Design of Instrumentation Sensor Networks for Non-Linear Dynamic Processes Using Extended Kalman Filter

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ABSTRACT: *This paper presents a methodology for design of instrumentation sensor networks in non-linear chemical plants. The method utilizes a robust extended Kalman filter approach to provide an efficient dynamic data reconciliation. A weighted objective function has been introduced to enable the designer to incorporate each individual process variable with its own operational importance. To enhance the evaluation accuracy of the weighted objective function, a true relative standard deviation measure has been employed in the presented formulation. A Genetic Algorithm (GA) has been used to solve the resulting constrained optimization problem due to cost-optimal and performance-optimal design objectives. The proposed method has been tested on a non-linear continuous-stirred tank reactor (CSTR) benchmark plant, illustrating its effective design capabilities.*

KEY WORDS: *Instrumentation sensor network, Non-linear dynamic processes, Extended Kalman filter, Genetic algorithm.*

INTRODUCTION

Sensors are necessary instrumentations in process plant to provide good quality process data for a variety of purposes. For a long time, the selection of sensors in chemical plants has been traditionally driven by the needs of basic control loop design. Efficient control and safe operation of chemical plants usually require the measurements of some specific process variables such as flow rates, temperatures, and compositions. In large

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1021-9986/08/3/11 13/$/3.30

process plants, however, hundreds of such variables may exist. But only a subset of them is normally used in routine operations. Moreover, some of the interested process variables might not be available through measurements. These unmeasured variables are usually estimated by exploiting the mass and energy balance relationships between different variables. The measured variables and all the unmeasured variables that can be uniquely estimated are denoted as observable. The observability of the process variables depends on the process structure as well as the locations of sensors. Consequently, the design of sensor networks in chemical plants to fulfill desired operational requirements has emerged as an important topic of interest. The engineering problem of sensor network design concerns the determination of which variable should be measured and with what precision, so that certain pre-specified quality of data is assured.

The problem of "selecting instruments for optimal state estimation" has been analyzed by several researchers. *Vaclavek* and *Loucka* [1] first explored this problem to guarantee observability of a required set of variables in multi-component process using graph theory. *Madron* [5] presented a strategy based on the construction of minimum spanning trees for sensor networks with minimum cost or maximum overall precision. *Ali* and *Narasimhan* [2] presented a method that maximizes reliability in linear non-redundant networks. This approach was extended to obtain redundant networks with fixed number of sensors larger than the minimum by *Ali* and *Narasimhan* [3]. *Sen, Narasimhan,* and *Deb* [4] presented a genetic algorithm approach to design redundant sensor networks using different objective functions.

Bagajewicz [6] proposed a different formulation based on mixed integer non-linear programming (MINLP) concept to obtain cost-optimal sensor network structures for linear systems, subject to constraints on precision and robustness. Many other similar research works have been presented, dating up to the year 2000, most of which are reviewed in the book by *Bagajewicz* [7]. Although all these works use different objective functions to accomplish maximum precision, minimum cost, maximum reliability, and so on, but it is shown that there is a connection between all the proposed models. Moreover they have been developed under the process steady state assumptions.

In this paper, the design methodology proposed in [13] is modified and extended to be used for non-linear dynamic processes. The resulting methodology incorporates the extended kalman filter (EKF) to perform the required dynamic data reconciliation . An appropriate implementation approach has been adopted to consider the time-updated effect of changing process noise covariance on the state covariance matrix leading to a more accurate and robust data reconciliation technique. In this work, a weighted objective function, including true relative standard deviation measures, has been proposed for evaluation of each candidate sensor network design solution which can provide a more practical and realistic design approach. Different simulation studies have been presented to apply the resulting design methodology to several sceneries. The results are finally compared with some existing methods.

PROPOSED DESIGN METHODOLOGY

Modified discrete-time EKF algorith

The Kalman filter [14] provides an efficient recursive procedure to estimate the hidden states $x \in R^{nx}$ of a discrete-time process that is governed by the following linear stochastic process and measurement model equations:

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \qquad (1)$$

$$z_k = Hx_k + v_k \qquad (2)$$

Where x_k denotes the hidden states, $u_k \in R^{nu}$ is the vector of external manipulated input variables, and $z_k \in R^{nz}$ represents the vector of noisy measured output variables at the kth discrete time. The random variables w_{k-1} and v_k represent the process and measurement noises, respectively. A is the state transition matrix, B is the control matrix and H is the output observation matrix. However, in most practical applications of interest, the process dynamics and the measurement equations obey the following non-linear relationships:

$$x_k = f(x_{k-1}, u_{k-1}, k) + w_{k-1} \qquad (3)$$

$$z_k = h(x_k, k) + v_k \qquad (4)$$

Where f and h are known non-linear functions. As a result, nonlinearity can come in either through process model and/or through the measurement model. Applying the standard Kalman filter on the linearized process and measurement equations about the nominal state values can introduce large errors leading to sub-optimal filter performance.

EKF gives a simple and effective remedy to overcome such problem. Its basic idea is to locally linearize the non-linear system described by Eqns. (3) and (4) at each time instant around the most recent state estimate as follows:

$$A_k = \left.\frac{\partial f}{\partial x}\right|_{x=\hat{x}_k} \tag{5}$$

$$H_k = \left.\frac{\partial h}{\partial x}\right|_{x=\hat{x}_k} \tag{6}$$

Where the state transition matrix A_k and the observation matrix H_k are the Jacobian matrices which are evaluated at $x = \hat{x}$, that is the estimated values of the states in real-time rather than its nominal values.

For estimating the output, however, the actual non-linear measurement equation is used as $\hat{y}_k = h\left(\hat{x}_k^-, k\right)$.

Then, the kalman filter is applied to the resulting time-varing linearized model. This can provide a more accurate implementation of the optimal recursive estimation procedure. More details on the complete EKF algorithm are found in the related textbooks and papers.

In practice, the process model in Eqn. (3) is of continuous-time nature. While, the measurements in Eqn. (4) are available through the common digital data-acquisition systems at discrete measurement time instants. Moreover, the EKF algorithm is implemented digitally to process all available measurements regardless of their precision in order to provide a quick and accurate estimate of the variables of interest. Therefore, an efficient formulation of the algorithm needs to be made for a real-time practical application to minimize the filter cycle time, while obtaining a reasonable accuracy in the filter implementation. The method used in this paper for numerical integration of the process model from one sample time to the next is the first-order Euler integration technique. Using the transition matrix technique [15], the time propagation equation for the state covariance matrix P can be solved as:

$$P_k^- = \Phi P_{k-1} \Phi^T + Q_d \tag{7}$$

where:

$$Q_d = \int_{(k-1)T_s}^{kT_s} \Phi(kT_s, \tau) Q(\tau) \Phi^T (KT_s, \tau) d\tau \tag{8}$$

Table 1: The modified discrete-time EKF algorithm.

Initial estimates for $\hat{x}_{k-1}$ and P_{k-1}

⇩

Time Update (“Predict”)

(I) Project the state ahead

$$\hat{x}_k^- = \hat{x}_{k-1} + T_s f(\hat{x}_{k-1}) \tag{11}$$

(II) create the Jacobean matrix

$$A_k = \partial f / \partial x \tag{12}$$

(III) Update the process covariance matrix

$$\Phi = I + T_s A_k \tag{13}$$

$$Q = (\Phi Q \Phi^T + Q)\frac{T_s}{2} \tag{14}$$

(IV) Project the error covariance ahead

$$P_k^- = \Phi P_{k-1} \Phi + Q_d \tag{15}$$

⇩ ⇧

Measurement Update (“Correct”)

(I) Compute the Kalman gain

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R)^{-1} \tag{16}$$

(II) Update estimate with measurement z_k

$$\hat{x}_k^- = \hat{x}_{k-1}^- + K_k (Y_k - H_k \hat{x}_k^-) \tag{17}$$

(III) Update the error covariance

$$P_k = P_k^- - K_k H_k P_k^- \tag{18}$$

In which T_s is the sampling period and Φ denotes the state transition matrix associated with A_k for all the time duration $\tau \in [(k-1)T_s, kT_s]$ which can be evaluated by:

$$\Phi = I + T_s A_k \tag{9}$$

As a result, Q_d can be obtained using the following trapezoidal integration scheme:

$$Q_d = \left(\Phi Q \Phi^T + Q\right)\frac{T_s}{2} \tag{10}$$

Table 1 summarizes the different steps needed for the efficient implementation of the modified discrete-time EKF algorithm.

Performance evaluation of the measurement system

The EKF algorithm gives the optimal process state estimate $\hat{x}_k$ and its associated error covariance matrix as the main two outcomes. As it is apparent from Eqn. (18), the computation of P_k is independent of the process variable measurements. Indeed, calculation of P_k requires the measurement covariance matrix (R), the process covariance matrix (Q), the observation matrix (H_k), and the state transition matrix (A_k). A_k and H_k are the Jacobian matrices, derived from the actual non-linear state-space models in Eqns. (3) and (4), which depend on the most recent state estimation. In this way, any possible time-varying dynamic variations in the process or measurement model equations can be introduced in the state estimation procedure.

R is the measurement error covariance matrix which indicates the intrinsic quality of the available measuring devices. Thus, its elements can be used as the design parameters for sensor placement. Assuming that measurement errors due to the individual measuring devices are independent, the matrix R takes a diagonal form. However, determining the process noise covariance matrix Q is a complex and difficult task due to the fact that their elements can not be observed directly. Thus, a diagonal matrix with positive and fixed elements has been assumed in [13].

Changing the covariance matrix Q, however, affects both the transient duration and the steady state operation of the Kalman filter. Increasing Q would indicate stronger noises driving the system or increased uncertainty in the model. This will increase the values of the state covariance elements. The Kalman filter gains will also increase thereby weighting the measurements more heavily, and the filter transient performance becomes faster. In this paper, the time evolution of the state covariance matrix P_k has been derived based on the transition matrix technique. This approach brings about two main advantages. First, it leads to a robust Kalman filter implementation by preserving both the crucial symmetrical and positive definiteness properties of the state covariance matrix P_k [14]. Secondly, it provides a reasonable way to increase the accuracy of the Kalman filter performance and hence the state covariance matrix estimation by time-updating the process noise covariance matrix through Eqn. (10).

This proposed procedure makes it possible to evaluate the impact of the measurement covariance matrix R on the estimated error state covariance matrix P_k more accurately. Therefore, P_k can be used as an inferential criterion to assess the performance of the measurement network. It is assumed that the measurement error elements in R are independent and normally distributed. This causes the EKF algorithm to provide state estimates that are optimal in sense of possessing minimum variance unbiased error.

For linear systems, the error state covariance matrix P_k may converge to a constant value over its recorded time-history, i.e. k=0,. . . ,n. In this case, the following asymptotic value of P_k can be used as the performance evaluating measure for the jth process variable [13] :

$$P_{c,1}^{j} = \lim_{k\to n}\left([P_k]_j\right) \tag{19}$$

But, this is not the usual circumstance encountered in the practical non-linear and/or time-varying systems. As a result, the performance of the sensor network can be evaluated by the following averaging measure for the jth process variable [13]:

$$P_{c,2}^{j} = \frac{1}{2}\left(\sum_{k=0}^{n}[P_k]_j\right) \tag{20}$$

In the Kalman filter, the state distribution is approximated by a Gaussian random variable which is then propagated analytically through a first-order linearization of the non-linear system. Terms neglected in the linearization may be relatively large which can introduce large errors in the true error state covariance matrix (P_k). Therefore, assessing the system performance based on diagonal elements of the estimated error covariance (P_k) may lead to erroneous results. In this paper, the true error covariance matrix has been used as a substitute assessing basis as follows:

$$P_{c,3}^{j} = \text{variance}\left([\hat{x}_k]_j - [x_k]_j\right) \tag{21}$$

Where $[x_k]_j$ is the time evolution (k=0, . . . , n) of the real jth process state variable obtained via the process simulation.

System performance objective function

Different sensor network designs have different performances. It is evident that performance of each possible network design should be evaluated in terms of all the process variables of interest. The performance measure considered in Eqn. (20), however, can be used to assess the estimation quality of only one particular process variable. Therefore, an objective function is needed to aggregate the combined performance of the whole system due to the performance of individual process variable measurement, given by Eqn. (20). A suitable option is based on the modification of the proposed measure in [13] which evaluates a weighted absolute distance of the selected sensor network $\left(P_{c,3}^{j}\right)$ from the best possible sensor network $\left(P_{b}^{j}\right)$ as follows:

$$d_P^S = \left(1 + \sum_{j \in S} \lambda_j \left| P_{c,3}^j - P_b^j \right| \right)^{-1} \quad (22)$$

Where S denotes the set of process variables of interest to be measured or observed, and λ_j is a weighting factor indicating the importance of the jth process variable in the network design. As shown, the obtained distance measure (d_p^s) has been normalized to give an evaluating measure value between 0-1, $d_p^s \in [0,1]$. The best sensor network $\left(P_b^j\right)$ is assumed to be the network in which all the process variables of interest can be completely measured with the minimum variances using the most accurate devices available.

Moreover, to increase the evaluation accuracy of the weighted function measure, the following true relative standard deviation of each process variable is used instead of the absolute covariance matrix $\left(P_{c,3}^{j}\right)$:

$$d_P^S = \left(1 + \sum_{j \in S} \lambda_j \frac{\left|\sqrt{P_{c,3}^j} - \sqrt{P_b^j}\right|}{\text{steady state value of the jth process variable}}\right) \quad (23)$$

This measure indicates a better representation for the error covariance of each individual process variable. Because, it provides a normalized measure irrespective of the magnitude of each individual process variable.

System model transformation for sensor placement

minimize the measurement errors (v_k) which are mainly due to the intrinsic quality of the available measuring devices. As a result, a possible connection between the sensor placement in the designed network and the Kalman filter has been suggested [13] via the diagonal elements of the measurement covariance matrix R. When a process variable is going to be measured by a physical sensor, the relevant measurement variance is placed in the matrix R, but when it is not measured or observed through other measurements, a high variance (usually 1000 %) is assigned. The design procedure is to choose sensors with different measurement variances to accomplish specified desired goals such as performance and/or cost.

In classical control, manipulated variables u_k are treated as known inputs with distinct entry in the system state-space model (Eqn. (1)). This distinction between state and manipulated variables, however, is not justified from the monitoring perspective using the Kalman filter estimation procedure for sensor placement. Therefore, an augmented state variable vector $x_k^* = [u_k, x_k]$ is developed by considering the manipulated variables as state variables. To implement this view, the manipulated inputs are assumed to be states without dynamics but governed by the following stochastic auto-regressive model equation:

$$u_k \approx u_{k-1} + w_{k-1} \quad (24)$$

This assumption changes the linearized model formulation, described by A_k, B_k, and H_k matrices, to the following augmented state-space model:

$$x_k^* = A^* x_{k-1}^* + w_{k-1}^* \quad (25)$$

$$y_k^* = H^* x_k^* + v_{k-1}^* \quad (26)$$

Where matrix B_k, has been dropped and the new transition state matrix is defined as follows:

$$A^* = \begin{bmatrix} I^{n_u \times n_u} & O^{n_u \times n_x} \\ B^{n_u \times n_u} & A^{n_u \times n_x} \end{bmatrix} \quad (27)$$

Where n_x and n_u denote the dimensions of the state (x_k) and manipulated variables (u_k), respectively.

States observation of the measurement system

Performance of a measurement system with any set of selected sensors can be evaluated by the sensor network measure given in Eqn. (23). This procedure, however, requires all the variances of the involved process variables in the set *S* to be already estimated through the Kalman filter algorithm. To ensure that the Kalman filter converges to an acceptable unique value, the process variables should be strictly observable. The Kalman filter can also provide the estimation of any unmeasured but observable variable with its variance. This is done by assigning initial infinite variance to the corresponding positions of R. It is additionally necessary to explicitly modify H^* for the unmeasured variables by setting the corresponding columns equal to zero. The error variance of an unobservable variable, however will tend towards infinity. Therefore, process variable observability condition has a direct impact on the Kalman filter performance whose proper operation is a perquisite to the sensor network evaluation procedure. A simple way to check the observability of all the desired process variables is to generate the observability matrix (obsv(A^*,H^*)). The rank of this observability matrix shows the number of observable state variables. If it is not full row rank, it indicates that the Kalman filter will not converge to a unique value for the error state covariance matrix from different initial conditions.

Sensor network design objective

A designer of sensor network has several potential objectives to choose from, depending on the desired process requirements. Some of the most important ones are as follows [7]:

1-Cost: The most common objective function used in every design is the sum of the annualized capital investment cost and the operating cost. The simplest objective in the case of sensor network design has been the overall annualized investment cost. Whereas, the operating costs that are related to electricity consumption have always been neglected, leaving maintenance costs as the primary factor.

2-Estimability: This term has usually been used to designate the ability to estimate a particular process variable using either hardware or software means. This objective can be used as a substitute goal in the absence of more sophisticated goals.

3-Precision: In many cases, specific levels of precision can be developed for particular process variables of interest in a measuring system. For example, accounting might be interested in closing the material balance within a certain percentage or even request that the precision of each stream crossing battery limits has a particular value. Parameter estimation is also a software-based activity for which the parameter precision depends on the set of sensors used.

Although all these candidate goals may seem equally important, cost has been the traditional objective function being used in every sensor network design case. The unknowns of this design problem are the number and location of measurements as well as the type, precision, and reliability of each instrument. An adequate cost function for a cost-optimal design of a precise sensor network, which is considered in this paper, is as follows:

$$\mathrm{Min}\left(\sum_{i\in S} C_i q_i\right) \tag{28}$$

$$\text{s.t.}\begin{cases}\psi(S)=1 \\ \sigma_{ii}(q)\le\sigma_{ii} \qquad \forall_i \in S\end{cases}$$

In this formulation, it is assumed that there is only one potential measuring device for each process variable, hence, $q_i \in [0,1]$ represents elements of a binary vector (q) indicating that a sensor is located in process variable i and C_i denotes the cost of each sensor. The precision constraint states that the variance (σ_{ii}=P_c^i) of the estimated value of variable $\hat{x}_i$ has to be kept lower than a certain threshold σ_{ii}^* for each variable or parameter in the set S.

Another objective which is used in this paper is to maximize the sensor network performance (d_p^s), given in Eqn. (23), by satisfying some desired cost and observability requirements as a set of additional constraints. This can be accomplished by the following conditional cost function [13]:

$$\max_{S_{ii}}\left(d_p^s\right) \tag{29}$$

$$\text{s.t.}\begin{cases}\psi(S)=1 \\ \sum_i \left(C_i S_{ij}\right)\le C_{max}\end{cases}$$

Where S_{ij} is an integer variable which indicates the placement of sensor type i at network location j ($j\in S$). When a process variable is measured, a sensor type with its corresponding variance error (σ_i^2) is allocated in R. But, when a process variable is not measured, a dummy sensor with an infinite variance ($\sigma_i^2 \to \infty$) is selected, which has a null cost.

As a consequence, the sensor network performance given in Eqn. (23) can be maximized by varying the diagonal elements of matrix R subject to a cost bound C_{max}. Associated C_{max} is used to obtain a curve that can show the best performance of the chosen sensor network according to a desired cost constraint. To draw this curve, C_{max} is increased repeatedly by a small amount up to the maximum cost available.

ψ(s) is another additional algorithmic constraint which relates to observability. Its value is equal to one when the set of feasible sensor networks allows one to observe all the selected process variables in the set S. However, if one of the selected variables is not observable, leading to the divergence of the estimated variance through the EKF algorithm, its value is set to zero.

Optimization using a Genetic searching algorithm

There are different approaches to solve a constrained optimization problem. Selecting an appropriate approach, however depends on the size of the problem. For instance, consider sensor network design problem for a plant with J sensor locations. Assuming that each candidate process variable at each sensor location can be measured by choosing among I types of sensors with different cost and accuracy. This leads to $(I+1)^J$ possible sensor network solutions. As a consequence, the size of the optimization problem becomes highly complex for medium and large scale plants, requiring a vast amount of computational time to proceed. In most real life problems, however, near optimal solutions that can be generated quickly are more preferred. Thus, genetic algorithm (GA) is an appropriate searching method to be used for solving the constrained optimization problem with highly complex combinatorial features.

In classical formulation of GA, a set of *N* candidate solutions are generated randomly as the initial population. To do this, each initial individual potential solution or chromosome is coded as a vector in the multi-dimensional search-space. The goodness of each resulting solution as individual is evaluated by using a pre-specified fitness criterion. Selecting all the proper chromosomes as the best possible solutions in the current population, a new generation of individuals is created from them by using crossover and mutation operators.

To implement the GA searching method for solution of the sensor network design problem, each gene in the chromosome is taken as a process variable which can be measured once using a single sensor selected from a specific set of sensors with different cost and accuracy. In this way, the length of each chromosome is equal to the number of variables that can be measured (J). For a location with no sensor, a dummy sensor with null cost and a very low accuracy is considered. Therefore, the value of each gene can vary from 0 to I, where I indicates the number of available sensor types in the design.

The fitness of each potential solution or individual in each generation is assessed based on the design performance criterion, that is Eqn. (28) or Eqn. (29).

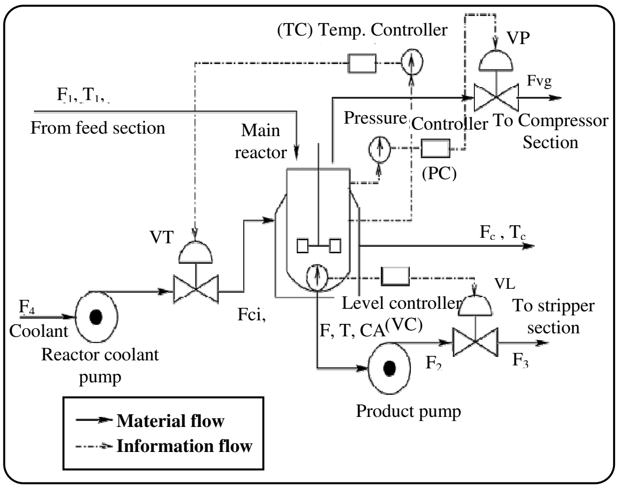


Fig. 1: CSTR process.

If a potential solution or sensor network is feasible, its corresponding objective function value is considered for fitness evaluation. However, if the individuals involve any unfeasibility, the objective function value is divided by a very small number (e.g. 10^{-10}), and the sensor network fitness is set near zero. It should be noted that the sensor type is associated with its characteristics (i.e., cost and accuracy)during the feasibility and fitness evaluation procedure.

In this research, the MATLAB GA toolbox developed by the University of Sheffield [16] has been used. In this algorithm, new populations are generated by selecting G individuals from the initial population using the roulette wheel selection scheme and two point crossover and mutation operators. The algorithm is terminated if the number of generations reaches a predefined maximum value (N_G).

SIMULATION CASE STUDY

The following CSTR benchmark problem, introduced by *Bhushan et al.* [17], has been considered in this research to study the performances of the presented sensor network design methods, under different conditions.

The process involves an exothermic liquid-phase reaction: $A_{(l)} \rightarrow B_{(l)} + C_{(g)}$. As shown, the temperature controller (TC) controls the temperature of the reactor by manipulating the inlet flow rate of the coolant flowing through the jacket. The level in the reactor is controlled by the level controller (VC) which manipulates the outlet flow rate from the reactor. The pressure in the reactor is controlled by changing the vent gas flow rate. Both the reactor and the jacket are modeled with perfectly mixed-tank dynamics. The CSTR model equations are as follows:

Global Mass Balance:

$$F_i - F = \frac{dV}{dt} \tag{30}$$

Component Mass Balance (C_A):

$$\frac{F_i}{V}(C_{Ai} - C_A) - r_A = \frac{dC_A}{dt} \tag{31}$$

Overall Heat Balance on the Reactor: Result obtained assuming constant heat capacities and densities:

$$\frac{F_i}{V}(T_i - T) + \frac{r_A(-\Delta H)}{\rho C_p} - \frac{UA(T - T_c)}{V\rho C_p} = \frac{dT}{dt} \tag{32}$$

Overall Heat Balance on the Jacket:

$$\frac{F_c}{V_j}(T_{ci} - T) + -\frac{UA(T - T_c)}{V_j \rho_j C_{pj}} = \frac{dT_c}{dt} \tag{33}$$

Gas Phase Balance:

$$r_A V - F_{vg} = \frac{dn}{dt} \tag{34}$$

Reaction Rate:

$$r_A = C_d C_A k_0 e^{-E/RT} \tag{35}$$

Elemental Mass Balances in Valves and Pumps: Assuming no accumulation:

$$\begin{cases} F_3 - F_2 = 0 \\ F_2 - F = 0 \\ F_4 - F_c = 0 \end{cases} \tag{36}$$

Pressure in the Reactor: where Vg is the vapor space and is assumed constant, assuming ideal behavior:

$$PV_g = nRT \tag{37}$$

The CSTR model parameters together with their nominal operating conditions are described in table 2.

As shown, process consists of 5 states (F, C_A, T, T_C, P) and 15 measurable variables (F_i, T_i, C_{Ai}, F_C, F, F_{vg}, T_{Ci}, F_2, F_3, F_4, V, C_A, T, T_C, P). As can be seen, the fifth state (n) has been replaced by pressure (P) for easier measurement.

Table 2: Nominal values for the CSTR.

Notation	Variable	Steady state / Constant value
V	Volume of reactor	48 ft^3
C_A	Reactant concentration in reactor	0.2345 lb.mol of A/ft^3
T	Reactor temperature	600°R
F	Outlet flow rate	40 ft^3/h
N	No. of moles of vapor	28.3657 lb. mol
P	Pressure in vapor space	2116.79 lb/ft^2
F_{vg}	Vent flow rate	10.6137 lb. mol/h
F_i	Inlet feed flow rate	40 ft^3/h
C_{Ai}	Inlet reactant concentration	0.5 lb. mol of A/ft^3
T_c	Jacket temperature	590.51°R
F_c	Coolant flow rate	56.626 ft^3/h
T_i	Inlet feed temperature	530°R
V_j	Volume of jacket	3.85 ft^3
K_0	Frequency factor	7.08× 10^{10} h^{-1}
C_d	Catalyst activity	1
E	Activation energy	29900 btu/lb. mol
R	Universal gas constant	1.99 btu/lb. mol °R
U	Heat-transfer coefficient	150 btu/h ft^2 °R
A	Heat-transfer area	150 ft^2
T_{ci}	Inlet coolant temperature	530°R
ΔH	Heat of reaction	-30,000 btu/lb. mol
C_p	Heat capacity (process side)	0.75 btu/lb$_m$ °R
C_{pj}	Heat capacity (coolant side)	1 btu/lb$_m$ °R
ρ	Density of process mixture	50 lb$_m$/ft^3
ρ_j	Density of coolant	62.3 lb$_m$/ft^3
V_g	Volume of vapor space	16 ft^3

Table 3: 1st Comparison of precision required for C_A,T,F (B1).

	F_i	T_i	C_{Ai}	F_c	F	F_{vg}	T_{ci}	F_2	F_3	F_4	V	C_A	T	T_c	P
Measurement				×	×						×	×	×	×	×
EKF	1.44	3.26	2.44	0.17	0.20	-	1.73	0.17	0.17	0.17	0.25	0.32	0.21	0.19	1.90
[17]	1	18.56	13.33	1	1	25.05	10.94	1	1	1	-	1	1	1	-

[a] *(Precision threshold)* $\sigma^*_{CA} \le 2\ \%$, $\sigma^*_T \le 1\ \%$, $\sigma^*_F \le 2\ \%$.

Table 4: 2nd Comparison of precision required for C_A,T,F (B2).

	F_i	T_i	C_{Ai}	F_c	F	F_{vg}	T_{ci}	F_2	F_3	F_4	V	C_A	T	T_c	P
Measurement			×	×	×	×					×	×	×	×	×
EKF	1.604	4.136	0.161	0.159	0.155	0.156	2.027	0.155	0.155	0.159	0.248	0.292	0.188	0.211	0.253
[17]	0.923	16.729	0.658	1	0.923	0.916	8.23	0.923	0.923	1	-	0.935	0.056	1	-

[a] *Precision threshold* $\sigma^*_{CA} \le 0.95\ \%$, $\sigma^*_T \le 0.95\ \%$, $\sigma^*_F \le 0.95\ \%$.

RESULTS AND DISCUSSION

Simulation study I

In this simulation study, the CSTR problem is used to test the observability and data reconciliation of the EKF algorithm. The process has already been used by *Bagajewicz et al.* [18] to design a sensor network for fault detection and precision objectives. However, the data used in that work has been based on the Jacobian matrices around the nominal process steady-state values. Although this reduces the computations, it can lead to erroneous results when the actual operating condition diverges from the nominal steady-state values beyond the limits with time evolution. This divergence phenomenon has shown itself when the state reconciliation test was performed using the proposed approach in [13] which is based on the standard Kalman filtering.

In this study, the data reconciliation is based on the EKF algorithm which continuously updates the linearized model at each time instant around the most recent operating condition estimates.

Two studies were run based on the same conditions reported in [17] requiring precision in three key variables (C_A, T, and F) with two different sets of sensors.

Table 3 shows the data reconciliation results for an assumed optimal set of sensors indicated by cross symbols (x) with the precision requirements of $\sigma^*_{CA} \le 2\ \%, \sigma^*_{CT} \le 1\ \%, \sigma^*_F \le 2\ \%$.

Table 4 shows similar data reconciliation results for a higher precision requirement of $\sigma^*_{CA} \le 0.95\ \%$, $\sigma^*_T \le 0.95\ \%, \sigma^*_F \le 0.9\ 5\%$.

The data shown in tables 3 and 4 demonstrates that more accurate results have been obtained via the data reconciliation of the EKF algorithm compared with the similar results reported in [17].

Simulation study II

The main objective in this simulation study is to design a sensor network with minimum cost which satisfies the observability constraint together with some desired precision thresholds in three key process variables (C_A, T, and F) under the same conditions declared in [17]. It is assumed that the cost of sensor for each variable is 100. The GA parameters were set to initial population N_{ind}=20, number of generations N_G=20, number of childs in each generation G=18, crossover probability P_C=0.7, mutation probability $P_m=P_C/L_{ind}$=0.0467, and scaling factor $S_f=10^{-10}$.

The GA algorithm took 32 seconds mean time to converge. In order to test the convergence property of the GA algorithm, the simulation study was repeated 10 times taking 5 min and 19 seconds. Table 5 summarizes the obtained results for the lower precision requirements specified by $\sigma^*_{CA} \le 2\ \%, \sigma^*_T \le 1\ \%, \sigma^*_F \le 2\ \%$.

Table 5: Minimum cost Genetic Algorithm results for precision threshold $\sigma^*_{CA} \le 2$ %, $\sigma^*_T \le 1$ %, $\sigma^*_F \le 2$ %.

No.		1	2	3	4	5	6	7	8	9	10
Measured Process variables		3,4,7,8, 12,14,15	2,4,6,7,9,12,15	3,8,10,11,12,14,15	2,7,9,12, 13,14,15	3,6,8,10, 12,13,15	2,7,8,10,12, 14,15	3,4,6,11,12, 13,15	3,4,7,9, 12,14,15	3,8,10,11,12,14,15	4,5,7,11, 12,13,15
Accuracy of desired process variables	F	0.1684	0.1227	0.1423	0.1477	0.0953	0.1694	1.8974	0.1285	0.1161	0.1135
	CA	0.2844	0.3592	0.2887	0.2961	0.2930	0.3138	0.2290	0.3039	0.3002	0.3273
	T	0.2524	0.1916	0.1735	0.2123	0.1698	0.1853	0.1546	0.1819	0.1572	0.1423
cost		700	700	700	700	700	700	700	700	700	700

Table 6: Minimum cost Genetic Algorithm results for precision threshold $\sigma^*_{CA} \le 0.95$ %, $\sigma^*_T \le 0.95$ %, $\sigma^*_F \le 0.95$ %.

No.		1	2	3	4	5	6	7	8	9	10
Measured Process variables		3,6,8,10, 11,12,15	7,9,11,12,13,14,15	4,5,6,7,12,13,15	1,3,6,10, 12,13,15	2,3,4,7,9,12,15	3,4,5,7, 12,13	1,2,4,6,7,13,15	2,7,8,11,12, 14,15	1,2,4,6, 12,13,15	6,7,9,10,12, 13,15
Accuracy of desired process variables	F	0.0709	0.0819	0.0657	0.1061	0.0821	0.0654	0.1172	0.0702	0.0972	0.0824
	CA	0.2432	0.1793	0.2096	0.2081	0.2141	0.2076	0.2859	0.1607	0.2011	0.2100
	T	0.1282	0.0579	0.0843	0.0866	0.0998	0.0857	0.1347	0.0661	0.0880	0.0843
cost		700	700	700	700	700	700	700	700	700	700

Similar simulation tests were run for the higher precision requirements specified by $\sigma^*_{CA} \le 0.95$ %, $\sigma^*_T \le 0.95$, $\sigma^*_F \le 0.95$ % . The obtained results have been summarized in table 6.

Examining the results shown in tables 5 and 6 leads to the following general basic observations:

The GA algorithm has been converged to 700 for both precision requirements, as the optimal solution in all the trials.

There are 2^{15}=32768 possible solutions in this simulation study. Examining all these possible sensor networks with an exhaustive search will take about 1 hour. This time duration is so high for such a medium-sized problem compared to 32 seconds mean time lasting in the GA search.

It is noted that the design could not be done via the proposed approach based on the standard Kalman filtering in [13] due to the state divergences in the data reconciliation stage.

Simulation study III

In this study, the main objective is to design a sensor network based on the maximum performance requirement with constraints on total instrument. For this purpose, five sets of sensors have been considered similar to the previous simulation case study. To make the design problem more practical, wide variety of available sensors has been utilized in the design.

The flow-meter variances are [0.1 %, 0.25 %, 0.5 %, 1 %, 2 %] and their corresponding costs are [40, 34, 16, 10, 5]. The temperature and pressure variances are [1 %, 2 %] with corresponding costs as [2,1.5]. The concentration variances are [1 %, 2 %] with corresponding costs as [20, 10]. Finally, the volume-meter variances and costs are [0.1 %, 0.5 %, 1 %, 2 %] and [10, 8, 5, 3], respectively.

Once the design is done assuming that all the interested process variables have the same degree of importance, indicated by λ_j=1, j∈S, as recommended in [13]. Next, the design procedure is repeated under the same conditions except that the process variables have different degrees of importance as proposed in this paper. The key process variables are indicated by λ_j=1 while the remaining variables are grouped into two different categories shown by λ_j=0.5 for low cost sensor types (temperatures and pressures) and λ_j=0.1 for the rest (flows and levels). Finally, the weighting factor vector

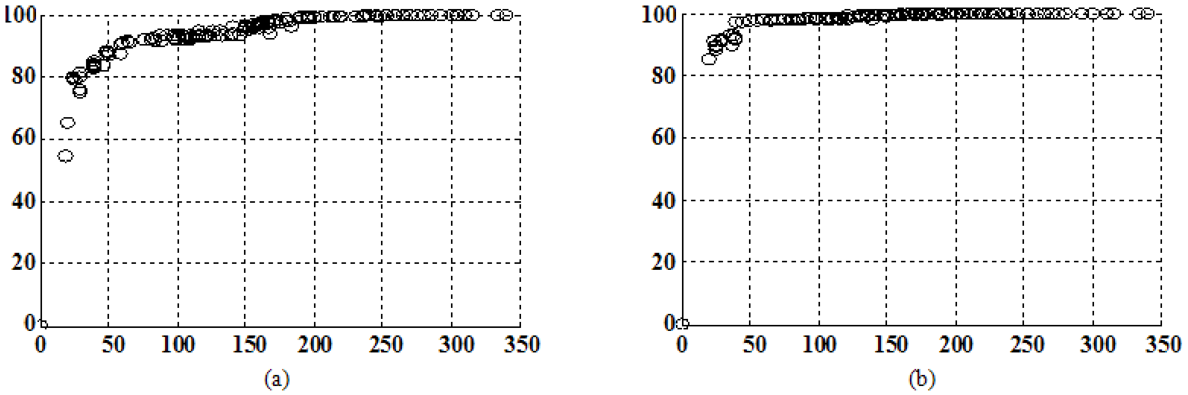


Fig. 2: Cost-Performance trade-off curve for constant λ (a) and variable λ (b).

will be set to λ=[0.1, 0.5, 0.1,0.1, 1, 0.1, 0.5, 0.1, 0.1, 0.1, 0.1, 1, 1, 0.5, 0.5].

According to the proposed sensor placement problem statement, the number of sensor network alternatives is so high ($6^7\times3^7\times5^1\cong3.06\times10^9$). In this case, the application of an exhaustive search is unpractical because it would take years. The optimization problem has been solved using the proposed methodology. The GA parameters were set to N_{ind}=50, N_g=50, G=48, $S_f=10^{-10}$, p_c=0.7 and $p_m=P_c/L_{ind}$=0.0467 for both simulation design tests.

Since the GA converges quickly, it allows to scan the restriction space, and in doing so to solve the problem, $Cost_{max}$ is increased from 10 to 340 in steps of 10. This scan has done 10 times in order to check the feasibility of the algorithm. The trade-off between cost and performance are shown in Figs. 2 (a) and (b). The time taken for each design test was 18 h and 37 s, which corresponds to a mean time of 3 min and 11 s for each $Cost_{max}$ to converge. The tradeoff curves help the user to select a sensor network with desired performance and investment.

By examining the results shown in Figs. 2 (a) and (b), the following key observations can be concluded:

The minimum achievable performance for a feasible design is about 85 % with the cost 20 for the variable λ_j design approach, while this performance will be about 55 % with the same cost for the constant λ_j design approach.

The convergence rate for the variable λ_j design performance to the ideal 100 % is faster than the rate corresponding the constant λ_j approach. Fig. 2 (b) indicates that the design performance curve reaches near 100 % for the cost of 120 while this maximum performance approached for the cost of 200 in Fig. 2 (a).

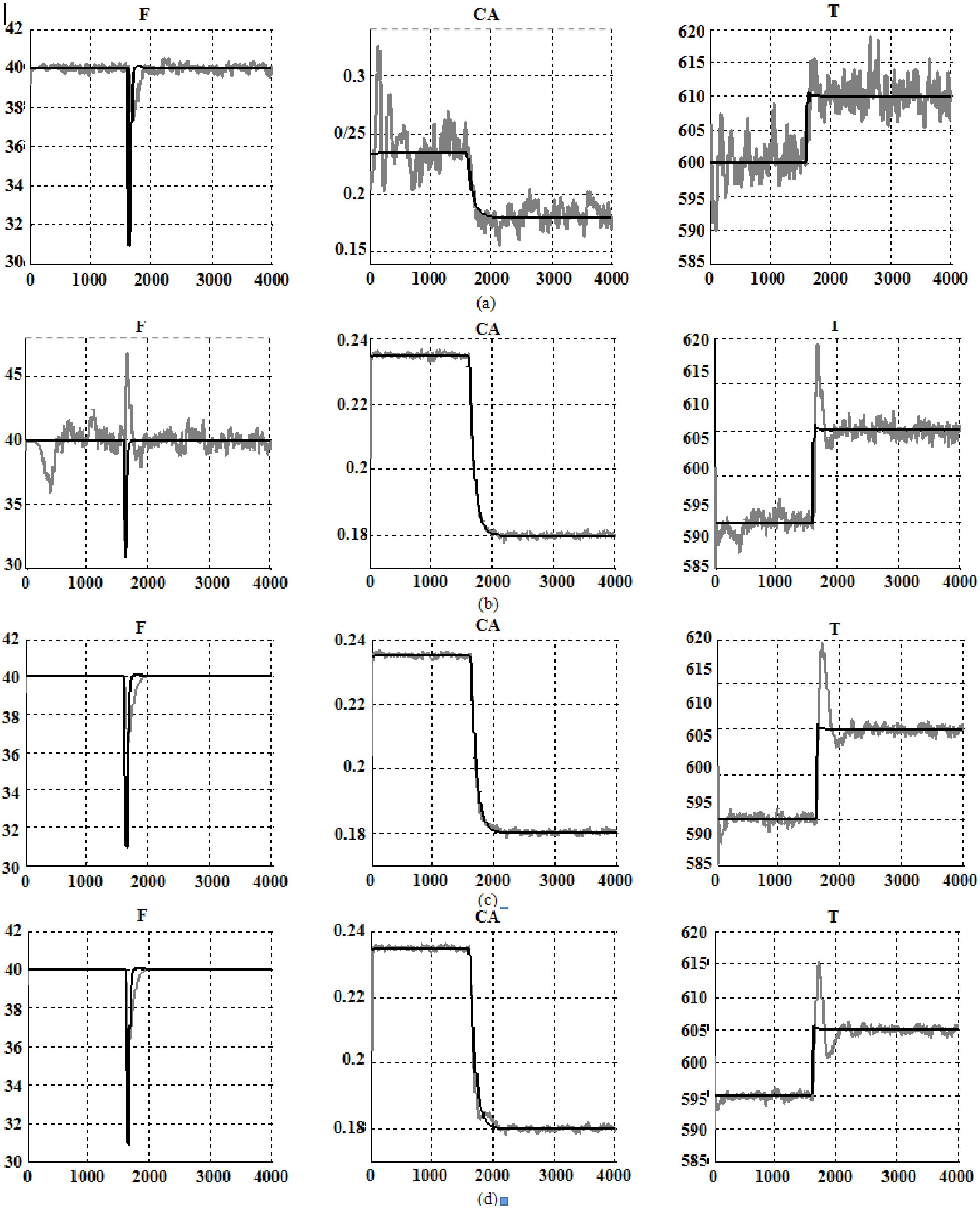
It is noted that higher cost solutions for the near maximum performance 100 % in the both design tests indicates higher redundancy in the designed measurement network.

In order to evaluate the performance of designed networks, four points corresponding to the variable λ trade-off curve has been selected at the maximum performance of investment constraints 20, 100, 170 and 340. Fig. 3 demonstrates the estimated key variables in the selected points versus real signals of key variables. The selected networks has been examined with CSTR case study with step changes (V=50, P=2126.8, T=610) on sample interval 1600. Examining these signal behaviors indicates that more investment on sensor network design leads to more accurate performance judged by lower error variance of these key variables.

CONCLUSIONS

The methodology proposed in [13] for sensor network design has been modified and extended to cater for non-linear dynamic systems. The resulting methodology incorporates a modified EKF filtering as its dynamic data reconciliation technique. This enables the design method to be applied for non-linear dynamic processes.

In the actual implementation of the Kalman filter, the measurement noise covariance R is given by the intrinsic quality of the measuring devices. However, the process noise covariance (Q) which affects the transient and steady-state operation of the data reconciliation can not be determined directly. Hence, its magnitude has been assumed to be fixed in [13]. This can lead to convergence problems due to possible errors in the model transition matrix [13]. In this paper, the time propagation equation for the covariance matrix (P_k) has been solved using the



(a)

(b)

(c)

(d)

Fig. 3: Estimated key variables (F, C_A, T) versus real signals. Cost constraint 20 (a), Cost constraint 100 (b), Cost constraint 170 (c), Cost constraint 340 (d).

transition matrix technique [14] which enhances the numerical robustness of the resulting discrete-time Kalman filter by preserving both the symmetry and the positive definiteness of covariance matrix (P_k). This implementation allows the process noise covariance matrix (Q) to be calculated at each sampling interval based on the most recent state model transition matrix.

To evaluate the performance of each individual potential design solution, a weighted function measure has been proposed. This approach is practically more attractive and provides the user the ability to exercise different weighting factors on process variables based on their operational and economical roles in the design objective. Moreover, to increase the evaluation accuracy of the weighted function measure, the true relative standard deviation of each involved variable has been used instead of the estimated error covariance matrix which can be accurate only up to the first-order for any system non-linearity. This provides a better indicating normalized measure which causes all the process variables to affect equally in design procedure irrespective of their absolute magnitudes. Simulation results demonstrate that the presented methodology provides a very promising sensor network design tool for non-linear dynamic processes. It was shown that the design can lead to the most suitable investment situation where higher performances can be obtained at lower costs.

Received : 14th May 2007 ; Accepted : 27th January 2008

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