Robust H-Infinity Control of Industrial Boiler Drum Level with Structured and Unstructured Uncertainty Model

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ABSTRACT: *The water level control in boiler drums is a crucial process in many process industries. In industries, water spillage or overheated tubes of the water boiler are the serious consequences of extremely high or low-level drum water level maintenance. The boiler drum is a MIMO system, consists of dead time nonlinearity and thereby there exists a transportation lag between the input and the system. Also, they possess high dynamic variations. Hence, the control of the boiler drum level is of great importance. Though, conventional PID controllers are employed in industries, due to the presence of nonlinearity, boiler drum performance can be affected when it is controlled by a PID controller. Moreover, the PID controller produces a larger settling time. Here, a robust controller for the boiler drum level control based on the H-infinity technique is designed. The first-principle mathematical model of the boiler drum is formulated. The uncertainties namely: structured uncertainty, unstructured uncertainty, and nonlinear uncertainty are modeled by incorporating the boiler drum dynamics including its inherent nonlinearity. The boiler drum level control is carried out using the H-infinity controller scheme with the uncertainties accounted for and the performance is compared with that of a conventional PID controller. The qualitative and quantitative comparison of performances of the above control schemes reveals that the H-infinity controller has a quick rise time and faster settling time.*

KEYWORDS: *Boiler drum level; Mathematical modeling; Structured uncertainty; Unstructured uncertainty; H-infinity controller.*

INTRODUCTION

Industrial boilers are key structures of the power plant sectors. The boiler drum is a subsystem of the boiler with paramount importance showing complex phase equilibrium and inverse response behavior. The measurement variables that can be obtained from the main system of the boiler are the temperature of the steam, the pressure of the steam, the

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concentration of exhaust gas, and the boiler drum level. The efficiency of the boiler drum is decreased due to the difference in load demands. The criticality of the drum level control depends on many factors. This can be due to increased response time of the system and random disturbances, difference in load demands, abrupt reduction in drum water level exposing boiler tubes which in turn cause overheating and failure, and also increase in drum water level that may pave the way for carryover of water into the turbine unit reducing the boiler efficiency. Considering these factors, the goal of this work is to control the drum level in the thermal power plant. The proposed work aims to model the boiler drum using first principle approach for the purpose of designing the controller. The change in drum level may occur due to the uncertainties and load disturbance. The uncertainty may arise from modeling error of plant, unknown dynamics and also due to linear approximation of the model. The uncertainty modeling of the boiler drum is developed as this controller deals with the uncertainties.

Several dynamic models of boiler drums have been developed in the literature. *Astrom* and *Bell* [1] developed the nonlinear model of boiler drum based on a mathematical modeling approach. Also, a lumped mixed standard model is developed [2] for boiler drum dynamics that also showed improved performances when implemented.

For the mathematical models developed, model-based controllers were designed. Conventionally, PID controllers were designed for the boiler drum level control. The performance analysis with one pulse, cascade and three pulse controllers are discussed in [3]. Following which, FOPID [4], Fuzzy logic controllers [5], optimized fuzzy controller [6] and hybrid fuzzy PID controllers [7], sliding mode control [8], IMC are also designed for the boiler drum level control. Also, improved versions of controllers like active disturbance rejection [9] and statistical algorithms [10] like k-means clustering and pattern matching, and Quantitative feedback theory [11] were implemented for the boiler drum level control. The above-discussed controller's performance was validated on the laboratory experimental setup. However, there are also in literature, real time industrial boiler drum control using artificial neural networks [12] and conventional PI controller with three element control [13]. Thus, many controller designs owing to conventional, intelligent and hybrid models are being developed in the recent times which prove that the boiler drum control is still a critical system requiring efficient and accurate control.

However, the uncertainty analysis was not shown much interest earlier. In recent times, more work on accounting to the uncertainties also have emerged. Uncertainties are of two types namely: structured and unstructured uncertainties. A comprehensive review of the uncertainty modeling is provided by the authors in [14]. Works accounting to only structured uncertainty for multidimensional array system [15] and boiler drum control [16] is also available in the literature. The parametric and unstructured multiplicative uncertainty approach has been developed in [17] to study the robustness of the controller developed. With the advent of technologies, enhancing the robustness of networked cascade control systems is also approached using augmented techniques [18].

In the present work, the modeling of the boiler drum is developed by using the first principle approach with the help of mass and energy balance equations. The robust control is designed for drum-level control using the H-infinity technique. As robust controller deals with the uncertainties, the uncertainty modeling based on the structured, unstructured and nonlinear uncertainties of boiler drum are also developed. Finally, the performance of H-infinity controller is compared with the conventional controller.

This article is organized as follows: In the process description section, a brief description of the boiler drum is presented. The development of mathematical modeling of boiler drum using first principle dynamic equations is presented in the next section followed by the uncertainty modeling of boiler drum. The outputs, comparisons and inferences of the proposed and developed models are presented in the results and discussion section. The conclusions based on the observations are presented at the end.

THEORETICAL SECTION

Process description

The boiler drum is also known as storage unit where the water and steam separation process occur. The steam is filled in the upper part of the boiler drum**.** The structure of the boiler drum is shown in Fig 1.

From Fig.1, it can be seen that, the boiler drum also includes the riser and down comers. The feed water is continuously circulated from the non-heating surface (down comer)

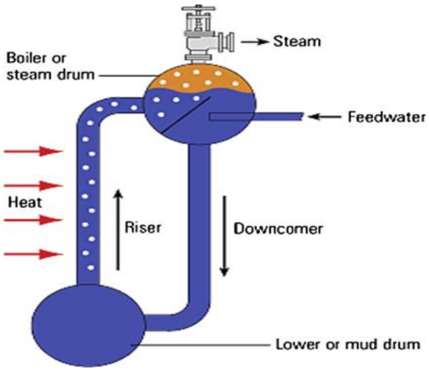


Fig. 1: Structure of boiler drum [4]

to the heating surface (riser) for conversion to steam by boiling. Maintaining the drum level at the desired position is critical, both for the equipment safety as well as efficient operation of the boiler. If the level is low, the boiler tubes become uncovered which led to overheat and get damaged. If the water level is high, the separation process of steam from the water steam mixture gets collapse, which in turn leads to carry moisture (water) into the turbine systems. Also, the dynamic shrink and swell phenomenon is a reason to produce variation in the level of the liquid. The shrink and swell phenomenon mean that simply a decrease or an increase in drum level is due to the formation of more or less steam bubbles in the water and not due to the change in the amount of water in the drum. Thus, the objective of drum level control system is to maintain the level of water steam mixture in the boiler drum at the desired level. The input parameters are: the feed water flow, steam flow, heat input and the output parameter is the drum level.

Modelling of boiler drum

The boiler drum is modeled using the first principle approach with the help of mass and energy balance equations.

First principle method of modelling of boiler drum

Boiler drum is one of the crucial parts of the system and there are many modelling efforts on it. Many variations of modeling are available in literatures. However, in this work, the design approach of *Astrom* and *Bell* (2000) is followed for mathematically modeling the boiler drum.

Mass Balance

The mass energy conservation equation of boiler drum model describes the dynamics of the following: total water volume ‘ V_{wt} ’, drum pressure ‘p’, steam quality ‘ α_r ’ and steam volume in drum ‘ V_{sd} ’ and also the response of boiler drum level ‘l’ to the steam flow q_s and feed water flow ‘ q_f ’. Therefore,

Rate of change of mass of steam and water in the drum = Feed water flow to the system –steam flow from the drum.

$$\frac{d}{dt}\left[\rho_s V_{st} + \rho_w V_{wt}\right] = q_f - q_s \qquad (1)$$

where,

q_f – Feed water mass flow rate (kg/sec)

q_s – Steam mass flow rate (kg/sec)

ρ_s – Density of steam

ρ_w – Density of water

V_{wt} – Total volume of water

V_{st} – Total volume of steam.

Energy balance

The global energy balance is given in Eq. (2).

$$\frac{d}{dt}\left[\rho_s u_s V_{st} + \rho_w u_w V_{wt} + m_t c_p t_m\right] = Q + q_f h_f - q_s h_s \qquad (2)$$

Also, in general, the internal energy is given by

$$u = h - {}^{p}/_{\rho} th$$

Now, the energy balance equation can be written as given in Eq. (3).

$$\frac{d}{dt}\left[\rho_s h_s V_{st} + \rho_w h_w V_{wt} - pV_t + m_t c_p t_m\right] = Q + q_f h_f - q_s h_s \qquad (3)$$

where, h_s –Enthalpy of steam

h_f –Enthalpy of feed water

m_t –Total mass of metal tubes and drum

t_m –Metal temperature

c_p–Specific heat of metal

The total volume of drum, down comer and risers, V_t is given in Eq. (4).

$$V_t = V_{st} + V_{wt} \qquad (4)$$

where,

V_{st} – Total volume of steam

V_{wt} – Total volume of water.

From Eqs. (1), (3) and (4), the second order model is developed with two states namely, drum pressure 'p' and total volume of water 'V_{wt}'. The state equations are given in Eq. (5) and (6).

$$e_{11}\frac{dV_{wt}}{dt}+e_{12}\frac{dp}{dt}=q_f-q_s \tag{5}$$

$$e_{21}\frac{dV_{wt}}{dt}+e_{22}\frac{dp}{dt}=Q+q_fh_f-q_sh_s \tag{6}$$

where,

$$e_{11}=\rho_w-\rho_s$$

$$e_{12}=V_{st}\frac{\partial\rho_s}{\partial p}+V_{wt}\frac{\partial\rho_w}{\partial p}$$

$$e_{21}=\rho_wh_w-\rho_sh_s$$

$$e_{22}=V_{st}\left(h_s\frac{\partial\rho_s}{\partial p}+\rho_s\frac{\partial h_s}{\partial p}\right)+V_{wt}\left(h_w\frac{\partial\rho_w}{\partial p}+\rho_w\frac{\partial h_w}{\partial p}\right)-V_t+m_tc_p\frac{\partial t_s}{\partial p}. \tag{7}$$

By using steam tables, h_s, h_w, ρ_s, ρ_w, t_s, $\frac{\partial\rho_s}{\partial p}$, $\frac{\partial h_s}{\partial p}$, $\frac{\partial\rho_w}{\partial p}$, $\frac{\partial h_w}{\partial p}$, $\frac{\partial t_s}{\partial p}$ are evaluated. The results are based on approximation of steam tables quadratic functions as given in Eq. (8) to (16).

$$h_s=2.7254E6+(-1.8992E4-1160.0(p-10))(p-10) \tag{8}$$

$$h_w=1.4035E6+(4.9339E4-880.0(p-10))(p-10) \tag{9}$$

$$\rho_s=53.1402+(7.673+0.36(p-10))(p-10) \tag{10}$$

$$\rho_w=691.35+(-18.672-0.0603(p-10))(p-10) \tag{11}$$

$$t_s=310.6+(8.523-0.33(p-10))(p-10) \tag{12}$$

$$\frac{\partial h_s}{\partial p}=-1.8992E4-1160.0(p-10) \tag{13}$$

$$\frac{\partial h_w}{\partial p}=4.9339E4-880.0(p-10) \tag{14}$$

$$\frac{\partial\rho_s}{\partial p}=7.673+0.36(p-10) \tag{15}$$

$$\frac{\partial\rho_w}{\partial p}=-18.672-0.0603(p-10) \tag{16}$$

The lumped parameter model is developed by using the mass and energy balance equation of the riser section.

The mass balance of riser section is given in Eq. (17).

$$\frac{d}{dt}(\rho_s\overline{\alpha_V}V_r+\rho_w(1-\overline{\alpha_V})V_r)=q_{dc}-q_r \tag{17}$$

where,

q_r –Total mass flow rate out of riser

q_{dc}–Total mass flow rate into riser.

The energy balance of riser section is given in Eq. (18)

$$\frac{d}{dt}\left(\rho_sh_s\overline{\alpha_V}V_r+\rho_wh_w(1-\overline{\alpha_V})V_r-pV_r+m_rc_pt_s\right)=Q+q_{dc}h_w-(\alpha_rh_c+h_w)q_r \tag{18}$$

The momentum balance for the down comer riser loop is given in Eq. (19)

$$(L_r+L_{dc})\frac{dq_{dc}}{dt}=(\rho_w-\rho_s)\overline{\alpha_V}V_rg-\frac{k}{2}\frac{{q_{dc}}^2}{2\rho_wA_{dc}} \tag{19}$$

where,

k – Dimensionless friction coefficient

L_r and L_{dc} – Lengths

A_{dc} – Area

The mass balance for the steam under the liquid level is given in Eq. (20)

$$\frac{d}{dt}(\rho_sV_{sd})=\alpha_rq_r-q_{sd}-q_{cd} \tag{20}$$

Where, q_{dc} - Condensation flow.

By using the above equations, the boiler drum model is developed with four states namely, drum pressure p, total water volume V_{wt}, steam quality α_r and steam volume V_{sd}. The state equations of boiler drum are given from Eq. (21) to (24).

$$e_{11}\frac{dV_{wt}}{dt}+e_{12}\frac{dp}{dt}=q_f-q_s \tag{21}$$

$$e_{21}\frac{dV_{wt}}{dt}+e_{22}\frac{dp}{dt}=Q+q_fh_f-q_sh_s \tag{22}$$

$$e_{32}\frac{dp}{dt}+e_{33}\frac{d\alpha_r}{dt}=Q-\alpha_rh_cq_{dc} \tag{23}$$

$$e_{42}\frac{dp}{dt}+e_{43}\frac{d\alpha_r}{dt}+e_{44}\frac{dV_{sd}}{dt}=\frac{\rho_s}{T_d}\left({V_{sd}}^0-V_{sd}\right)+\frac{h_f-h_w}{h_c}q_f \tag{24}$$

Where,

$$e_{11}=\rho_w-\rho_s$$

$$e_{12}=V_{wt}\frac{\partial\rho_w}{\partial p}+V_{st}\frac{\partial\rho_s}{\partial p}$$

$$e_{21}=\rho_wh_w-\rho_sh_s$$

$$e_{22}=V_{st}\left(h_s\frac{\partial\rho_s}{\partial p}+\rho_s\frac{\partial h_s}{\partial p}\right)+V_{wt}\left(h_w\frac{\partial\rho_w}{\partial p}+\rho_w\frac{\partial h_w}{\partial p}\right)-V_t+m_tc_p\frac{\partial t_s}{\partial p}$$

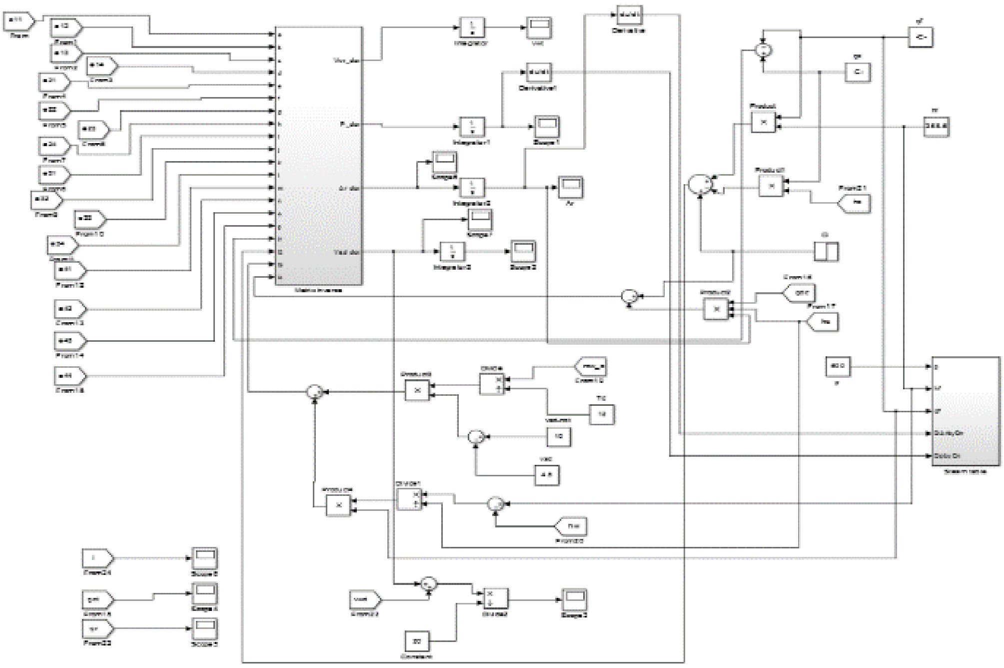


Fig. 2: Block diagram of open loop Boiler Drum Model

$$e_{32} = \left(\rho_w \frac{\partial h_w}{\partial p} - \alpha_r h_c \frac{\partial \rho_w}{\partial p}\right)(1-\overline{\alpha_V})V_r + \left[(1-\alpha_r)h_c \frac{\partial \rho_s}{\partial p} + \rho_s \frac{\partial h_s}{\partial p}\right]$$

$$\overline{\alpha_V}V_r + (\rho_s+(\rho_w-\rho_s)\alpha_r)h_c V_r \frac{\partial \overline{\alpha_V}}{\partial p} - V_r + m_r c_p \frac{\partial t_s}{\partial p}$$

$$e_{33} = [(1-\alpha_r)\rho_s + \alpha_r \rho_w]h_c V_r \frac{\partial \overline{\alpha_V}}{\partial \alpha_r}$$

$$e_{42} = V_{sd}\frac{\partial \rho_s}{\partial p} + \frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - V_{sd} - V_{wd} + m_d c_p \frac{\partial t_s}{\partial p}\right) +$$

$$\alpha_r(1+\beta)V_r\left(\overline{\alpha_V}\frac{\partial \rho_s}{\partial p} + (1-\overline{\alpha_V})\frac{\partial \rho_w}{\partial p} + (\rho_s-\rho_w)\frac{\partial \overline{\alpha_V}}{\partial p}\right)$$

$$e_{43} = \alpha_r(1+\beta)(\rho_s-\rho_w)V_r\frac{\partial \overline{\alpha_V}}{\partial \alpha_r}$$

$$e_{44} = \rho_s$$

The partial derivative of the steam volume fraction with respect to pressure and steam quality is obtained as given in Eq. (25) and (26), respectively.

$$\frac{\partial \overline{\alpha_V}}{\partial \alpha_r} = \frac{\rho_w}{\rho_s \eta}\left(\frac{1}{\eta}\ln(1+\eta) - \frac{1}{1+\eta}\right) \tag{25}$$

$$\frac{\partial \overline{\alpha_V}}{\partial p} = \frac{1}{(\rho_w-\rho_s)^2}\left(\rho_w \frac{\partial \rho_s}{\partial p} - \rho_s \frac{\partial \rho_w}{\partial p}\right)\left(1 + \frac{\rho_w}{\rho_s}\frac{1}{1+\eta} - \frac{\rho_s+\rho_w}{\eta \rho_s}\ln(1+\eta)\right) \tag{26}$$

Where,

$$\eta = \alpha_r(\rho_w - \rho_s)/\rho_s$$

$$\overline{\alpha_V} = \frac{\rho_w}{\rho_w-\rho_s}\left(1 - \frac{\rho_s}{(\rho_w-\rho_s)\,\alpha_r}\ln\left(1 + \frac{\rho_w-\rho_s}{\rho_s}\alpha_r\right)\right)$$

$$T_d = \frac{\rho_s V_{sd}{}^0}{q_{sd}}$$

$$q_{ct} = \frac{h_w-h_f}{h_c}q_f + \frac{1}{h_c}\left(\rho_s V_{st}\frac{\partial h_s}{\partial p} + \rho_w V_{wt}\frac{\partial h_w}{\partial p} - V_t + m_t c_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt}$$

$$q_r = q_{dc} - V_r\left(\overline{\alpha_V}\frac{\partial \rho_s}{\partial p} + (1-\overline{\alpha_V})\frac{\partial \rho_w}{\partial p} + (\rho_w-\rho_s)\frac{\partial \overline{\alpha_V}}{\partial p}\right)\frac{dp}{dt} + (\rho_w-\rho_s)V_r\frac{\partial \overline{\alpha_V}}{\partial \alpha_r}\frac{d\alpha_r}{dt}$$

Drum level model

The drum level can be modelled from the distribution of the steam below the drum level. The volume of water in the drum is given in Eq. (27).

$$V_{wd} = V_{wt} - V_{dc} - (1-\overline{\alpha_V})V_r \tag{27}$$

The deviation of the drum level l can be measured from the normal operating level and is given by Eq. (28).

Table 1: Nominal values of boiler drum variables [7]

Parameter Notation	Nominal Value
V_r	$37 m^3$
V_{dc}	$11 m^3$
V_t	$88 m^3$
A_d	$20 m^2$
A_{dc}	$0.355 m^2$
m_r	160e3 kg
m_t	300e3 kg
m_d	100e3 kg
K	25
β	0.3
T_d	12 sec
$V_{sd}{}^0$	$10 m^3$
g	$9.8 m/sec^2$
V_d	$40 m^3$

$$l = \frac{V_{wd}+V_{sd}}{A_d} \tag{28}$$

The above equation can also be written as

$$l = l_w + l_s \tag{29}$$

where,

l_w – Level variation caused by change of the amount of water in drum

l_s – Level variation caused by the change of steam in the drum.

Open Loop Boiler Drum Model

Thus, the mathematical model of boiler drum is obtained by solving the differential equations and simulation results are verified with the help of the nominal values of boiler drum variables as listed in Table 1. The Simulink block diagram of the boiler drum in open loop is shown in Fig. 2.

The nominal values of boiler drum variables are listed in the Table 1. The basic boiler dynamic equations were simulated for the nominal values.

Instead of mathematically modeling, a data driven approach is also discussed by the same authors in [12].

Uncertainty modelling

Uncertainty represents the unexpected behavior of plants during operation. It can be modeled by using statistical and probability-based techniques. In control system theory, as well as in many methodologies, mathematical model of a controlled system of the plant is used for the purpose of controller design. The major problem is that, the assumed ideal mathematical model does not match with the real behavior of the plant due to many reasons. Considering this fact into account, the utilization of the uncertainty model is required in order to overcome the mismatches of the real and modelled plant. Basically, uncertainty can be classified into three types. They are structured uncertainty, unstructured uncertainty and nonlinear uncertainty.

Structured uncertainty

Structured uncertainty also known as parametric uncertainty, represents the parameter variations in plant dynamics such as, uncertainties in the entries of state space models, uncertainties in gains of control systems and uncertainties in the location of poles and zeros of the transfer functions. In short, the structured uncertainty is defined such that the structure of the system is known and the parameters of the system are unknown or varying in a range.

For developing the uncertainty model, the differential equation of the boiler drum as given in Eqs. (21) to (24) is represented in the form of matrix in Eq. (30)

$$\begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ e_{21} & e_{22} & e_{23} & e_{24} \\ e_{31} & e_{32} & e_{33} & e_{34} \\ e_{41} & e_{42} & e_{43} & e_{44} \end{bmatrix} \begin{bmatrix} \frac{dV_{wt}}{dt} \\ \frac{dp}{dt} \\ \frac{d\alpha_r}{dt} \\ \frac{dV_{sd}}{dt} \end{bmatrix} = \begin{bmatrix} q_f - q_s \\ Q + q_f h_f - q_s h_s \\ Q - \alpha_r h_c q_{dc} \\ \frac{\rho_s}{T_d}\left(V_{sd}{}^0 - V_{sd}\right) + \frac{h_f - h_w}{h_c} q_f \end{bmatrix} \tag{30}$$

where,

$$e_{13} = e_{14} = e_{23} = e_{24} = e_{31} = e_{34} = e_{41} = 0.$$

The structured uncertainty due to parameter variations can be estimated using the Eq. (31) and (32).

$$\Delta_{e_{(x,y)}} = \frac{\left(e_{(x,y)_{max}} - e_{(x,y)_{min}}\right)}{e_{(x,y)}} \tag{31}$$

$$\begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} \Delta_{e_{(1,1)}} & \cdots & \Delta_{e_{(1,m)}} \\ \vdots & \ddots & \vdots \\ \Delta_{e_{(n,1)}} & \cdots & \Delta_{e_{(n,m)}} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \tag{32}$$

Table 2: Simulated values of parameter and its uncertain range

Parameter Notation	Simulated value	Uncertain range (Δ)
ρ_w	-5.0332e3	± 2.0461
ρ_s	1.4507e4	± 3.01
V_{wt}	57	± 0.008
V_{st}	8	± 0.01
$\frac{\partial \rho_w}{\partial p}$	-23.01	-0.01
$\frac{\partial \rho_s}{\partial p}$	-10.12	± 0.017
h_w	-20990090	±2.140
h_s	-42759080	± 3.102
$\frac{\partial t_s}{\partial p}$	-123.4770	± 0.8
$\frac{\partial h_s}{\partial p}$	-239392	± 0.8
V_t	160.8	1.2
V_{sd}	4.5	± 0.001
h_c	-21768990	±2

The uncertain range of each parameter of the boiler drum is estimated with the help of the mathematical model of boiler drum in the nominal operating condition and tabulates in Table 2.

By using this maximum and minimum range of the parameters of boiler drum the uncertainty model is developed as follows:

$$\Delta_{e_{11}\max} = \left((\rho_w + \Delta_w) - (\rho_s + \Delta_s)\right) - (\rho_w - \rho_s) = \Delta_{w_{\max}} - \Delta_{s_{\max}}$$

$$\Delta_{e_{11}\min} = \Delta_{w_{\min}} - \Delta_{s_{\min}}$$

$$\Delta_{e_{11}} = (\Delta_{e_{11}\max} - \Delta_{e_{11}\min})/e_{11} = 6.2764\text{ e-}6.$$

Similarly,

$$\Delta_{e_{12}} = (V_{wt} + \Delta V_{wt}) * \left(\frac{\partial \rho_w}{\partial P} + \Delta \frac{\partial \rho_w}{\partial P}\right) + (V_{st} + \Delta V_{st}) * \left(\frac{\partial \rho_s}{\partial P} + \Delta \frac{\partial \rho_s}{\partial P}\right) - \left(V_{wt} * \frac{\partial \rho_w}{\partial P} + V_{st} * \frac{\partial \rho_s}{\partial P}\right)$$

$$\Delta_{e_{12}} = (\Delta_{e_{12}\max} - \Delta_{e_{12}\min})/e_{12} = -2.4145\text{e-}11$$

$$\Delta_{e_{21}} = (\rho_w + \Delta\rho_w) * (h_w + \Delta h_w) + (\rho_s + \Delta\rho_s) * (h_s + \Delta h_s) - (\rho_w h_w + \rho_s h_s)$$

$$\Delta_{e_{21}} = (\Delta_{e_{21}\max} - \Delta_{e_{21}\min})/e_{21} = 0$$

$$\begin{aligned}\Delta_{e_{22}} = {} & (V_{wt} + \Delta V_{wt})(h_w + \Delta h_w)\left(\frac{\partial \rho_w}{\partial P} + \Delta \frac{\partial \rho_w}{\partial P}\right) \\ & + (V_{wt} + \Delta V_{wt})(\rho_w + \Delta \rho_w)\left(\frac{\partial h_w}{\partial P} + \Delta \frac{\partial h_w}{\partial P}\right) \\ & + (V_{st} + \Delta V_{st})(h_s + \Delta h_s)\left(\frac{\partial \rho_s}{\partial P} + \Delta \frac{\partial \rho_s}{\partial P}\right) \\ & + (V_{st} + \Delta V_{st})(\rho_s + \Delta \rho_s)\left(\frac{\partial h_s}{\partial P} + \Delta \frac{\partial h_s}{\partial P}\right) \\ & - (V_t + \Delta V_t) + m_t C_p\left(\frac{\partial t_s}{\partial P} + \Delta \frac{\partial t_s}{\partial P}\right) \\ & - (V_{wt} h_w \frac{\partial \rho_w}{\partial P} + V_{wt}\rho_w \frac{\partial h_w}{\partial P} + V_{st} h_s \frac{\partial \rho_s}{\partial P} \\ & + V_{st}\rho_s \frac{\partial h_s}{\partial P} - V_t + m_t C_p \frac{\partial t_s}{\partial P})\end{aligned}$$

$$\Delta_{e_{22}} = (\Delta_{e_{22}\max} - \Delta_{e_{22}\min})/e_{22} = 1.8904\text{e-}06$$

$$\begin{aligned}\Delta_{e_{32}} = {} & V_r(\rho_w + \Delta\rho_w)\left(\frac{\partial h_w}{\partial P} + \Delta \frac{\partial h_w}{\partial P}\right) - V_r(\alpha_r + \\ & \Delta\alpha_r)(h_c + \Delta h_c)\left(\frac{\partial \rho_w}{\partial P} + \Delta \frac{\partial \rho_w}{\partial P}\right) - V_r(\overline{\alpha_v} + \Delta\overline{\alpha_v})(\rho_w + \\ & \Delta\rho_w)\left(\frac{\partial h_w}{\partial P} + \Delta \frac{\partial h_w}{\partial P}\right) + (\alpha_r + \Delta\alpha_r)(h_c + \Delta h_c)\left(\frac{\partial \rho_w}{\partial P} + \right. \\ & \left.\Delta \frac{\partial \rho_w}{\partial P}\right)(\overline{\alpha_v} + \Delta\overline{\alpha_v}) + (h_c + \Delta h_c)\left(\frac{\partial \rho_s}{\partial P} + \Delta \frac{\partial \rho_s}{\partial P}\right)(\overline{\alpha_v} + \\ & \Delta\overline{\alpha_v})V_r - (\alpha_r + \Delta\alpha_r)(h_c + \Delta h_c)(\frac{\partial \rho_s}{\partial P} + \Delta \frac{\partial \rho_s}{\partial P})(\overline{\alpha_v} + \\ & \Delta\overline{\alpha_v})V_r + (\rho_s + \Delta\rho_s)(\frac{\partial h_s}{\partial P} + \Delta \frac{\partial h_s}{\partial P})(\overline{\alpha_v} + \Delta\overline{\alpha_v})V_r + (\rho_s + \\ & \Delta\rho_s)(h_c + \Delta h_c)V_r(\frac{\partial \overline{\alpha_v}}{\partial P} + \Delta \frac{\partial \overline{\alpha_v}}{\partial P}) + (\rho_w + \Delta\rho_w)(\alpha_r + \\ & \Delta\alpha_r)(h_c + \Delta h_c)V_r(\frac{\partial \overline{\alpha_v}}{\partial P} + \Delta \frac{\partial \overline{\alpha_v}}{\partial P}) - (\rho_s + \Delta\rho_s)(\alpha_r + \\ & \Delta\alpha_r)(h_c + h_c)V_r(\frac{\partial \overline{\alpha_v}}{\partial P} + \Delta \frac{\partial \overline{\alpha_v}}{\partial P}) - V_r + m_r C_p(\frac{\partial t_s}{\partial P} + \Delta \frac{\partial t_s}{\partial P}) \text{-} \\ & (V_r\rho_w \frac{\partial h_w}{\partial P} \text{-} V_r\alpha_r h_c \frac{\partial \rho_w}{\partial P} \text{-} \\ & V_r\overline{\alpha_v}\rho_w \frac{\partial h_w}{\partial P} + \alpha_r h_c \frac{\partial \rho_w}{\partial P}\overline{\alpha_v} + h_c \frac{\partial \rho_s}{\partial P}\overline{\alpha_v}V_r \text{-} \alpha_r h_c \frac{\partial \rho_s}{\partial P}\overline{\alpha_v}V_r + \\ & \rho_s \frac{\partial h_s}{\partial P}\overline{\alpha_v}V_r + \rho_s h_c V_r \frac{\partial \overline{\alpha_v}}{\partial P} + \rho_w \alpha_r h_c V_r \frac{\partial \overline{\alpha_v}}{\partial P} \text{-} \rho_s \alpha_r h_c V_r \frac{\partial \overline{\alpha_v}}{\partial P} \text{-} \\ & V_r + m_r C_p \frac{\partial t_s}{\partial P})\end{aligned}$$

$$\Delta_{e_{32}} = (\Delta_{e_{32}\max} - \Delta_{e_{32}\min})/e_{32} = 8.0580\text{e-}11$$

$$\begin{aligned}\Delta_{e_{33}} = {} & (\rho_s + \Delta\rho_s)(h_c + \Delta h_c)(V_r + \Delta V_r)\left(\frac{\partial \overline{\alpha_v}}{\partial \alpha_r} + \Delta \frac{\partial \overline{\alpha_v}}{\partial \alpha_r}\right) - (\rho_s \\ & + \Delta\rho_s)(\alpha_r + \Delta\alpha_r)(h_c + \Delta h_c)V_r\left(\frac{\partial \overline{\alpha_v}}{\partial \alpha_r} + \Delta \frac{\partial \overline{\alpha_v}}{\partial \alpha_r}\right) \\ & + (\alpha_r + \Delta\alpha_r)(\rho_w + \Delta\rho_w)(h_c + \Delta h_c)V_r\left(\frac{\partial \overline{\alpha_v}}{\partial \alpha_r}\right. \\ & \left. + \Delta \frac{\partial \overline{\alpha_v}}{\partial \alpha_r}\right) - (\rho_s h_c V_r \frac{\partial \overline{\alpha_v}}{\partial \alpha_r} - \rho_s \alpha_r h_c V_r \frac{\partial \overline{\alpha_v}}{\partial \alpha_r} \\ & + \alpha_r \rho_w h_c V_r \frac{\partial \overline{\alpha_v}}{\partial \alpha_r})\end{aligned}$$

$$\Delta_{e_{33}} = (\Delta_{e_{33}\max} - \Delta_{e_{33}\min})/e_{33} = 0$$

Similarly,

Δe_{42} = -2.4771e-09

$\Delta_{e_{43}} =$ 4.3129e-04

$\Delta_{e_{44}} =$ 6.8918e-07

Now, the structured uncertainty model is developed as,

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_3 \end{bmatrix} = \begin{bmatrix} 6.2764\text{ e-6} & \text{-2.4145e-11} & 0 & 0 \\ 0 & \text{1.8904e-06} & 0 & 0 \\ 0 & \text{8.0580e-11} & 0 & 0 \\ 0 & \text{-2.4771e-09} & \text{4.3129e-04} & \text{6.8918e-07} \end{bmatrix}$$

$$[B]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

where,

$$B = \begin{bmatrix} 0.0628 & -8.1e-08 & 2.1565 & 0.0069 \\ 0.0628 & -8.1e-08 & 2.1565 & 0.0069 \\ 0.0628 & -8.1e-08 & 2.1565 & 0.0069 \\ 0.0628 & -8.1e-08 & 2.1565 & 0.0069 \end{bmatrix}$$

Thus, the structured uncertainty model is developed with the help of mathematical model of boiler drum in nominal operation. The results are presented in the form of error magnitude plot and discussed in the forthcoming sections.

Unstructured uncertainty

The unstructured uncertainty is defined as the uncertainty that arises when the structure of the model is unknown i,e., the unmodelled dynamics or truncation of high frequency model or the nonlinearities. The unstructured uncertainty model is of different types namely: additive model, multiplicative model, inverse model, etc. The most commonly employed uncertainty model is the multiplicative uncertainty and the model resembles a MIMO system [19].

The unstructured uncertainty is developed with the help of the weighting function which denotes the unmodelled dynamics or the high frequency model. The weighting function can be obtained by using the Eq. (33) and (34), respectively.

$$W_h = G - G_{low} \tag{33}$$

$$W_l = G - G_{high} \tag{34}$$

Where, W_h and W_l - weighting functions, G - Nominal transfer function, G_{low} and G_{high}-Low and high frequency transfer function.

Finally, the unstructured uncertainty is estimated as in Eq. (35).

$$\begin{bmatrix} du_1 \\ du_2 \end{bmatrix} = \begin{bmatrix} W_h & 0 \\ 0 & W_l \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} * \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \tag{35}$$

Where, x – State variables

Δ - Uncertainty matrix

Frequency related dynamics gives more impact on the performance of the drum boiler. Hence, it is necessary to model the unstructured uncertainty accurately.

To cancel the frequency related uncertainties, first, we need to compute the high and low frequency components of the drum boiler using properly designed weighting function. The weighting function to estimate the low frequency component is given in Eq. (36).

$$\frac{\text{Output}}{\text{Input}} = W_l = K\frac{1}{\tau s+1} \tag{36}$$

Where, K=1 is the gain, $\tau = \frac{1}{2\pi f_c}$ is the time constant ($f_c = \frac{1}{2\pi}$).

Similarly, the weighting function to estimate the high frequency component is given by Eq. (37).

$$\frac{\text{Output}}{\text{Input}} = W_h = K\frac{\tau s}{\tau s+1} \tag{37}$$

Where, K = 1 is the gain and cut of frequency $f_c = \frac{1}{24\pi}$

Now, the unstructured uncertainty to be estimated is given by Eq. (38).

$$\begin{bmatrix} du_1 \\ du_2 \end{bmatrix} = \begin{bmatrix} W_h & 0 \\ 0 & W_l \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} * \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \tag{38}$$

For lower frequency,

$$\begin{bmatrix} du_{l1} \\ du_{l2} \\ du_{l3} \\ du_{l4} \end{bmatrix} = \begin{bmatrix} W_l & 0 & 0 & 0 \\ 0 & W_l & 0 & 0 \\ 0 & 0 & W_l & 0 \\ 0 & 0 & 0 & W_l \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} * \begin{bmatrix} 0.00000001 \\ 0.00000001 \\ 0.00000001 \\ 0.00000001 \end{bmatrix}$$

For high frequency,

$$\begin{bmatrix} du_{h1} \\ du_{h2} \\ du_{h3} \\ du_{h4} \end{bmatrix} = \begin{bmatrix} W_h & 0 & 0 & 0 \\ 0 & W_h & 0 & 0 \\ 0 & 0 & W_h & 0 \\ 0 & 0 & 0 & W_h \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} * \begin{bmatrix} 8.3300e-06 \\ 8.3300e-06 \\ 8.3300e-06 \\ 8.3300e-06 \end{bmatrix}$$

Thus, for the Total frequency,

$$\begin{bmatrix} du_1 \\ du_2 \\ du_3 \\ du_h \end{bmatrix} = \begin{bmatrix} du_{l1} \\ du_{l2} \\ du_{l3} \\ du_{l4} \end{bmatrix} + \begin{bmatrix} du_{h1} \\ du_{h2} \\ du_{h3} \\ du_{h4} \end{bmatrix} \tag{39}$$

Uncertainty due to system nonlinearity

The water level and heat transducer and environment noises are modeled by white noise processes with power P as given by Eq. (40).

$$P = \frac{N}{4\pi}\omega \qquad (40)$$

where, N = 9.8875e-08, $\omega = 10$ are the intensity and bandwidth of the white noise. Thus, the uncertainty model is developed for the boiler drum with the help of differential equations.

Control of drum level

In general, the conventional controller is used for the control of drum level because of its simplicity and easy to design. Due to the presence of nonlinearities in the system, the boiler drum performance is affected when controlled by the conventional controller. In this work, the robust controller is designed for the purpose of drum level control and also to deal with uncertainties.

The robust control is an approach which is designed to deal with the uncertainties in the system. The motive of robust control is to function optimally when provided with the uncertainties in the parameters that are found to be varying over a range. Here, a robust controller design is proposed based on H-infinity technique. The aim of H-infinity control is to reduce the error between the reference input and the process output even though in the presence of uncertainty in the system. In this work, the objective function of the H- infinity control is considered as the algebraic Riccati equation for the purpose of reducing the error. The algebraic Riccati equation is given by Eq. (41).

$$A^TX + XA - XBR^{-1}B^TX + Q = 0 \qquad (41)$$

This equation is further simplified and written as given in Eq. (42).

$$P = A^TP + PA - Q \qquad (42)$$

Let us assume $Q = \begin{bmatrix} 0.3 & 0.4 & 0.2 & 1 \\ 0.5 & 0.3 & 0.8 & 0.2 \\ 0.1 & 0.7 & 0.3 & 0.2 \\ 0.1 & 0.3 & 0.5 & 0.8 \end{bmatrix}$

By using the above equations, the H-infinity control is designed and the control of drum level is attained. The conventional controller (PID) is also designed for the control of drum level and the performance of it compared with the robust control.

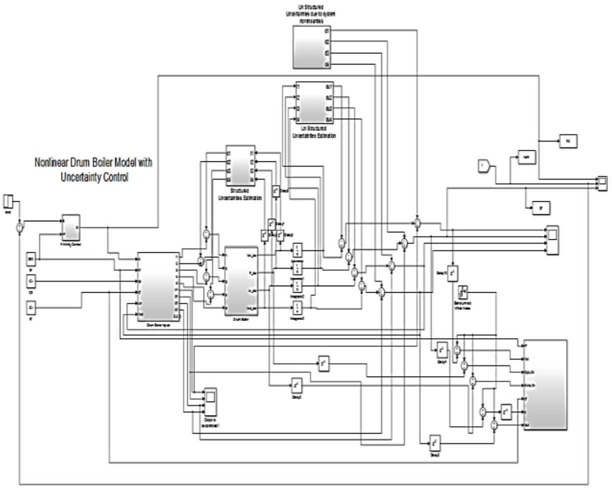


Fig. 3: H-infinity control for drum level

The Simulink representation of designed h-infinity controller for drum level control is shown in Fig. 3. Here, the drum level is controlled by providing steam flow rate and feed water flow rate as control efforts.

RESULTS AND DISCUSSIONS

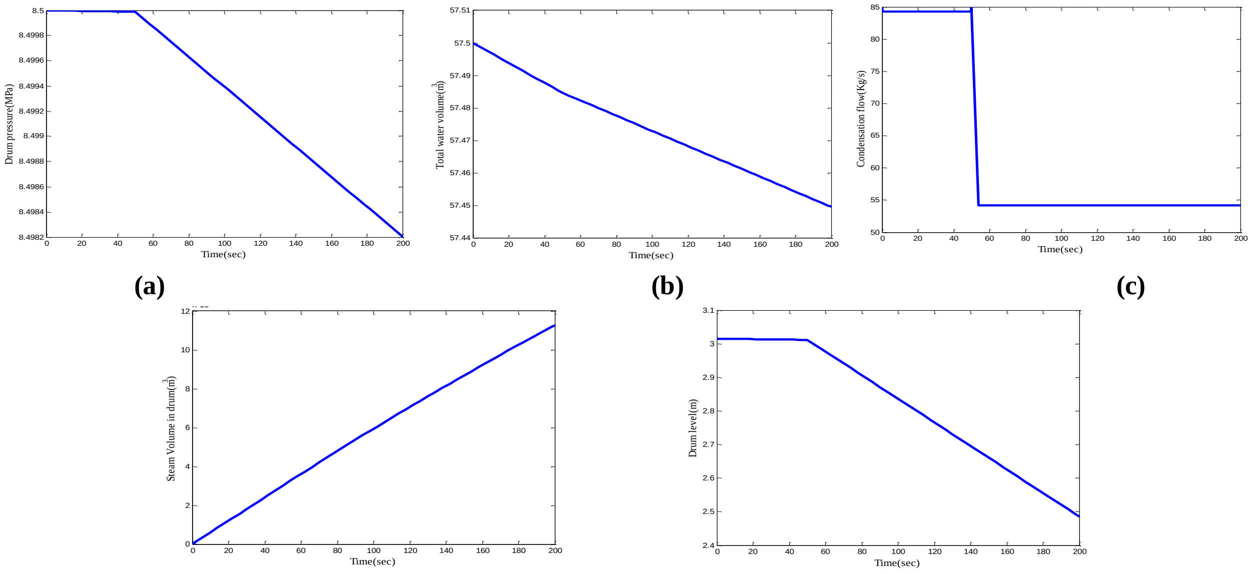
In this section, the response of the drum parameters subjected to the robust control is discussed. The results are also discussed w.r.t the uncertainties model. Both open loop and closed loop performances are evaluated systematically. Finally, the response of the proposed H-infinity controller is compared with a conventional PID controller and the results are evaluated.

Open loop response of boiler drum level

Step change in steam flowrate from nominal value.

Fig.4 shows the responses of the different parameters of Boiler Drum Model for a step change in steam flow rate from nominal value.

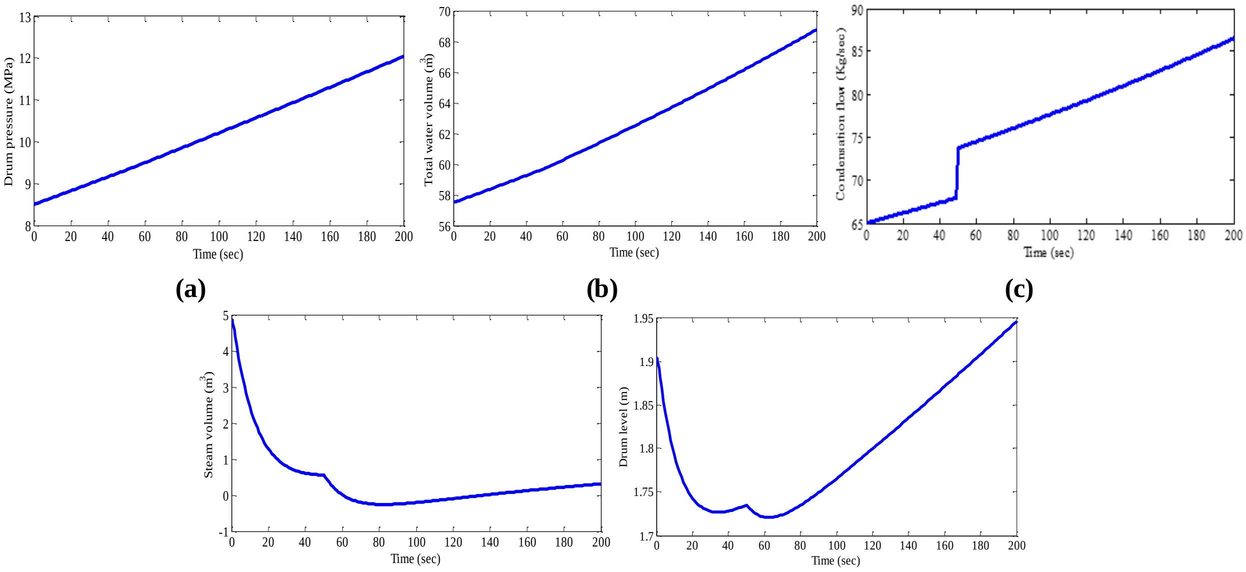
For the step change in steam flow rate, there is a gradual decrease in drum pressure as shown in Fig. 4(a). Also, the decrease in total water volume due to the step change in steam flow rate is as shown in Fig. 4(b). From Figs. 4(a) and 4(b), it is observed that, there is an inverse relationship existing between drum pressure and total water volume against the steam flow rate. i.e., the decrease in drum pressure and total water volume is due to the increase in steam flow rate. The decrease in condensation flow due to step change in steam flow rate is shown in Fig. 4(c). Also, in Fig. 4(d), it can be seen that, there is an increase in steam volume in drum due to step change



(a) (b) (c)

(d) (e)

***Fig.4:** Response of (a) Drum pressure (b) Total water volume (c) Condensation flow (d) Steam volume (e) drum level, for a step change of steam flow rate from nominal value*



(a) (b) (c)

(d) (e)

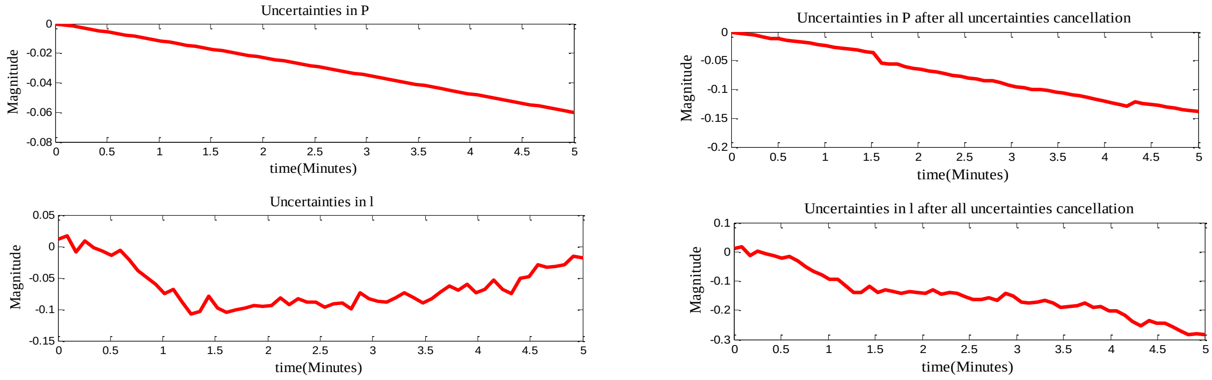
***Fig.5:** Response of (a) Drum pressure (b) Total water volume (c) Condensation flow (d) Steam volume (e) Drum level, for a step change of feed water flow from nominal value*

in steam flow rate. Hence, it can be interpreted that, due to the decrease in drum pressure, the steam volume gets increased and the condensation flow decreased in a step like manner. Fig. 4(e) shows the response of drum level for a step change in steam flow rate. is observed that, the drum level decreases because of decrease in drum pressure.

Response of Boiler Drum Model for a step change in feed water flow rate from nominal value

Fig.5 shows the responses of the different parameters of Boiler Drum Model for a step change in feed water flow from nominal value.

For the step change in feed water flow rate, the drum pressure increases gradually as shown in Fig. 5(a). The increase



(a) **(b)**

Fig. 6: Uncertainties in drum pressure and level for step change of feed water flow rate (a) for Structured uncertainty (b) after cancellation of structured uncertainty

in total water volume due to change in feed water flow rate is shown in Fig.5(b). From this, it is observed that, the drum pressure increases in a constant rate and also total water volume increases because of condensation occurring due to increased drum pressure. Fig.5(c) shows the increase in condensation flow due to the step change in feed water flow rate. The steam volume in drum decreases due to step change in feed water flow rate as shown in Fig. 5(d). The response of drum level is shown in Fig. 5(e). From this, it is observed that, the drum level initially decreases due to swell effect and then gets increased.

Open loop response of uncertainty models

The uncertainties are modelled based on model parameters. The open loop response of structured uncertainty model of boiler drum for different parameter variations namely: feed water flow rate and steam flow rate are shown in Fig. 6 and Fig. 7 respectively.

Structured uncertainty modelling of boiler drum

Fig. 6(a) shows the uncertainties in drum pressure and level which occurs due to the structured uncertainty for the step change in feed water flow rate. From the Fig. 6(a), it is observed that, the uncertainty in drum pressure is linearly decreasing where the drum level has some oscillation. The uncertainty in drum pressure and level which is due to the structured uncertainty is cancelled as shown in Fig. 6(b). From this, it is observed that, the uncertainties in drum pressure and level are not reduced by cancelling the structured uncertainty.

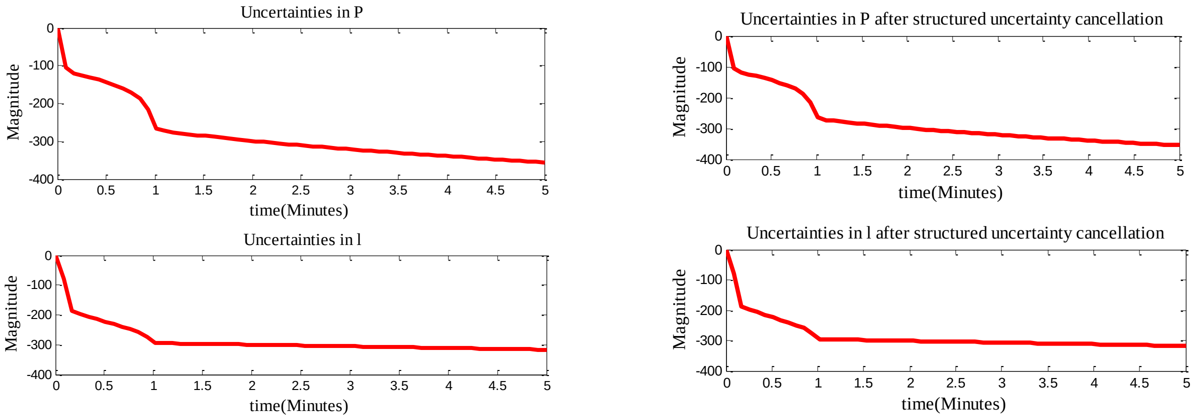
Fig. 7(a) shows the uncertainties in drum pressure and level which occurs due to the structured uncertainty for the step change in steam flow rate. From this, it is observed that, the uncertainty in drum level is large when compared with uncertainty produced for step change of feed water flow rate. The uncertainty in drum pressure and level due to the structured uncertainty cancellation is shown in Fig. 7(b). From this, it is observed that, the uncertainty in drum pressure and level is not reduced after cancellation of structured uncertainty.

Unstructured uncertainty modelling of boiler drum

Fig.8 & Fig. 9 shows the uncertainties in drum pressure and level which occurs due to the unstructured uncertainty for the step change in feed water flow rate and steam flow rate, respectively.

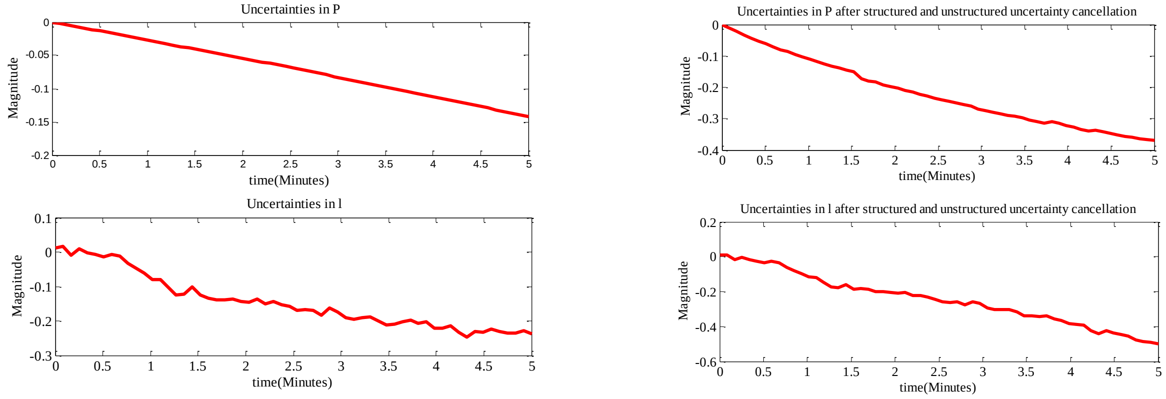
From Fig. 8(a), it is observed that, the uncertainty in drum pressure is slightly less when compared with drum level. The uncertainty in drum pressure and level which is due to the unstructured uncertainty is cancelled as shown in Fig. 8(b). From this it is observed that, the uncertainty in drum pressure and level is not reduced even after cancellation of unstructured uncertainty.

Fig. 9(a) shows the uncertainties in drum pressure and level which occurs due to the unstructured uncertainty for the step change in steam flow rate. It is observed that, the uncertainty in drum pressure and level are similar. The uncertainty in drum pressure and level which is due to the unstructured uncertainty is cancelled as shown in Fig. 9(b). From this, it is observed that, the uncertainty in drum pressure and level remains same even after cancellation of unstructured uncertainty.



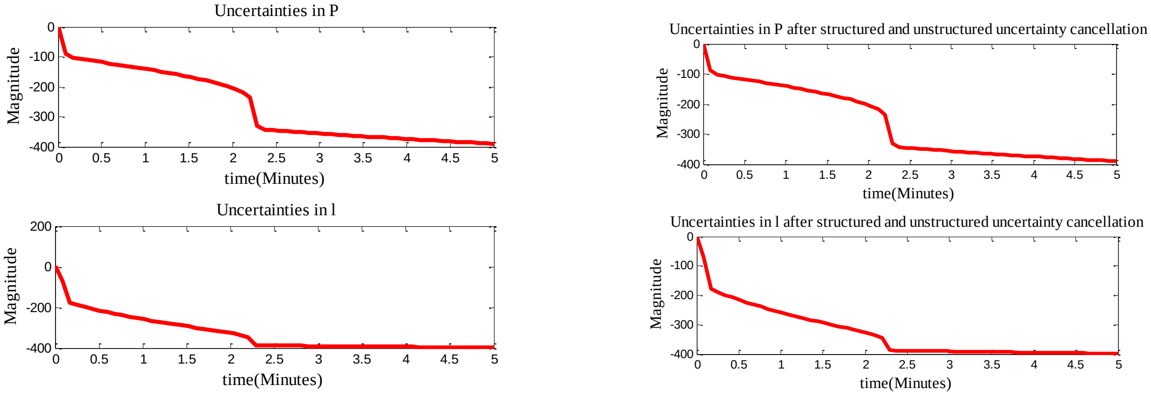
(a) **(b)**

Fig.7: Uncertainties in drum pressure and level for step change of steam flow rate (a) for Structured uncertainty (b) after cancellation of structured uncertainty



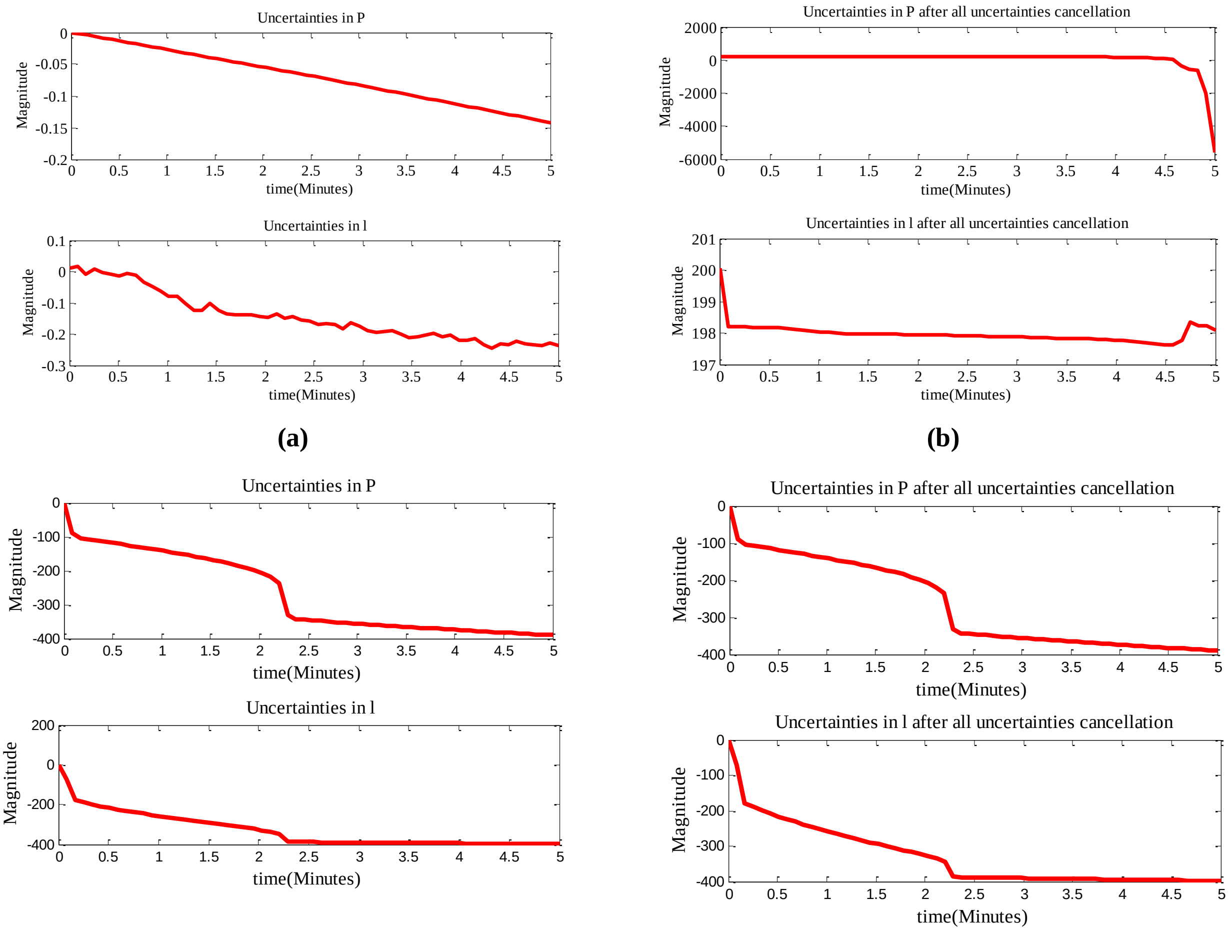
(a) **(b)**

Fig.8: Uncertainties in drum pressure and level for step change of feed water flow (a) for unstructured uncertainty (b) after cancellation of unstructured uncertainty



(a) **(b)**

Fig.9: Uncertainties in drum pressure and level for step change of steam flow (a) for unstructured uncertainty (b) after cancellation of unstructured uncertainty



(a) (b)

(c) (d)

Fig. 10: Uncertainties in drum pressure and level for all uncertainties (a) for step change of feed water flow rate (b) after cancellation for step change of feed flowrate (c) for step change of steam flow rate (d) after cancellation for step change of steam flow rate

Uncertainty due to system nonlinearity

The uncertainties in drum pressure and level for all uncertainties with step change of feed water flow rate and steam flow rate with the effect of their cancellations are shown in Fig. 10.

Fig. 10(a) shows the uncertainties in drum pressure and level this occurs due to all uncertainties for the step change in feed water flow rate. It is observed that, the uncertainty in drum pressure is slightly less when compared with drum level. The uncertainty in drum pressure and level due to all uncertainties is cancelled as shown in Fig. 10(b). From this, it is observed that, the uncertainty in drum pressure and level is not reduced by cancelling all uncertainties. Fig.10(c) shows the uncertainties in drum pressure and level that occurs due to all uncertainties for the step change in steam flow rate. From this, it is observed that, the uncertainty in drum pressure and level are similar. The uncertainty in drum pressure and level due to all uncertainties is cancelled as shown in Fig. 10(d). From this, it is observed that, there is no change in the uncertainties in drum pressure and level even after cancellation of all uncertainties.

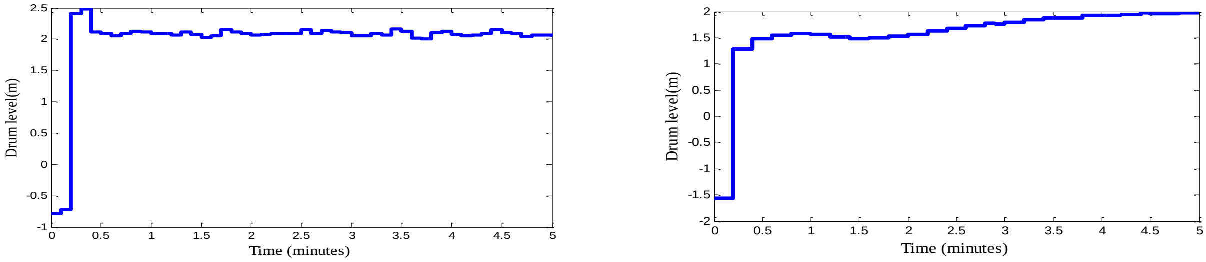
Closed loop response of boiler drum

The robust controller is designed for the drum level control based on H-infinity technique. The closed loop response of drum level is shown in Fig. 11.

Fig. 11(a) shows that the response of drum level for the step changes in feed water flow rate. From this, it is observed that, for the step change in feed water flow rate the H-infinity controller controls the drum level to its reference input. Fig. 11(b) shows that the response of drum level for the step changes in steam flow rate. It is observed that, for the step change in steam flow rate the H-infinity controller controls the drum level to its reference input.

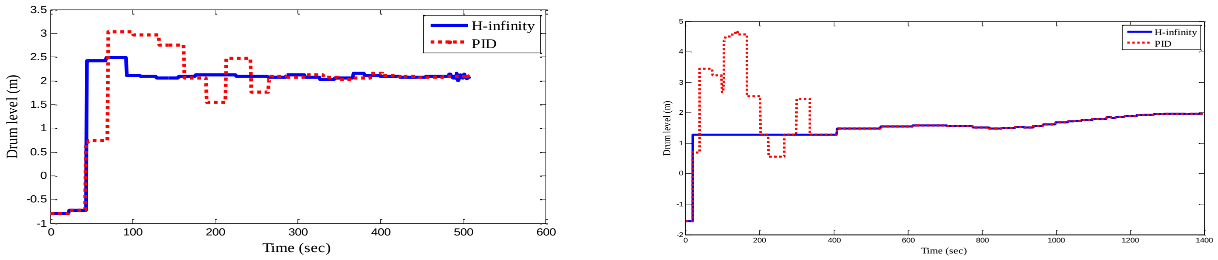
Table 3: Comparison of PID and H-infinity controller

Controller	Operating condition	ISE	IAE
H-infinity	Step change of feed water flow	12.76	5.926
	Step change of steam flow	2.121	3.132
PID	Step change of feed water flow	134.5	24.05
	Step change of steam flow	22.86	10.27



(a) **(b)**

Fig. 11: Response of drum level for the step change in (a) feed water flow rate (b) steam flow rate



(a) **(b)**

Fig. 12: Comparison of drum level response of PID and H-infinity controller for step change of (a) feed water flow rate (b) steam flow rate

Fig. 12(a) shows the comparison between PID and H-infinity controller for the drum level response for the step change in feed water flow rate. It is observed that, H-infinity controller has the better performance when compared with PID controller, as PID controller takes more time to attain the reference input whereas the H-infinity controller takes less time. Fig. 12(b) shows the comparison between PID and H-infinity controller for the drum level response for the step change in steam flow rate. From this, it is observed that, the H-infinity controller has the better performance when compared with PID controller, as PID controller takes more time attain the reference input whereas the H-infinity controller takes less time.

The performance of the PID and H-infinity controller are compared in terms of ISE and IAE value and is presented in Table 3.

From Table 3, it is observed that, the H-infinity controller is quite acceptable for drum level control as compared with PID controller.

CONCLUSIONS

In this work, the boiler drum model is developed by using the mathematical modelling approach. The open loop responses of different parameters of boiler drum model is obtained by providing a step change to steam flow rate and feed water flow rate from the nominal values. The uncertainty models are developed based on variation in model parameter, known dynamics and nonlinearity in the system. The results of uncertainty model are presented in the form of error magnitude plot. The robust controller is designed for the drum level control based on H-infinity technique. The performance of the robust control is compared with the conventional controller (PID) to analyze the best control for drum level. In this work, the drum level is controlled by designing the robust controller based on H-infinity technique. From the results, it is observed that, the H-infinity controller is quite promising to control the drum level as compared with PID controller. As a future direction, the robust control can be designed based on Quantitative Feedback Theory (QFT) for the drum level. More load variables can be added with the available model to improve the accuracy of the model.

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