An Irregular Lattice Pore Network Model Construction Algorithm

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ABSTRACT: *Pore network modeling uses a network of pores connected by throats to model the void space of a porous medium and tries to predict its various characteristics during multiphase flow of various fluids. In most cases, a non-realistic regular lattice of pores is used to model the characteristics of a porous medium. Although some methodologies for extracting geologically realistic irregular networks from pore space images have been presented, these methods require some experimental data which are either unavailable or costly to obtain. 3-dimensional image or 2-dimensional SEM images are among these types of data. In this paper a new irregular lattice algorithm for the construction of these models is proposed. Furthermore, based on some statistical and analytical studies, a fast and reliable procedure is suggested to find the optimum system parameters which may lead to the construction of the smallest cubic irregular pore network model that can be a representative of the target porous medium. The performance of the proposed method has been studied through the construction of an irregular lattice model representing a core plug based on its available experimental data. This study shows that the obtained model can reliably predict the network construction parameters.*

KEY WORDS: *Pore network modeling, Porous media, Irregular lattice networks, Statistical analysis.*

INTRODUCTION

Most of macroscopic properties of a porous medium, such as porosity, absolute and relative permeability and capillary pressure are determined based on its micro-structure. In principle, if we can construct a reasonable representation of the geometry and topology of the pore space, we will be able to predict those properties reliably. The most difficult task in construction of pore network model for a porous medium is to construct a more realistic model. Even though the importance of constructing a realistic pore network is known, most pore networks are still based on a regular cubic lattice. In most cases, this is due to the lack of information regarding spatial correlations, connectivity within the network, accurate pore size and throat size data, the distribution of pores and throats within the network and sometimes modeling and programming difficulties. Since the real porous medium has an irregular structure, the models based on a regular distribution of pores and throats will

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not be able to predict those properties reliably. Therefore a better way to represent the real porous medium is an active area of research. There are a lot of papers describing attempts to construct a more realistic model Even though the importance of constructing a realistic pore network is known, most pore networks are still based on a regular cubic lattice. In most cases, this is due to the lack of information regarding spatial correlations, connectivity within the network, accurate pore size and throat size data, the distribution of pores and throats within the network and sometimes modeling and programming difficulties. Since the real porous medium has an irregular structure, the models based on a regular distribution of pores and throats will not be able to predict those properties reliably. Therefore a better way to represent the real porous medium is an active area of research. There are a lot of papers describing attempts to construct a more realistic model [1,2]

Modeling of fluid flow through porous media using pore network models was initiated by *Fatt* in the 1950s [3-5]. He was able to produce capillary pressure and relative permeability curves with respect to the average saturation for drainage process in a pore network model built based on a 2D lattice of pores and throats, filled in a sequential manner according to the inscribed radius of network elements.

The next major advancement in network modeling occurred in the late 1970s when *Chatzis & Dullien* [6] by focusing on the assumptions made earlier by *Fatt*, illustrated that 2D networks could not reliably predict the behavior of porous medium in 3D. They compared their results from regular networks to experimental mercury injection data from sandstones, and noted that regular networks combined with circular capillaries did not yield a realistic description of real sandstones. Later the concept of invasion percolation was introduced by *Wilkinson & Willemsen* [7] which is the theoretical foundation for drainage. This model was capable of allowing clusters of defending fluid to get surrounded by the invader and become trapped.

Most of the common networks are based on a regular lattice. It is a cubic network with a coordination number of six for three dimensions or four for two dimensions. The network includes both pores and throat that are also called nodes and links, respectively. The pore size and throat size are populated based on random distributions. The coordination number could be varied by eliminating or adding throats in the network. [8, 9] The regular lattice networks can be distorted by allowing the length of pores and throats to vary while the positions of the pore centers are fixed [10]. With an arbitrary distribution of throat lengths the resultant network may not even be physically realizable in three dimensions. However, in all these cases the network is still based on a regular topology, whereas the target porous medium has a more irregular structure. To mitigate this shortcoming, studies have been made using networks based on square or cubic lattices but with extra coordination [11], *Voronoi* networks [10,12,13], *Delaunay* triangulations [12,13] and irregular networks that allow a variable coordination number [14].

In this study, a new algorithm for construction of irregular pore network models is introduced in details. This algorithm does not require any 3-dimensional image from porous media or spatial correlation and the network will be constructed based on available experimental data such as pore size distribution data and absolute and effective porosity.

IRREGULAR CUBIC PORE NETWORK CONSTRUCTION ALGORITHM

The required input data for building the structure of an irregular cubic pore network model are as follows:

Length of cube side (L)

Total number of pores in the network (N_p)

Total number of throats in the network (N_t)

Pore size distribution (a probability distribution function for example Gaussian or Weibull) ($R_{p,i}$)

Aspect ratio distribution (a fix number or a probability distribution function for example *Gaussian* or *Weibull*) ($R_{asp,i}$)

Fraction of pores located at each boundary (f_B)

Geometrical shape of pores and throats

The proposed irregular cubic pore network construction algorithm consists of the following steps:

Locating the position of pores

Suppose that the flow direction is in x-direction. Use a uniform random number generator to generate two arrays of N_p random numbers between 0 and L for y- and z-coordinates of pores and an array of (1-2 f_B)N_p random numbers between 0 and L for x-coordination of pores. Note that $f_B \times N_p$ pores must be located at inlet boundary (x=0) and $f_B \times N_p$ pores must be located at outlet boundary (x=L). Since the pore locations are selected randomly, to prevent overlapping of pores with each other, before creating every new pore, its distance from all previously created pores can be checked to be greater than a prescribed value ($l_{t,min}$).

Specification of pore properties

Assign pore properties such as radius and geometrical shape to each pore by using the related probability distribution functions. Assume pore length as a function of pore radius. (In this work the length of each pore is assumed to be 2 times [10] of its radius or $l_{p,i} = 2 \times r_{p,i}$)

Locating the position of throats

Every throat connects two pores. Selection of pore pairs to be connected by throats plays an important role in final network properties such as absolute permeability, capillary pressure and relative permeability. The simplest way for locating the throat positions in an irregular lattice is random selection of pore pairs and connecting them by throats. But in this method there is no limitation on the length of the created throats and there is no guarantee that every pore will only connect to its adjacent pores. For example it is possible for a throat to connect a pore located at the inlet boundary directly to another one located at the outlet boundary and create a shortcut between two boundaries which can drastically affect all of the system properties.

In this study, another approach has been proposed to overcome this problem. In this approach, the smallest possible throats in the system are selected and created. This will cause every pore to be just connected to its nearest neighboring pores and therefore the system connectivity will be much more similar to the nature of porous medium in comparison with the random throat selection approach.

Throat generation consists of the following two steps:

(i) Considering all possible pairs of pores and calculating the distance between their centers (the length of possible throat) and adding the pair to the list of all possible throats. This list should be sorted by the length of possible throats. If there are N_p pores in the system, the number of all possible throats will be equal to $N_p \times (N_p - 1)/2$

(ii) Selection of first N_t pairs from all possible throats list (N_t smallest possible throats) and creating a throat to connect these pairs

Specification of throat properties

The radius of every throat can be calculated based on the radii of the two pores as follows [15]:

$$R_{t,ij} = \min\left\{R_{p,i}, R_{p,j}, \frac{R_{p,i} + R_{p,j}}{2 \times R_{asp}}\right\} \quad (1)$$

Where R_{asp} is the aspect ratio defined as the ratio of pore radius to the throat radius and can be either considered as a constant value or obtained from a probability distribution function.

The other properties of throats such as contact angle and geometrical shape can be assigned according to same specified probability distribution functions.

NETWORK EVALUATION

When the network is completely constructed, its prediction must be checked to make sure that it is an acceptable representation of the real porous media. The evaluation process should be performed using available experimental data such as total and effective porosity (ϕ_t and ϕ_{eff}), absolute permeability (k_{abs}), capillary pressure (P_c), and relative permeability (k_r). Total and effective porosity and absolute permeability are functions of both network structure and properties of fluids flowing inside the network and the interaction between fluids and rock (pores and throats).

In pore network models, number of pores and throats (N_p and N_t) play the most important role in required CPU time to run the model. On the other hand, lager networks (networks with greater number of pores and throats) are capable of better prediction of system behavior while trying to match the model predictions with experimental data. Therefore, using the optimum number of pores and throats in the pore network model will result in construction of a network which can predict the system behavior as fast and reliable as possible. In this study a new approach based on some analytical equations and statistical experiments has been proposed to find the optimum values for N_p and N_t.

STATEMENT OF THE PROBLEM

Pore network models must be capable of predicting a wide range of common experimental tests which are performed on rock samples. These tests are mainly divided into two groups; routine and special tests. Routine tests mainly focus on physical properties of rock sample such as its total and effective porosity and absolute permeability. In special tests the hydraulic properties of sample in presence of different fluids are studied. The major measured properties in special tests are capillary pressure and relative permeability. These values depend on the process by which the porous medium is influenced; i.e. the invader and defender fluids in simultaneous flow of two fluids through the porous medium. These flow processes are known as drainage and imbibition. In the drainage process, the invader fluid is the non-wetting phase and the defender fluid is the wetting phase, while in the imbibition process the invader fluid is the wetting phase and defender fluid is the non-wetting phase.

For acceptable simulation of special tests by network models, first the model should be capable of reliable prediction of the results of the routine tests. For construction of a cubic irregular pore network model for simulation of rock total porosity (ϕ_t), effective (connected) porosity (ϕ_{eff}) and absolute permeability (k_{abs}) by using the described irregular lattice algorithm, the following parameters should be defined:

Total number of pores (N_p)

Total number of throats (N_t)

Bulk volume of the model (V_b) or length of cube side (L)

Pore size distribution (V_{pi})

Aspect ratio distribution ($R_{asp,i}$)

Fraction of pores located on each boundary (f_B).

For prediction of special test results (hydraulic properties) of the rock sample one also needs to know the wettability and geometrical shape of each pore and throat and the fluids' properties such as viscosity and interfacial tension.

In this study, a new algorithm is proposed to match the system properties (ϕ_t, ϕ_{eff}, k_{abs}) by optimizing the static parameters of network (Np, N_t and L) and then in the next step the hydraulic properties (k_r and P_c) can be matched by optimizing other system variables such as wettability and geometrical shape distribution. This approach is based on statistical analysis of irregular lattice pore network models built by the proposed algorithm.

Final goal of this approach is to find the optimum values of Np, N_t and L with respect to experimentally measured values of ϕ_t, ϕ_{eff}, k_{abs} in a fast and reliable way. Since during optimization of pore network models with experimental data there are too many uncertain parameters which can be selected as optimization variables, finding the solution may take a long time. In this study we try to reduce the number of these parameters by finding some analytical and statistical correlations between system parameters and available experimental data.

STATISTICAL STUDIES

Throat length distribution

To find the distribution function of all possible throat length in a network built by the described irregular algorithm, some statistical analysis was performed. In this analysis, to simplify the problem, first it was assumed that there is a line segment with length of l. Then two uniformly distributed random points were selected on it for 20000 times and the distance between these points were recorded and the length distribution function was obtained. This study showed that the approximate average distance between two points on a line segment of length l is 0.332×l.

In the next step, the same problem was extended to 2- and 3-dimensional space. In the 2-dimensional case, two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ were selected uniformly inside a square with characteristic length of l for 20000 times and like the previous case the distribution of the distance between these two points $\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\right)$ was found as shown in Fig. 1. Fig. 1 shows that the distribution of distance between two points inside a square with side length of l is a nearly normal probability distribution function with an approximate average of 0.521×l.

Finally, the same procedure was repeated in 3-dimensional space for a square of length l and two points inside it as $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ and the distribution of distance between these two points $\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}\right)$ was plotted again (Fig. 2). Again it shows a normal distribution behavior with an average of 0.661×l.

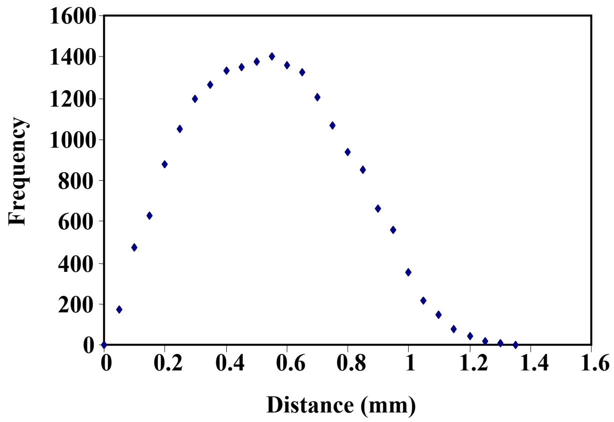

Fig. 1: Length frequency distribution of 20000 two randomly selected points in a square with side length of 1.

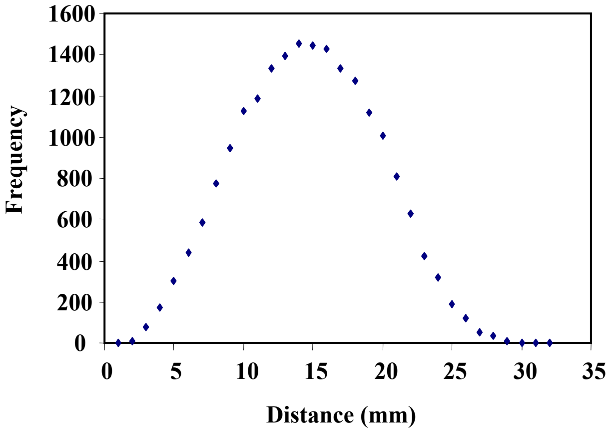

Fig. 2: Length frequency distribution of 20000 two randomly selected points in a cube with side length of 1.

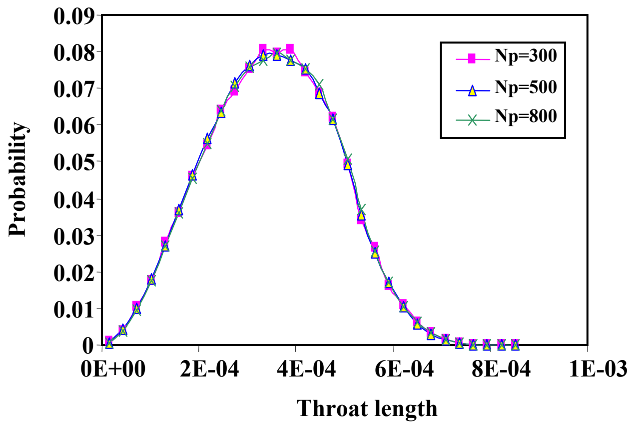

Fig. 3: Effect of Np in throat length probability distribution in a fixed volume.

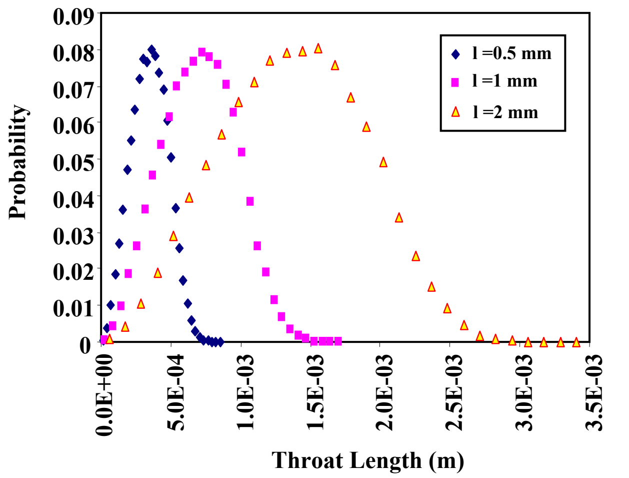

Fig. 4: Effect of cube side length (l) on throat length probability distribution for a system with N_p=500.

In the next step, the number of points inside a cube of length l was increased and the distance between every two points inside the cube was recorded. These points can be regarded as the center of pores inside the irregular pore network model and the distance between every two points is the length of throat connecting the pores. This procedure was repeated for different number of points (pores), or N_p, and the obtained distribution functions were compared to each other (Fig. 3). As it can be inferred from Fig. 4, for relatively larger number of points (pores), the throat length distribution does not depend on the number of points (pores) located inside the cube and in all cases the average throat length and standard deviations are nearly the same.

To test the effect of cube side (l) on the throat length distribution function at constant N_p, the same procedure was repeated for cubes with side length of 1, 2, 3 and 5 and N_p=500. The result is plotted in Fig. 4 which shows that increasing in l will lead to the increase of average and standard deviation of the distribution function. Fig. 5 shows that the average throat length and the standard a dimensionless throat length $(l_{td} = l_t / l)$, all the distribution functions will become the same as shown in Fig. 6. The probability distribution shown in Fig. 6 can be modeled by a normal probability distribution function with the following mathematical formula:

$$f(l_{td}) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{1}{2}\left(\frac{l_{td}-\mu}{\sigma}\right)^2} \qquad (2)$$

deviation are linear functions of l. Therefore, according to where, μ is the average dimensionless throat

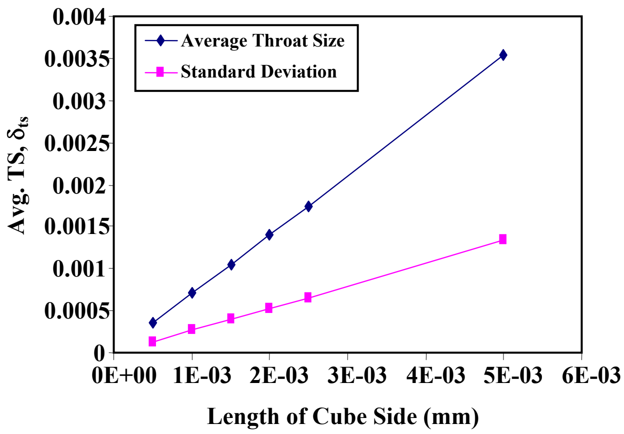

Fig. 5: Average and standard deviation of throat size distribution function vs. length of cube side.

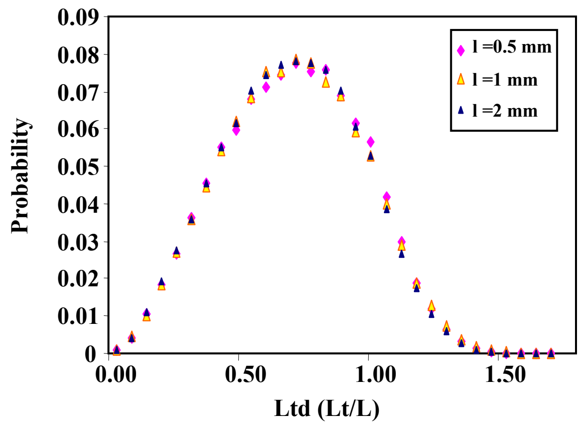

Fig. 6: Throat length probability distribution vs. dimensionless length (l_{td}) for a system with N_p=500.

length ($l_{td,ave}$) and σ is the standard deviation. To match the probability distribution function shown in Fig. 7 the following values (obtained by curve fitting) should be used:

$$\mu = 0.7066 \text{ and } \sigma = 0.2667. \quad (3)$$

The cumulative distribution function for normal function is:

$$F_c(l_{td}) = \frac{1}{2}\left[\operatorname{erf}\left(\frac{l_{td}-\mu}{\sqrt{2}\sigma}\right)+1\right] \quad (4)$$

where erf(x) is the error function and is defined as:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \quad (5)$$

and its value can be estimated by using the following Maclaurin series:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{n!(2n+1)} \quad (6)$$

Although the domain of normal probability distribution function is (-∞, +∞), the domain of throat size distribution is limited. So for more precise predictions, instead of normal probability distribution function, the following Beta-type probability distribution function has been used in this study:

$$f(l_{td}) = 5.2886\times l_{td}^2\times(1.4150-l_{td})^2 \quad (7)$$

and its cumulative distribution function is:

$$F_c(l_{td}) = 1.0577\times l_{td}^5 - 3.7417\times l_{td}^4 + 3.5296\times l_{td}^3 \quad (8)$$

Correlation between porosity ratio and throat-pore ratio

Porosity ratio (ϕ_{eff}/ϕ_t) depends on the number of throats connecting pores or coordination number of pores. Average coordination number ($N_{coord,ave}$) relates to the throat-pore ratio (N_t/N_p) according to the following equation:

$$N_{coord,ave} = 2\times\frac{N_t}{N_p} \quad (9)$$

Increasing the number of throats (N_t) at a constant number of pores (N_p) results in increase of average coordination number ($N_{coord,ave}$) and therefore the number of the dead end pores and throats will decrease which will cause the effective porosity to be increased and became closer to the absolute porosity.

To find the correlation between throat-pore ratio (N_t/N_p) and porosity ratio (ϕ_{eff}/ϕ_t), a statistical study was performed. In this study, for several values of N_p and N_t/N_p, the model was built for 100 realizations and average value of porosity ratio was calculated. As it is shown in Fig. 7, for a typical case of network construction with N_p = 200 and N_t = 280 (N_t/N_p = 1.4), after 100 realizations, there is almost no fluctuation in the average values of absolute and effective porosity.

The final result of this study is represented in Fig. 8. This figure shows that the value of porosity ratio (ϕ_{eff}/ϕ_t) is correlated to the throat-pore ratio (N_t/N_p). According to the results of this study, the following correlation can be used to match the statistical data:

$$\frac{\phi_{conn}}{\phi_t} = 1-\frac{1}{1+e^{a\frac{N_t}{N_p}-b}} \quad \text{for } N_t/N_p < 4 \quad (10)$$

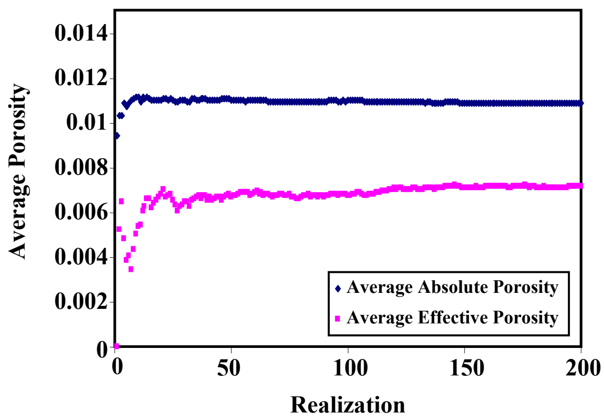

Figure7: Fluctuation of average absolute and effective porosity for a typical case of network construction with $N_p = 200$ and $N_t = 280$ ($N_t/N_p = 1.4$).

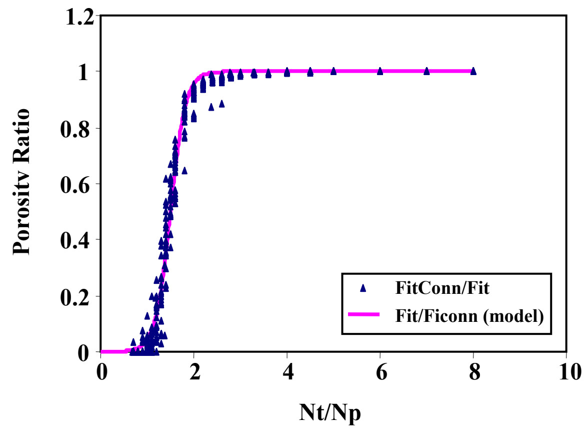

Figure8: Correlation between porosity ratio (ϕ_{eff}/ϕ_t) and throat-pore ratio (N_t/N_p).

$$\frac{\phi_{conn}}{\phi_t} = 1 \qquad \text{for } N_t/N_p > 4 \qquad (11)$$

where a and b are constant parameters whose appropriate values have been obtained (by curve fitting) to be 6.1480 and 9.2173, respectvely.

ANALYSIS OF MODEL PARAMETERS DEPENDENCE

For a cube with side length of L, the following equation correlates N_p, N_t and ϕ_t:

$$N_p \times V_{p,ave} + N_t \times V_{t,ave} = \phi_t \times L^3 \qquad (12)$$

where, $V_{p,ave}$ and $V_{t,ave}$ are average pore and throat volume respectively. Since $V_{p,ave}$ can be determined from pore size distribution data, Eq. (12) can be used if there is an estimated value for $V_{t,ave}$.

Estimation of $V_{t,ave}$

$V_{t,ave}$ can be calculated from the following equation:

$$V_{t,ave} = A_{t,ave} \times l_{t,ave} \qquad (13)$$

where $A_{t,ave}$ and $l_{t,ave}$ are the average cross sectional area and average length of throats, respectively. $A_{t,ave}$ can be determined from pore size and aspect ratio distribution data. The cross sectional area of a throat connecting i'th and j'th pores, $A_{t,i}$, can be determined from:

$$A_{t,ij} = \pi R_{t,ij}^2 \qquad (14)$$

where $R_{t,ij}$ is obtained from Eq. (1).

To find $l_{t,ave}$, the general probability function is used. First we have:

$$l_{t,ave} = l_{td,ave} \times L \qquad (15)$$

where L is the side length of a cube and $l_{td,ave}$ is the average dimensionless length of the throats.

In order to create N_t throats in the system, according to the described irregular pore network construction algorithm, we need to create all possible throats smaller than a specific length named $l_{td,max}$. The average imensionless length of the throat can be obtained as follows:

$$l_{td,ave} = \frac{\int_0^{l_{td,max}} f(l_{td})dl_{td}}{l_{td,max} - 0} = \frac{F_c(l_{td,max})}{l_{td,max}} \qquad (16)$$

where f(x) and $F_c(x)$ are probability distribution function (Eq.(7))and its cumulative distribution function ((Eq.(8)).

Now the problem is to find $l_{td,max}$. This parameter can be calculated th rough the solution of the following equation:

$$\frac{N_t}{N_{t,max}} = \int_0^{l_{td,max}} f(l_{td})dl_{td} = F_c(l_{td,max}) \qquad (17)$$

where $N_{t,max}$ is the maximum possible number of throats and is equal to:

$$N_{t,max} = \frac{N_p(N_p - 1)}{2} \qquad (18)$$

Table 1: Weibull distribution function parameters used in model.

Parameters	Min.	Max.	γ	δ
Pore Radius (R_p), μm	0.01	15	3.0	0.2
Aspect Ratio ($R_{asp,i}$)	1.0	3.0	3.0	0.5

Weibull distribution function for pore radius (R_p) is:

$$R_p = (R_{p,max} - R_{p,min})\left[-\delta \ln\left(x\left(1-e^{-1/\delta}\right)+e^{-1/\delta}\right)\right]^{1/\gamma} + R_{p,min}$$

and fore aspect ratio ($R_{asp,i}$) is:

$$R_{asp,i} = (R_{asp,max} - R_{asp,min})\left[-\delta \ln\left(x\left(1-e^{-1/\delta}\right)+e^{-1/\delta}\right)\right]^{1/\gamma} + R_{asp,min}$$

Finally, $l_{td,max}$ can be obtained through the solution of the following equation:

$$G(l_{td,max}) = \frac{2N_t}{N_p(N_p-1)} - F_c(l_{td,max}) = 0 \qquad (19)$$

Replacement of $V_{p,ave}$ and $V_{t,ave}$ in Eq.(12) and rewriting the equation in terms of L will result in the following cubic polynomial which can be solved to find the value of L for the specified N_p and N_t.

$$L^3+AL+B=0 \qquad (20)$$

where

$$A = -\frac{N_t A_{t,ave} L_{td,ave}}{\phi_t} \qquad (21)$$

and

$$B = -\frac{N_p V_{p,ave}}{\phi_t} \qquad (22)$$

Eq. (20) can be solved to find the value of L. An appropriate initial estimate for numerical solution of Eq. (20) is as follows:

$$L_{init} = \sqrt[3]{\frac{2N_p V_{p,ave}}{\phi_t}} \qquad (23)$$

A compact recipe to find network parameters

The following procedure suggests the appropriate values for number of throats (N_t) and length of cube side (L) at a given number of pores (N_p), to build an irregular pore network model through the proposed algorithm.

1- By using pore size distribution data, find $V_{p,ave}$ and $A_{t,ave}$.

2- Calculate porosity ratio from experimental data based on which throat-pore ratio (N_t/N_p) and the number of throats (N_t) can be calculated via equation (10).

3- $l_{td,max}$, $l_{td,ave}$ and L will be obtained through equations (19), (16) and (20), respectively.

Repeating the irregular network construction algorithm with the values obtained from the above procedure for some realizations and calculation of their average properties, will certainly reproduce the experimental data used in their development. However, it is not required to repeat the procedure for several times to match the experimental data. In most of the cases, after a few realizations, depending on the tolerance of the acceptable error for static data (E_{static}), a good representative of the target porous media will be produced. Furthermore, for each realization with an acceptable error for static data, the error in the prediction of the dynamic behavior ($E_{dynamic}$) is obtained via simulation with a dynamic simulator in order to ensure the accuracy of the predication. The dynamic data may contain experimental results of every kind of multi-phase flow through porous media such as drainage and imbibition.

Increasing the number of pores and throats in a pore network model, will lead to the increase of the number of optimization parameters and therefore the network flexibility to match the experimental data. On the other side, using larger number of pores and throats in a model results in a drastic increase of computational demand for the construction and optimization of the network. Therefore, to find the optimum representative network, it is recommended to take a constructive approach in the development of the pore network model. This means, one should start with a small number of pores (N_p) and increase it incrementally until the desired match is obtained.

Case study

To evaluate the proposed algorithm, the pore network model representing a core sample of an Iranian reservoir rock was constructed. The reported values of absolute and effective porosity of this sample are 17.2% and 21.6%, respectively. The pore size distribution data from SEM image analysis and capillary pressure data are reported in Table 1 and the absolute permeability of the system is 4 md.

For the first trial N_p was assumed as 50. Using the proposed method, the values of N_t and L were obtained

Table 2: First 10 Realization of a Pore Network Model with N_p=50.

Number	Effective Porosity, ϕ_{eff}	Total Effectiveϕ_t	Absolute Permeability, k_{abs}	Relative Error, %
2	18.44509	20.89226	11.41229	5.257721
3	22.33069	23.17814	21.75592	18.56789
4	21.25494	22.08397	38.65244	12.90791
5	0	21.34033	0	50.60108
6	16.61683	22.11585	7.810875	2.889358
7	16.83715	21.94564	4.19826	1.854895
8	20.7638	22.57711	4.38691	12.62172
9	19.92844	22.08004	14.21511	9.042724
10		21.58504	0	50.03462

Table 3: First 10 Realization of a Pore Network Model with N_p=200.

Number	Effective Porosity, ϕ_{eff}	Total Effective, ϕ_t	Absolute Permeability, k_{abs}	Relative Error, %
1	22.75271	24.41093	17.28041	22.64837
2	16.84791	22.40653	3.345332	2.890471
3	20.86848	22.92587	8.730177	13.73334
4	17.50653	22.47042	9.501975	2.90596
5	0	22.60405	0	52.32419
6	20.73693	23.87161	22.86993	15.54011
7	17.82216	22.17414	8.399178	3.137651
8	16.62354	23.47732	3.526713	6.021408

as 86 and 0.000114 m. Then these values are used in the development of the network based on the proposed algorithm for irregular network construction. The results for first 10 realizations are given in Table 2. The same procedure was repeated for N_p = 200. This time the values of N_t and L were obtained as 273 and 0.000171m, respectively. The network was built by the same algorithm and again the results of first 10 realizations are given in Table 3. Comparing Table 2 and Table 3 shows that despite the drastic difference in the computational demand required for both cases, there is at least one realization with an average relative error of less than 2% for both values of N_p (i.e, N_p = 50 and N_p = 200). Hence, considering the fact that the required time for construction of the network with 50 pores is 5 msec. and the time to build the network with 200 pores is 572 msec. on a Pentium 4, 3.0 GHz, with 1 GB of RAM, it does not make sense to start with a large number of pores for the construction of the network representing these rock sample.

To match the other network predictions with experimental data, we need to use the optimum network which can make a good prediction of both static and dynamic data. Some of the network predictions depend on both network structure (number and location of pores and throats) and other network input data. For example, absolute permeability is dependant upon the pore and throat size distribution data as well as the network structure, or capillary pressure, and relative permeability depends on wettability and geometrical shape of pores and throats in addition to the mentioned parameters. Before trying to match these properties, it is necessary to ensure that the values of the effective parameters including network input data such as pore size distribution, wettability and shape factor data are appropriately tuned. In the case of unsuccessful attempt to match experimental data with tuning of uncertain input data, enlargement of network (selecting a larger value for N_p) may solve the problem by increasing the network flexibility to mimic the system behavior.

CONCLUSIONS

In this study a new approach for construction of cubic irregular pore network models was introduced in details. Furthermore, based on some statistical and analytical studies, a fast and reliable procedure has been suggested to find the optimum system parameters without any requirement to run the network construction algorithm for several times. The network constructed by this approach is the smallest cubic irregular pore network model that can represent the target porous medium. The proposed network construction algorithm has been tested to build a model for a reservoir rock sample and the optimization procedure has been used to find the system parameters at two different network sizes (e.g., N_p = 50 and N_p = 200). The case study verified the reliability of the proposed algorithm to build an irregular pore network model for a real rock sample.

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