{5.6146C_p}{2\pi \phi_c \tau_w} = \left( \frac{C_p}{\alpha_D} + \frac{1}{C_D} \right)^{-1} \] (1.6)

\[ C_D = \frac{5.6146C_p}{2\pi \phi_c \tau_w} \] (1.7)

\[ C_{\phi D} = \frac{K_h C_p}{141.2q \beta \mu} \] (1.8)

\[ P_{\phi D} = \frac{K_h p_w}{141.2q \beta \mu} \] (1.9)

\[ P_{\varepsilon D} = \frac{K_h \Delta P_w}{141.2q \beta \mu} \] (1.10)

\[ r_D = \frac{r_w}{\tau_w} \] (1.11)

\[ t_D = \frac{0.000264K_A t}{\phi_m r_w^2} \] (1.12)

\[ \alpha_D = \frac{0.000264K_A}{\phi_m r_w^2} \] (1.13)

2. Determination of Dimensionless Wellbore Pressure

The diffusivity equation that has been found to
describe the fractured reservoirs behavior in
Laplace space is [9]:

\[ L \left( \frac{dP_{\phi D}}{dt} \right) + \frac{1}{r_D} L \left( \frac{dP_{m}}{dt} \right) = sf(s)L(P_m) \] (2.2)

where \( f(s) \) is defined by Eq. (2.1) for the
case of zero state interporosity flow condition:

\[ f(s) = \frac{\omega (1 - \omega) s + \lambda}{(1 - \omega) s + \lambda} \] (2.2)

where \( \omega \) is dimensionless storativity and \( \lambda \) is
dimensionless interflow parameter.

The general solution to Eq. (2.1) in Laplace
space is given by [1]:

\[ L(P_m) = A_0(r_D \sqrt{s}) + Bk_0(r_D \sqrt{s}) \] (2.3)

Where \( I_0 \) and \( K_0 \) are modified Bessel function
of the first and second kind respectively and of zero
order. \( A \) and \( B \) are constants and are evaluated
according to the reservoir boundary conditions.

To obtain dimensionless pressure solutions
for use in the analysis of pressure buildup tests,
it is necessary to incorporate the effects of
wellbore phase redistribution into the diffusivity
equation.

\[ P_m(r_D, \theta) = 0 \] (2.4)

\[ r_D \rightarrow \lim P_m(r_D, \theta) = 0 \] (2.5)

\[ (c_P \frac{dP_m}{dt}) = 1 + C_D (\frac{dP_{\phi D}}{dt}) - \frac{dP_{\varepsilon D}}{dt} \] (2.6)

and in Laplace space, these equations become:

\[ r_D \rightarrow \lim L(P_m) = 0 \] (2.7)

\[ L \left( \frac{dP_m}{dt} \right) = -\frac{1}{s} + C_D [L \left( \frac{dP_{\phi D}}{dt} \right) - \frac{dP_{\varepsilon D}}{dt}] \] at \( r_D = 1 \) (2.8)

By using the inner and outer boundary
conditions without wellbore storage and phase redistribution,
constants \( A \) and \( B \) may be obtained
and by substituting into Eq. (2.3) yields:

\[ L(P_m) = \frac{K_0(r_D \sqrt{s})}{s[\sqrt{s}f(s)]} \] (2.9)

This equation is Laplace space is the solution
for the constant rate and infinite acting fractured
reservoir. \( P_m \) is the dimensionless sandface
wellbore pressure.

The effects of wellbore storage and phase
redistribution and skin may be taken into
account, the dimensionless pressure drop at the
wellbore in Laplace space becomes [2,6,10]:

\[ L(P_{\phi D}) = \frac{S[L(P_m)(1 + C_D \Delta^2 L(P_m)) + C_D \Delta^2 L(P_m)S + S}{s[1 + C_D \Delta^2 (S + sL(P_m))] \] (2.10)

Here the Laplace transform of the dimensionless
wellbore pressure is related to Laplace transform of
the dimensionless sandface pressure. \( S \) is the
skin factor, \( s \) is the Laplace variable and \( C_D \) is
the dimensionless wellbore storage.

\[ L(P_{\phi D}) = \frac{C_{\phi D}}{s} - \frac{C_{\phi D}}{s + 1/\alpha_D} \] (2.11)

Substituting \( L(P_m) \) from Eq. (2.9) and \( L(P_{\phi D}) \)
from Eq. (2.11) into Eq. (2.10) yields:
\[
L(P_{\omega D}) = \left[ \frac{K_0(V\sqrt{f(s)})}{V\sqrt{f(s)}K_1(V\sqrt{f(s)})} + S \right] \left[ 1 + C_D C_{\phi D} \left( \frac{1}{s} - \frac{1}{s + 1/\alpha_D} \right) \right]

s(1 + C_D \left( \frac{K_0(V\sqrt{f(s)})}{V\sqrt{f(s)}K_1(V\sqrt{f(s)})} + S \right))
\]

(2.12)

3. Application of Optimization Methods for Phase Redistribution Data

Optimization techniques have been found to be effective in a variety of areas throughout the scientific world. The focus here is on application of this technique for the estimation of optimal values of reservoir properties. Suppose that one wants to fit N data points \((x_i, y_i)\) \(i = 1, ..., N\) to a model which has M adjustable parameters \(a_j\), \(j = 1, ..., M\). If each data point \((x_i, y_i)\) has its own standard deviation \(s_i\), so that chi-square may be defined as follows:

\[
x^2 = \sum_{i=1}^{N} \left( \frac{y_i - y(x_i; a_1, ..., a_M)}{s_i} \right)^2
\]

(3.1)

the best values of the model parameters are obtained when the chi-square is minimized.

The dimensionless pressure in the Laplace space for both fractured and conventional reservoirs are non-linear equations. Thus an automated procedure of non-linear regression analysis using Marquardt method is developed for reservoir parameter estimation. The procedure was proposed by marquart as an extension of the Gauss Newton method to allow for convergence with relatively poor starting guesses for the unknown coefficients. The dimensionless pressure in Laplace space is given in appendix 2 (f(s) = 1 for conventional reservoirs), thus with an initial guess for the \(C_D\), \(C_{\phi D}\), \(S\), \(K\), \(\alpha_D\): Eq.(2.12) may be inverted numerically by Stehfest [6] algorithm and \(P_{\omega D}\) may be obtained in real time domain, that is used in Marquardt’s routine. In addition, Marquardt’s routine requires the vector of derivatives \(\partial P/\partial C_D\), \(\partial P/\partial C_{\phi D}\), \(\partial P/\partial \alpha_D\), \(\partial P/\partial S\), \(\partial P/\partial K\) that must be calculated. Differentiation of Eq.(2.12) with respect to \(C_D\), \(C_{\phi D}\), \(S\), \(K\) and \(\alpha_D\) gives:

\[
L(\frac{\partial P_{\omega D}}{\partial C_D}) = \left[ \frac{K_0(s^2)}{s^3 K_1(s^2)} + S \right] \left[ \frac{K_0(s^2)}{s^3 K_1(s^2)} + S \right] \left( 1 + C_D \left( \frac{K_0(s^2)}{s^3 K_1(s^2)} + S \right) \right)^2
\]

(3.2)

\[
L(\frac{\partial P_{\omega D}}{\partial C_{\phi D}}) = \left[ \frac{K_0(s^2)}{s^3 K_1(s^2)} + S \right] \left( 1 + C_D \left( \frac{K_0(s^2)}{s^3 K_1(s^2)} + S \right) \right)^2
\]

(3.3)

\[
L(\frac{\partial P_{\omega D}}{\partial S}) = \left[ 1 + C_D \left( \frac{K_0(s^2)}{s^3 K_1(s^2)} + S \right) \right]^2
\]

(3.4)

\[
L(\frac{\partial P_{\omega D}}{\partial K}) = \left[ \frac{K_0(s^2)}{s^3 K_1(s^2)} + S \right] \left( 1 + C_D \left( \frac{K_0(s^2)}{s^3 K_1(s^2)} + S \right) \right) \left( \beta_1 K\alpha + 1 \right)^2
\]

(3.5)

\[
L(\frac{\partial P_{\omega D}}{\partial \alpha_D}) = \left[ \frac{K_0(s^2)}{s^3 K_1(s^2)} + S \right] \left[ \beta_1 C_D h_1 C_{\phi D} \right]^2
\]

(3.6)

where:

\[
h_1 = h/141.2 q \beta \mu , \beta_1 = 0.000264/(\phi \mu c_i r_i^2)
\]

\[
\alpha_D = \beta_1 K\alpha , C_{\phi D} = h_1 K C_{\phi}
\]

Eqs.(3.2), (3.3), (3.4), (3.5), (3.6) are inverted by Stehfest algorithm and \(\partial P/\partial C_D\), \(\partial P/\partial C_{\phi D}\), \(\partial P/\partial \alpha_D\), \(\partial P/\partial S\), \(\partial P/\partial K\) are calculated.