AUTOMATED ANALYSIS OF PRESSURE BUILD UP TESTS AFFECTED BY PHASE REDISTRIBUTION

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ABSTRACT: *Analytical solutions and type curves for the constant rate radial flow of fluid in both conventional and naturally fractured reservoirs including the effect of wellbore phase redistribution are presented. An automated procedure for non-linear least square minimization using the analytical solutions and their derivatives with respect to the unknown parameters developed to analyze the pressure build up data affected by phase redistribution. Field examples and analysis are also presented.*

KEY WORDS: *Pressure build up test, Wellbore phase redistribution, Type curve, Optimization, Non-linear regression.*

INTRODUCTION

Theoretically the Horner plot of a pressure build up test in a radial infinite porous media should be a straight line. Some of the variations from this behavior, such as the curved portion immediately after shut-in which results from after production and skin effects, and the flattened end portion due to boundary effects are known. The bottom hole pressure (BHP) response in a pressure build up test is controlled by many factors in both the reservoir (i.e. heterogeneities and boundaries) and the wellbore conditions such as limited entry, skin and wellbore contents effect.

The effect of the wellbore contents on a pressure build up test has been described by two phenomena; wellbore storage and phase redistribution. Wellbore storage, noted by *Van Everdingen* and *Hurst* [1], has been described in

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detail by *Ramey* [2] and *Agarwal* et al [3]. When a producing oil well is shut-in at the surface, due to compressible nature of the content of the well, fluids continue to flow from the reservoir into the wellbore (after flow) for a time. If after shut-in gas and liquid are simultaneously flowing into the wellbore, gravity effects will cause segregation of phases. Since the gas cannot expand on its way to the top of the well and because of the difference between gas and oil compressibilities, it causes a net increase in the wellbore pressure.

General analysis of wellbore phase redistribution was first presented by *Stegmeier* and *Matthews* [4] and *Pitzer* et al [5]. *Fair* [6] later showed the effect of wellbore phase redistribution to be a storage effect and presented analytical solutions for the radial infinite reservoir. *Witerfeld* [7] examined the effect of wellbore on pressure build up tests by solving the flow equations in the wellbore/ reservoir system with finite difference approximation and found that after a sufficiently long time, gas and liquid phases were completely segregated.

In this work we propose, new type curves for analyzing pressure transient data affected by phase redistribution in naturally fractured reservoirs. We have also treated Fair's conventional model differently and have obtained type curves in terms of P_D vs t_D/C_D similar to those presented by *Gringarten* et al [8] but including the effect of phase redistribution. We have considered the Laplace space solutions and their derivatives with respect to the unknown reservoir parameters with an automated procedure and have been successful in analyzing a pressure build up test conducted on a well located in southwest of Iran.

ANALYTICAL SOLUTIONS TO THE MATHEMATICAL MODELS

The dimensionless wellbore pressure in Laplace space for a well, producing with a constant rate from an infinite system including the effects of skin, wellbore storage and phase redistribution, as derived in the Appendix, is:

$$L(P_{wD}) = \frac{\left[\frac{K_0(\sqrt{sf(s)})}{\sqrt{sf(s)}K_1(\sqrt{sf(s)})} + S\right]\left[1+C_D C_{\phi D} s^2\left(\frac{1}{s} - \frac{1}{s+1/\alpha_D}\right)\right]}{s\left[1+C_D s\left(\frac{K_0(\sqrt{sf(s)})}{\sqrt{sf(s)}K_1(\sqrt{sf(s)})} + S\right)\right]} \quad (1)$$

where s is the Laplace variable, $f(s)=1$ for conventional, and for naturally fractured systems $f(s)$ is a function of the two characteristic parameters λ and ω as introduced by *Warren* and *Root* [9] for the pseudo-steady state flow between matrix and fractures as follows:

$$f(s) = \frac{\omega(1-\omega)s + \lambda}{(1-\omega)s + \lambda} \quad (2)$$

According to *Bourdet* and *Gringarten* [10] the function $[S+K_0(\sqrt{x})/\sqrt{x}K_1(\sqrt{x})]$ can always be approximated $Ln(2/(\gamma\sqrt{x}.e^{-2S}))$ where $\gamma=1.781$ and is the exponential of the Euler constant. By using this approximation and transforming the time base from t_D to $t_D/(C_D)_{m+f}$ Eqs. 1 and 2 for all practical purposes may be written as:

$$L(P_{wD}) = \frac{\left[1+\frac{C_{\phi D}}{(C_D)_{f+m}} s^2\left(\frac{1}{s/(C_D)_{f+m}} - \frac{1}{s/(C_D)_{f+m}+1/\alpha_D}\right)\right]}{s\left[s + \left(Ln\left(\frac{2}{\gamma\sqrt{sf(s)/(C_D)_{f+m}}.e^{2S}}\right)\right)^{-1}\right]} \quad (3)$$

and

$$f(s) = \frac{\omega(1-\omega)s + \lambda(C_D)_{f+m}}{(1-\omega)s + \lambda(C_D)_{f+m}} \quad (4)$$

At early times ($s \longrightarrow \infty$) the f(s) function is equal to ω and Eq.(3) becomes:

$$L(P_{wD}) = \frac{\left[1+\frac{C_{\phi D}}{(C_D)_{f+m}} s^2\left(\frac{1}{s/(C_D)_{f+m}} - \frac{1}{s/(C_D)_{f+m}+1/\alpha_D}\right)\right]}{s\left[s + \left(Ln\left(\frac{2}{\gamma\sqrt{s\omega/(C_D)_{f+m}}.e^{2S}}\right)\right)^{-1}\right]} \quad (5)$$

or as shown in Eq. 6 :

$$L(P_{wD}) = \frac{\left[1+\frac{C_{\phi D}}{(C_D)_{f+m}} s^2\left(\frac{1}{s/(C_D)_{f+m}} - \frac{1}{s/(C_D)_{f+m}+1/\alpha_D}\right)\right]}{s\left[s + \left(Ln\left(\frac{2}{\gamma\sqrt{s/(C_D)_{f}}.e^{2S}}\right)\right)^{-1}\right]} \quad (6)$$

where $(C_D)_f$ represents the dimensionless wellbore storage based on the storativity of the fracture system.

At late times ($s \longrightarrow 0$), the f(s) function is equal to unity, therefore Eq.(3) reduces to:

$$L(P_{wD}) = \frac{\left[1+\frac{C_{\phi D}}{(C_D)_{f+m}} s^2\left(\frac{1}{s/(C_D)_{f+m}} - \frac{1}{s/(C_D)_{f+m}+1/\alpha_D}\right)\right]}{s\left[s+\left(\mathrm{Ln}\left(\frac{2}{\gamma\sqrt{s/(C_D)_{f+m}}\,.e^{2S}}\right)\right)^{-1}\right]} \quad (7)$$

Eqs. 6 and 7 may be also written as follows:

$$L(P_{wD}) = \frac{\left[1+s^2\left(\frac{C_{\phi D}}{s} - \frac{1}{\frac{s}{C_{\phi D}}+\frac{1}{C^2_{\phi D}}\left(\frac{C_D}{C_{AD}}-1\right)}\right)\right]}{s\left(s+\left(\mathrm{Ln}\left(\frac{2}{\gamma\sqrt{s/(C_D)_f}\,.e^{2S}}\right)\right)^{-1}\right)} \quad (8)$$

and

$$L(P_{wD}) = \frac{\left[1+s^2\left(\frac{C_{\phi D}}{s} - \frac{1}{\frac{s}{C_{\phi D}}+\frac{1}{C^2_{\phi D}}\left(\frac{C_D}{C_{AD}}-1\right)}\right)\right]}{s\left[s+\left(\mathrm{Ln}\left(\frac{2}{\gamma\sqrt{s/(C_D)_{f+m}}\,.e^{2S}}\right)\right)^{-1}\right]} \quad (9)$$

where :

$C_{AD} = (C_{\phi D} / \alpha_D + 1/C_D)^{-1}$

Eqs.(8) and (9) are identical, except for the value of the dimensionless wellbore storage constant.

In order to obtain the real time dimensionless pressure, Eqs. 8 and 9 have been numerically inverted by using *Stehfest* algorithm [11] and new type curves for several values of the wellbore phase redistribution coefficient $C_{\phi D}$ and $C_D\ e^{2S}$ have been generated. Results are presented in Figs. 1 and 2 which indicate that under certain conditions the pressure functions may show a "hump". These type curves may be used in the same manner as described by *Gringarten* et al [8] to obtain permeability, skin, and wellbore storage. In addition to these parameters phase redistribution coefficient and the two parameters characteristic of dual porosity systmes may be obtained as follows.

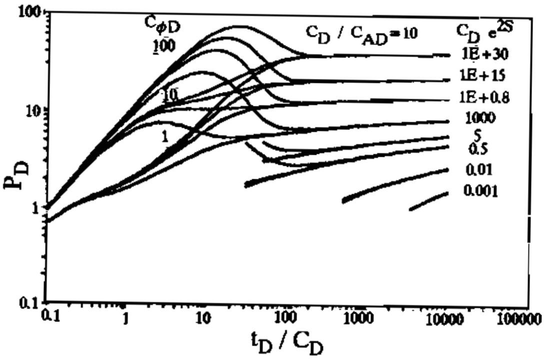

Fig. 1: Type curve with phase redistribution in fractured reservoirs.

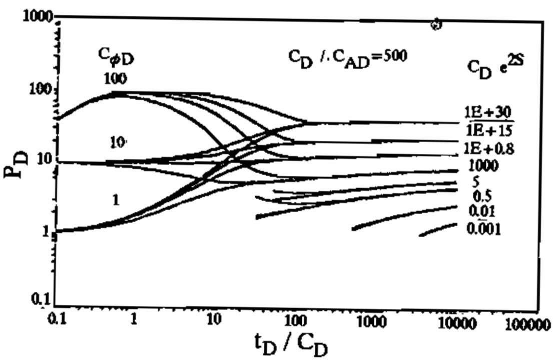

Fig. 2: Type curve with phase redistribution in fractured reservoirs.

λ may be obtained from the begining of the radial flow of the total system [12,13].

$$t_D \times \frac{\lambda}{4} = 1.0 \quad (10)$$

and ω from the ratio of the $C_D\ e^{2S}$ value for the late time total system (corresponding to fissure+blocks) to the $C_D\ e^{2S}$ value for the fracture system. Where ω is the storativity ratio and λ is the interporosity flow coefficient.

$$\omega = \frac{(C_D\ e^{2S})_{f+m}}{(C_D\ e^{2S})_f} \quad (11)$$

The Laplace transform of the dimensionless wellbore pressure for conventional reservoirs may be written as follows:

$$L(P_{wD}) = \frac{\left[1+s^2\left(\frac{C_{\phi D}}{s} - \frac{1}{\frac{s}{C_{\phi D}}+\frac{1}{C^2_{\phi D}}\left(\frac{C_D}{C_{AD}}-1\right)}\right)\right]}{s\left[s+\left(\mathrm{Ln}\left(\frac{2}{\gamma\sqrt{s/C_D}\,.e^{2S}}\right)\right)^{-1}\right]} \quad (12)$$

This equation is also numerically inverted into real time domain for several values of the wellbore phase redistribution coefficient $C_{\phi D}$ and C_D e^{2S}. Results are shown in Figs. 3 and 4.

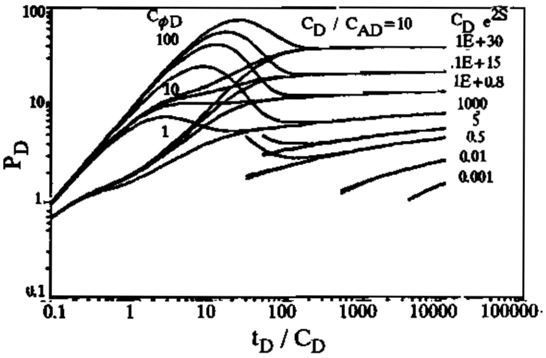

Fig. 3: Type curve with phase redistribution in conventional reservoirs.

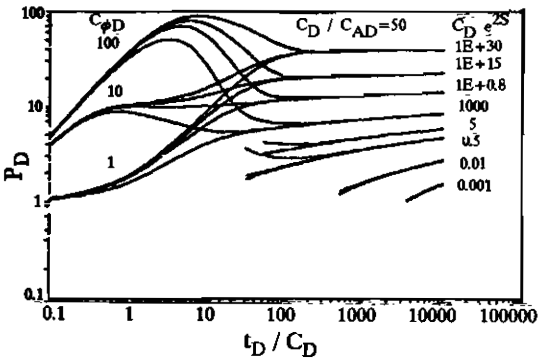

Fig. 4: Type curve with phase redistribution in conventional reservoirs.

FIELD EXAMPLES

Example 1

This example is a pressure build up test conducted on an oil well producing from a sandstone reservoir located in southwest of Iran. The transient analysis indicates a homogeneous reservoir with no indication of double porosity or double permeability behavior. The actual pressure buil up data are shown in Table 1.

Table 1 : Pressure buil up data

q = 5777 STB/D
μ = 0.42 cp.
β = 1.51 RB/STB
h = 42.65 ft.
r_w = 0.3542 ft.
ϕ = 0.22
c_t = 1.32×10^{-5} psi^{-1}
$P_{ws}(\Delta t=0)$= 3615.36 psia

Δt (hr)	ΔP_w (psi)	Δt (hr)	ΔP_w (psi)
0.0022	5.4570	0.0602	10.2660
0.0027	5.9670	0.0666	10.4850
0.0033	6.4280	0.0738	10.7120
0.0038	6.7540	0.0819	10.9590
0.0044	7.0060	0.0902	11.1920
0.0049	7.2530	0.0999	11.4430
0.0055	7.4260	0.1285	12.0640
0.0060	7.5910	0.1655	12.7090
0.0066	7.6300	0.2130	13.3590
0.0069	7.6040	0.2741	13.9900
0.0074	7.5410	0.3524	14.6190
0.0077	7.5290	0.4533	15.2150
0.0083	7.5320	0.5830	15.7670
0.0085	7.5260	0.7499	16.3490
0.0091	7.4940	0.9647	16.8320
0.0097	7.4710	1.2408	17.3600
0.0102	7.4220	1.7647	18.0460
0.0105	7.3970	2.5092	18.6960
0.0110	7.3570	2.6391	18.788
0.0119	7.3590	2.7752	18.878
0.0124	7.3950	2.9185	18.966
0.0130	7.4560	3.0691	19.063
0.0138	7.5800	3.2274	19.144
0.0144	7.6670	3.3941	19.243
0.0160	7.9530	3.5694	19.328
0.0177	8.1880	3.7535	19.429
0.0197	8.3410	3.9472	19.509
0.0219	8.4510	4.1508	19.604
0.0241	8.5270	4.3549	19.668
0.0269	8.6950	4.5902	19.761
0.0297	8.9000	4.8272	19.867
0.0327	9.0840	5.0763	19.933
0.0363	9.2520	5.3383	20.027
0.0402	9.4340	5.6135	20.108
0.0444	9.6380	5.9032	20.180
0.0494	9.8480	6.2080	20.253
0.0544	10.0470		

The model and field data were optimized by using non-linear least square technique [14, 15, 16, 17]. The output [18] is shown in Fig. 5 and the reservoir parameters are given in Table (2) (Appendix 3).

Table 2 : Reservoir parameters

C_D=112.428	$C_{\phi D}$=148.96	α_D=731.73
K=2458.7 md		S= -4.5

As can be seen there is an excellent match between field data and model output (Fig. 5).

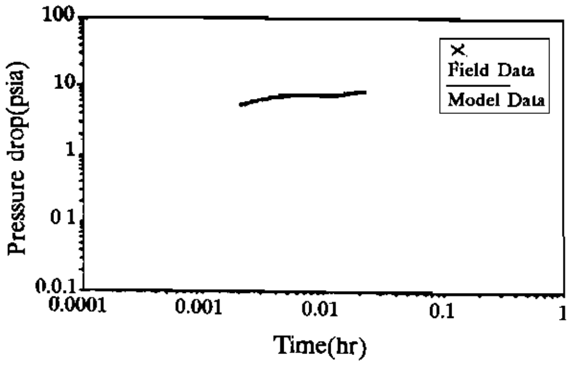

Fig. 5 : Optimization match. Example 1

Example 2

Example 2 is an actual set of pressure build up data measured in a gas lifted oil well in southeast Louisiana [6]. Fig. 6 compares the model output with the field input data. The results of the analysis are presented in Table (3).

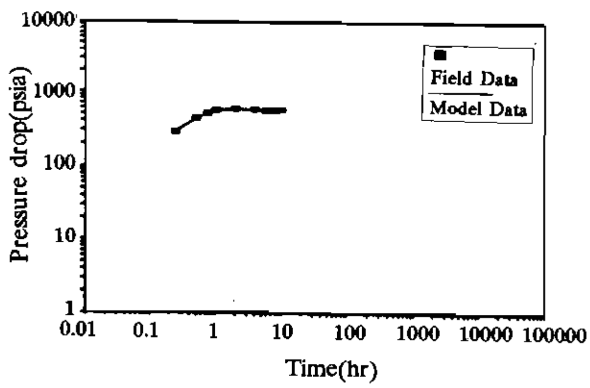

Fig. 6 : Optimization match. Example 2 (southeast Louisiana).

Table 3 : Results of optimization match for example 2

	K(md)	S	C_D	$C_{\phi D}$	α_D
Type curve matching[6]	134	0	750	10	8571.43
Model data	132.7	-0.004	647	9.94	8430.35

Here again there is a satisfactory agreement between the two sets of data.

Example 3

Example 3 consists of data obtained from a well in southeast Louisiana [6] producing at low rate. The field data and model results are shown in Fig.7. Results of analysis are given in Table 4.

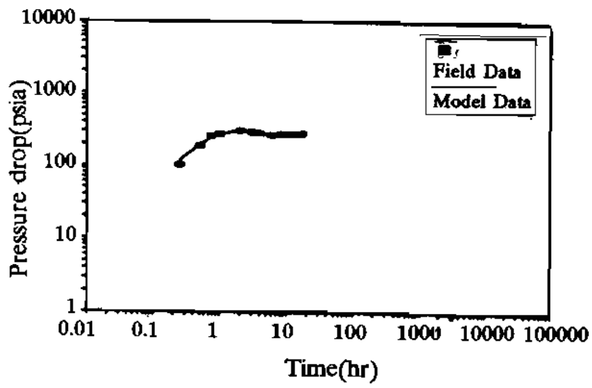

Fig. 7 : Optimization match. Example 3 (southeast Louisiana).

Fig. 7 shows good agreement between the field and model data. It also reveals that the last three data points in the example are approaching the semilog straight line. Using semilog analysis, the permeability and skin are estimated to be 13.41 md and 4.51 respectively (Fig.8). These values are in adequate agreement with those obtained from the optimization technique.

Table 4 : Results of optimization match for example 3

	K(md)	S	C_D	$C_{\phi D}$	α_D
Type curve matching[6]	1.45	5	100	10	250
Semi-Log	13.41	4.51			
Model data	14.016	4.95	51.064	11.34	202.6

Comparing the permeabilities which have been obtained by semi-Log analysis and the optimization method with that obtained from type curve matching, one can see that the permeability from type curve and reported by *Fair* [6] is in error.

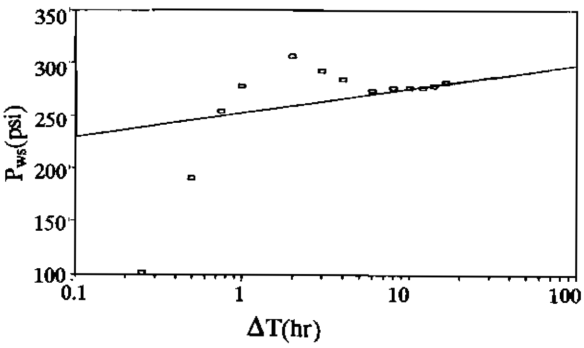

Fig. 8 : Semi-Log analysis (example 3)

DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS

Mathematical analysis of phase redistribution has been extended and a new type curve for Log-Log analysis of conventional reservoirs with phase redistribution has been constructed.

The diffusivity equation in fractured reservoirs has been solved including the phase redistribution boundary conditions and solutions in terms of dimensionless well bore pressure has been presented in Laplace space.

A new type curve for Log-Log analysis of fractured reservoirs with phase redistribution has been constructed by considering definition of dimensionless time t_D / C_D.

In order to use the new type curves for both conventional and fractured reservoirs, the apparent dimensionless storage coefficient (C_{AD}) and the dimensionless wellbore storage coefficient (C_D) need to be calculated. It is therefore recommended that in conjunction with the pressure build up data the fluid gradient under flowing and static conditions be measured.

From the type curve for fractured reservoirs in addition to the reservoir parameters (permeability, skin, wellbore storage constant), the two parameters characteristic of the fissuration: ω the storativity ratio and λ, the interporosity flow coefficients may also be determined.

An automated procedure of non-linear regression analysis using Marquardt method has been developed for reservoir parameter estimation [18]. A computer program has been provided on the IBM-PC to estimate optimal values of permeability (K), skin(S), wellbore storage (C_D), phase redistribution pressure parameter (C_ϕ) and phase redistribution time parameter (α) in conventional and fractured reservoirs. In the option of fractured reservoirs in addition to K, S, C_D, C_ϕ and α, ω, the storativity ratio and λ the interporosity flow coefficient are also calculated.

The optimization program has been examined for two generated data, two field examples in Louisiana and a field example in southwest of Iran. Results from semi-Log analysis and the optimization program showed to be in good agreement.

NOMENCLATURE

English Letter Symbols

A	constant
a	array of parameters (unknown)
B	constant
C	wellbore storage coefficient, bbl/psi
C_a	apparent storage coefficient, bbl/psi
C_{AD}	apparent dimensionless storage coefficient
C_f	compressibility of fluid within the fissure system
c_m	compressibility of fluid within the matrix
c_t	total compressibility, psi^{-1}
C_D	dimensionless wellbore storage coefficient
C_ϕ	phase redistribution pressure parameter, psi
$C_{\phi D}$	dimensionless phase redistribution pressure parameter
h	reservoir thickness, ft
K	reservoir permeability, md
L(f)	Laplace transform of function f
M	number of parameter (unknown)
md	milli-darcy
N	number of data points
P	pressure, psi

P_{fD} dimensionless fissure pressure
P_w wellbore pressure, psi
P_{wD} dimensionless wellbore pressure
P_D dimensionless pressure
P_ϕ phase redistribution pressure, psi
$P_{\phi D}$ dimensionless phase redistribution pressure
ΔP pressure difference, psi
q flow rate, B/D
q_{sf} sand-face flow rate, B/D
r radius, ft
RB reservoir barrel
r_w wellbore radius, ft
r_D dimensionless radius, $r_D=r/r_\omega$
s Laplace transform variable
S skin factor
t time, hours
t_D dimensionless time
Δt shut-in time
V_w wellbore volume

Greek Letter Symbols

α phase redistribution time parameter, hours
α_D dimensionless phase redistribution time parameter
β oil formation volume factor RB/STB
ψ Eulers costant= 0.57721566
λ Interporosity flow parameter
μ fluid viscosity, cp
ϕ porosity (fraction)
ω storativity ratio

Subscript Letter Symbols

b bulk
D dimensionless
f fissure
fb bulk fissure
fD dimensionless fissure
m matrix
ma matrix block, matrix average
mb bulk matrix
s surface
sf sandface
t total
w well
wf flowing bottom hole
ws shut-in bottom hole

Mathematical Notations

I_0 modified Bessel function of the first kind and of order zero
I_1 modified Bessel function of the first kind and of order one
K_0 modified Bessel function of the second kind and of order zero
K_1 modified Bessel function of the second kind and of order one
L Laplace transform
L^{-1} Laplace inversion operation
Ln Logarithm to the base e(natural logarithm)
∂_x partial derivative of x
Δ finite increment
Σ summation

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REFERENCES

[1] Van Everdingen, A. F., and Hurst, W., "*Application of the Laplace Transform to Flow Problems in Reservoirs*", *AIME*, **186**, 305 (1949).

[2] Ramey, H. J. Jr., "*Non-Darcy Flow and Wellbore Storage Effects in Pressure Build up and Draw Down of Gas Wells*", *JPT*, 223 (Feb. 1965).

[3] Agarwal, R.G., Al-Hussainy, R., and Ramey, H.J. Jr., "*An Investigation of Wellbore Storage and Skin Effect in Unsteady Liquid Flow: 1. Analytical Treatment*", *SPE*, 279 (Sept. 1970).

[4] Stegemeier, G. L., and Mathews, C. S., "*A Study of Anomalous Pressure Build up Behavior*", *Trans. AIME*, **44**, 213 (1958).

[5] Pitzer, S. C., Rice, J. D., and Thomas, C. E., "*A Comparison of Theoretical Pressure Build up Curves with Field Curves Obtained from Bottom-Hole Shutin Tests*", *Trans. AIME*,

216, 416.

[6] Fair, W. B. Jr., "*Pressure Build up Analysis with Wellbore Phase Redistribution*", *SPE*, 259 (1981)

[7] Winterfeld, P. H., "*Simulation of Pressure Build up in a Multiphase Wellbore/Reservoir System*", *SPE Formation Evalution*, 247 (June. 1989).

[8] Gringarten, A. C., Bourdet, D., Landel, P. A., and Kniazeff, V., "*A Comparison between Different Skin and Wellbore Storage Type Curves for Early Time Transient Analysis*", *Paper SPE 8205 Presented at the SPE-AIME 54th Annual Technical Conference and Exhibition*, Las Vegas, Nev., 23 (Sept. 1979).

[9] Warren, J. E., and Root, P. J., "*Behavior of Naturally Fractured Reservior*", *SPE*, 245 (1963).

[10] Bourdet, D., and Gringarten, A. C., "*Determination of Fissure Volume and Block Size in Fractured Reservoirs by Type Curve Analysis*", *Paper SPE 9293 presented at the SPE-AIME 55th Annual Technical Conference and Exhibition*, Dallas, Texas, 21 (Sep. 1980).

[11] Stehfest, H., "*Algorithm 368- Numerical Inversion of Laplace Transform*", *Communications of the ACM*, **13**. No. 1, 47 (1970).

[12] Sobbi, F. A., *Ph.D. Thesis*, Heriot Watt University, Edinburgh, U.K., (1988).

[13] Stewart, G., and Sobbi, F. A., "*Well Test Interpretation for Naturally Franctured Reservoirs* ",*Paper SPE 18173 Presented at the 63rd Annual Technical Conference and Exhibition of SPE*, Houston., Texas, 2-5 (Oct. 1988).

[14] Flannery, B. P., Teukolsky, S. A., and Wetterling, W. T., "*Numerical Recipes in Pascal (The Art of Scientific Computing)*, William H. Press, 547 (1989).

[15] Kuester, J. L., and Mize, J. H., "*Optimization Techniques with Fortran*", 240 (1973).

[16] Marquardt. D.M., "*An Algorithm for Least Squares Estimation of Non-linear Parameters*", *J. Soc. Indust. Appl. Math.*, **11**, 431 (1963).

[17] Abramowitz, M., and Stegun, I.A., "*Handbook of Mathematical Function with Formulae, Graphs and Mathematical Tables*", (1972).

[18] Mohebbi, A., "*M. Sc. Thesis*", *University of Petroleum Industry*, Ahwaz, Iran, (1992).

APPENDIX

1. Mathematical Analysis of Phase Redistribution

For a well where wellbore storage occurs, the contribution of the expansion of the wellbore liquid may be written as:

$$q_s - q_{sf} = cV_w \frac{dP_w}{dt} \qquad (1.1)$$

where c is the compressibility of the fluid in the wellbore. V_w is the wellbore volume, q_s is the surface flow rate and q_{sf} is the sandface flow rate. Defining the wellbore storage constant C= c.V_w and introducing the dimensionless parameters shown below, Eq.(1.1) may be written as:

$$\frac{dP_{wD}}{dt_D} = \frac{1}{C_D}\left(1 - \frac{q_{sf}}{q}\right) \qquad (1.2)$$

where C_D , t_D and P_{wD} are dimensionless wellbore storage, dimensionless time and dimensionless wellbore pressure respectively.

Fair [6] modified Eq. (1.2) by adding a term describing the pressure change caused by phase redistribution as follows:

$$\frac{dP_{wD}}{dt_D} = \frac{1}{C_D}\left(1 - \frac{q_{sf}}{q}\right) + \frac{dP_{\phi D}}{dt_D} \qquad (1.3)$$

This equation may be rearranged to show the sandface flow rate depenency:

$$\frac{q_{sf}}{q} = 1 - C_D\left(\frac{dP_{wD}}{dt_D} - \frac{dP_{\phi D}}{dt_D}\right) \qquad (1.4)$$

where

$$P_{\phi D} = C_{\phi D}\left(1 - e^{(-t_D/\alpha_D)}\right) \qquad (1.5)$$

$P_{\phi D}$ is the dimensionless wellbore phase redistribution pressure, $C_{\phi D}$ and α_D are dimensionless phase redistribution pressure and time parameters defined below:

$$C_{AD} = \frac{5.6146C_a}{2\pi\phi c_t hr^2_w} = \left(\frac{C_{\phi D}}{\alpha_D} + \frac{1}{C_D}\right)^{-1} \quad (1.6)$$

$$C_D = \frac{5.6146C}{2\pi\phi c_t hr^2_w} \quad (1.7)$$

$$C_{\phi D} = \frac{KhC_\phi}{141.2q\,\beta\mu} \quad (1.8)$$

$$P_{\phi D} = \frac{KhP_\phi}{141.2q\,\beta\mu} \quad (1.9)$$

$$P_{wD} = \frac{Kh\Delta P_w}{141.2q\,\beta\mu} \quad (1.10)$$

$$r_D = \frac{r}{r_w} \quad (1.11)$$

$$t_D = \frac{0.000264K\Delta t}{\phi\mu c_t r^2_w} \quad (1.12)$$

$$\alpha_D = \frac{0.000264K\alpha}{\phi\mu c_t r^2_w} \quad (1.13)$$

2. Determination of Dimensionless Wellbore Pressure

The diffusivity equation that has been found to describe the fractured reservoirs behavior in *Laplace* space is [9]:

$$L\left(\frac{d^2P_{fD}}{dr^2_D}\right) + \frac{1}{r_D} \cdot L\left(\frac{dP_{fD}}{dr_D}\right) = sf(s)L(P_{fD}) \quad (2.2)$$

where f(s) is defined by Eq.(2.1) for the pseudosteady state interporosity flow condition:

$$f(s) = \frac{\omega(1-\omega)s+\lambda}{(1-\omega)s+\lambda} \quad (2.2)$$

where ω is dimensionless storativity and λ is dimensionless interflow parameter.

The general solution to Eq.(2.1) in Laplace space is given by [1]:

$$L(P_{fD})=AI_0(r_D\sqrt{sf(s)})+BK_0(r_D\sqrt{sf(s)}) \quad (2.3)$$

Where I_0 and K_0 are modified Bessel function of the first and second kind respectively and of zero order. A and B are constants and are evaluated according to the reservoir boundary conditions.

To obtain dimensionless pressure solutions for use in the analysis of pressure build up tests, it is necessary to incorporate the effects of wellbore phase redistribution into the diffusivity equation.

$$P_{fD}(r_D,0) = 0 \quad (2.4)$$

$$\lim_{r_D \to \infty} P_{fD}(r_D, t_D) = 0 \quad (2.5)$$

$$-\left(\frac{dP_{fD}}{dr_D}\right) = 1 - C_D\left(\frac{dP_{wD}}{dt_D} - \frac{dP_{\phi D}}{dt_D}\right) \quad r_D=1 \quad (2.6)$$

and in Laplace space, these equations become:

$$\lim_{r_D \to \infty} L(P_{fD}) = 0 \quad (2.7)$$

$$L\left(\frac{dP_{fD}}{dr_D}\right) = -\frac{1}{s} + C_D\left[L\left(\frac{dP_{wD}}{dt_D}\right) - L\left(\frac{dP_{\phi D}}{dt_D}\right)\right] \text{ at } r_D=1 \quad (2.8)$$

By using the inner and outer boundary conditions without wellbore storage and phase redistribution, constants A and B may be obtained and by substituting into Eq. (2.3) yields:

$$L(P_{fD}) = \frac{K_0[r_D\sqrt{sf(s)}]}{s[\sqrt{sf(s)}]K_1[\sqrt{sf(s)}]} \quad (2.9)$$

This equation in Laplace space is the solution for the constant rate and infinite acting fractured reservoir. P_{fD} is the dimensionless sandface wellbore pressure.

The effects of wellbore storage and phase redistribution and skin may be taken into account, the dimensionless pressure drop at the wellbore in Laplace space becomes [2,6,10]:

$$L(P_{wD}) = \frac{sL(P_{fD})(1+C_Ds^2L(P_{\phi D}))+C_Ds^2L(P_{\phi D})S+S}{s[1+C_Ds(S+sL(P_{fD}))]} \quad (2.10)$$

Here the Laplace transform of the dimensionless wellbore pressure is related to Laplace transform of the dimensionless sandface pressure. S is the skin factor, s is the Laplace variable and C_D is the dimensionless wellbore storage.

$$L(P_{\phi D}) = \frac{C_{\phi D}}{s} - \frac{C_{\phi D}}{s+1/\alpha_D} \quad (2.11)$$

Substituting $L(P_{fD})$ from Eq.(2.9) and $L(P_{\phi D})$ from Eq.(2.11) into Eq.(2.10) yields:

$$L(P_{wD}) = \frac{\left[\frac{K_0(\sqrt{sf(s)})}{\sqrt{sf(s)}K_1(\sqrt{sf(s)})} + S\right]\left[1+C_D C_{\phi D} s^2\left(\frac{1}{s} - \frac{1}{s+1/\alpha_D}\right)\right]}{s\left(1+C_D s\left[\frac{K_0(\sqrt{sf(s)})}{\sqrt{sf(s)}K_1(\sqrt{sf(s)})} + S\right]\right)} \tag{2.12}$$

3. *Application of Optimization Methods for Phase Redistribution Data*

Optimization techniques have been found to be effective in a variety of areas throughout the scientific world. The focus here is on application of this technique for the estimation of optimal values of reservoir properties. Suppose that one wants to fit N data points (x_i,y_i) i= 1,...,N to a model which has M adjustable parameters a_j, j=1,...,M. If each data point(x_i,y_i) has its own standard deviation ∂_i, so that chi-square may be defined as follows:

$$x^2 = \sum_{i=1}^{N} \left(\frac{y_i - y(x_i;\, a_1...a_M)}{\partial_i}\right)^2 \tag{3.1}$$

the best values of the model parameters are obtained when the chi-squre is minimized.

The dimensionless pressure in the Laplace space for both fractured and conventional resevoirs are non-linear equations. Thus an automated procedure of non-linear regression analysis using Marquardt method is developed for reservoir parameter estimation. The procedure was proposed by marqurdt as an extension of the Gauss Newton method to allow for convergence with relatively poor starting guesses for the unknown coefficients. The dimensionless pressure in Laplace space is given in appendix 2(f(s)=1 for conventional reservoirs), thus with an initial guess for the C_D, $C_{\phi D}$, S, K, α_D: Eq.(2.12) may be inverted numerically by Stehfest [6] algorithm and P_{wD} may be obtained in real time domain, that is used in Marquardt's routine. In addition, Marquardt's routine requires the vector of derivatives $\partial\Delta P/\partial C_D$, $\partial\Delta P/\partial C_{\phi D}$, $\partial\Delta P/\partial\alpha_D$, $\partial\Delta P/\partial S$, $\partial\Delta P/\partial K$ that must be calculated. Differentiation of Eq.(2.12) with respect to C_D, $C_{\phi D}$, S, K and α_D gives:

$$L\left(\frac{\partial P_{wD}}{\partial C_D}\right) = \frac{\left[\frac{K_0(s^{\frac{1}{2}})}{s^{\frac{1}{2}}K_1(s^{\frac{1}{2}})} + S\right]C_\phi h_1 K s\left(\frac{1}{s} - \frac{1}{s+1/\beta_1 K\alpha}\right) - \left[\frac{K_0(s^{\frac{1}{2}})}{s^{\frac{1}{2}}K_1(s^{\frac{1}{2}})} + S\right]^2}{\left(1+ C_D s\left[\frac{K_0(s^{\frac{1}{2}})}{s^{\frac{1}{2}}K_1(s^{\frac{1}{2}})} + S\right]\right)^2} \tag{3.2}$$

$$L\left(\frac{\partial P_{wD}}{\partial C_\phi}\right) = \frac{\left[\frac{K_0(s^{\frac{1}{2}})}{s^{\frac{1}{2}}K_1(s^{\frac{1}{2}})} + S\right]C_D h_1 K s\left(\frac{1}{s} - \frac{1}{s+1/\beta_1 K\alpha}\right)}{\left(1+ C_D s\left[\frac{K_0(s^{\frac{1}{2}})}{s^{\frac{1}{2}}K_1(s^{\frac{1}{2}})} + S\right]\right)^2} \tag{3.3}$$

$$L\left(\frac{\partial P_{wD}}{\partial S}\right) = \frac{1+ C_D C_\phi h_1 K s^2\left(\frac{1}{s} - \frac{1}{s+1/\beta_1 k\alpha}\right)}{s\left(1+C_D s\left[\frac{K_0(s^{\frac{1}{2}})}{s^{\frac{1}{2}}K_1(s^{\frac{1}{2}})} + S\right]\right)^2} \tag{3.4}$$

$$L\left(\frac{\partial P_{wD}}{\partial K}\right) = \frac{\left[\frac{K_0(s^{\frac{1}{2}})}{s^{\frac{1}{2}}K_1(s^{\frac{1}{2}})} + S\right]C_D h_1 C_\phi}{\left(1 + C_D s\left[\frac{K_0(s^{\frac{1}{2}})}{s^{\frac{1}{2}}K_1(s^{\frac{1}{2}})} + S\right]\right)(s\beta_1 K\alpha+1)^2} \tag{3.5}$$

$$L\left(\frac{\partial P_{wD}}{\partial \alpha}\right) = \frac{\left[\frac{K_0(s^{\frac{1}{2}})}{s^{\frac{1}{2}}K_1(s^{\frac{1}{2}})} + S\right]s C_D h_1 C_\phi \beta_1 K^2}{\left(1 + C_D s\left[\frac{K_0(s^{\frac{1}{2}})}{s^{\frac{1}{2}}K_1(s^{\frac{1}{2}})} + S\right]\right)(s\beta_1 K\alpha+1)^2} \tag{3.6}$$

where :

$$h_1 = h/141.2q\,\beta\mu\ ,\ \beta_1 = 0.000264/(\phi\mu\, c_t\, r_w^2)$$
$$\alpha_D = \beta_1 K\alpha\ ,\ C_{\phi D} = h_1 K C_\phi$$

Eqs.(3.2), (3.3), (3.4), (3.5), (3.6) are inverted by Stehfest algorithm and $\partial\Delta P/\partial C_D$, $\partial\Delta P/\partial C_{\phi D}$, $\partial\Delta P/\partial\alpha_D$, $\partial\Delta P/\partial S$, $\partial\Delta P/\partial K$ are calculated.