Modeling of Inflow Well Performance of Multilateral Wells: Employing the Concept of Well Interference and the Joshi's Expression

Shadizadeh, Seyed Reza; Kargarpour, Mohammad Ali; Zoveidavianpoor, Mansoor*+

Abadan Faculty of Petroleum Engineering, Petroleum University of Technology, Abadan, I.R. IRAN

ABSTRACT: *The use of multilateral well is becoming an emerging method to improve oil recovery efficiently and to drain reservoirs more efficacious. By developing drilling technology, completing the oil wells as multilateral wells become more interesting especially from economic point of view. On the other hand, lack of any means for forecasting the performance of this type of wells causes drilling of them economically a risky job. The major objective of this work is to present a simple and effective means to estimate the performance of a multilateral well. In a simple approach to the multilateral well, one can consider it as several horizontal wells flowing into a common well string. By employing the concept of well interference and the Joshi's expression for horizontal well performance, a mathematical model for computing multilateral wells performance has been developed. Two correlations for estimating the multilateral well performance with odd and even branches have been presented by utilizing the concept of well interference in conjunction with a horizontal well performance expression. Consequently, the generated correlations along with the concept of equivalent length have been used in this work to present a general method for predicting a multilateral well performance. Also, economic analysis developed model for a multilateral well is presented in this paper.*

KEY WORDS: *Multilateral well, Inflow performance, Horizontal well, Interference.*

INTRODUCTION

Lack of desirable hydrocarbon recovery and decreasing the oil production from mature and unconventional reservoirs causes a big challenge to the industry nowadays. Multilateral well technology has gained strong momentum in the past ten years and can provide innovative solutions to word's oil shortages and prove to be the effective tool to impelling the industry in the next century.

Drilling several horizontal sections from a single vertical wellbore has improved the drilling and production economics on many wells. Multilateral wells reduce drilling time and wellhead and casing costs because only one main vertical bore is drilled.

The complexity of multilateral wells ranges from simple to extremely complex. They may be as simple as a vertical wellbore with one sidetrack or as complex as a horizontal extended-reach well with multiple lateral and sub-lateral branches [1].

Cost experts agree that horizontal wells have become

* *To whom correspondence should be addressed.*

+ *E-mail: Mzoveidavian@put.ac.ir*

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a preferred method of recovering oil and gas from reservoirs in which these fluids occupy strata that are horizontal, or nearly so, because they offer greater contact area with the productive layer than vertical wells [2]. Of primary importance is the increased well production compared to similar single horizontal wells and vertical wells. The use of a single vertical well bore minimizes location, access road, and cleanup costs [3].

Multilateral wells have potential benefits in reservoir exploitation. Some reservoir applications of multilateral technology have been discussed [4-10], and the need to identify and quantify the reservoir benefits of multilateral wells has received more attention.

Joshi presented an equation to calculate the productivity of horizontal wells. His equation may be also used to account for reservoir anisotropy and well eccentricity [11]. *Raghavan & Joshi* presented an analytical solution of well productivity for symmetric horizontal radials defined as horizontal drainholes of equal-length kicked off from the same depth in symmetrical directions. The result was an inflow equation, i.e., the effect of wellbore flow to the common kick off point was not considered [12].

Larsen presented closed-form expressions of skin factors and productivity indices of radial symmetric multilateral wells. Well Inflow was analyzed based on the distances between the midpoints of the laterals. Wellbore flow was not considered [13]. *Retnanto et al.* studied the optimal configurations of the multilateral well. They investigated several configurations of multilateral well, and discussed the advantages and drawbacks of them. They studied the performance of six different configurations, and found that the length and number of branches could be optimized [4]. *Salas et al.* used analytic and numeric modeling techniques. Their results showed how multilateral well productivity depends on wellbore geometry. Reservoirs with greater heterogeneity were shown to have greater potential benefits from adding multilateral side-branches to an existing wellbore. They also mentioned the gas coning and water flood in reservoirs, which is useful for future study of new multilateral well technology [9].

During the last decade, several attempts have been made to increase the oil production per well in the National Iranian South Oil Company (NISOC). One of the most interesting ways of achieving this goal is drilling horizontal wells. This type of wells opens a larger area to

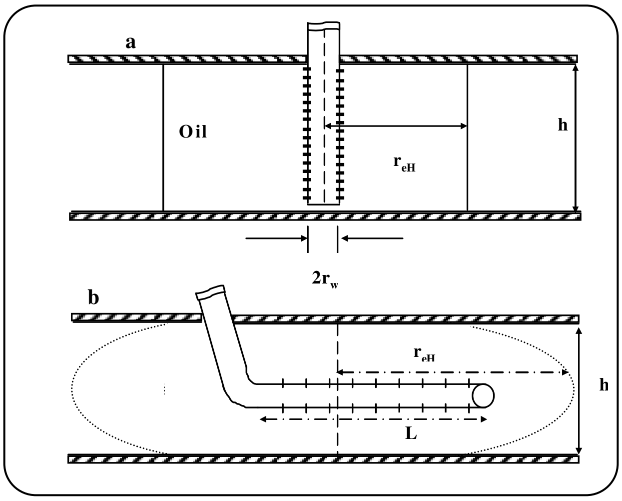

Fig. 1: Schematic of vertical and horizontal- well drainage volume.

the sandface of reservoir and therefore, makes greater chance of oil flow to the well. After relatively successful campaign jobs in this category, to enhance the productivity of this type of wells, complete the new ones as multilateral have become advisable. As the first try, this paper has been raised and defined to take over a mean for predicting the performance of multilateral horizontal well. Based on the simple analytical solution, by utilizing the concept of well interference in conjunction with a horizontal well performance expression, a multilateral well architecture was conducted in this work; two correlations for estimating the multilateral well performance with odd and even branches have been presented. This work also provides economic analysis of a multilateral well on the basis of the rate of return on the investment.

JOSHI WORK

Joshi presents an equation to calculate the productivity of horizontal wells and a derivation of that equation using potential-fluid theory. Fig. 1 shows that a horizontal well of length L drains an ellipsoid, while a conventional vertical well drains a right circular cylindrical volume. Both wells drain a reservoir of height h, but their drainage volumes are different. To calculate oil production from a horizontal well mathematically, the three-dimensional (3D) equation ($\nabla^2 P = 0$) needs to be solved first. If constant pressure at the drainage boundary and at the well bore is assumed, the solution would give a pressure distribution within a reservoir. Once the pressure

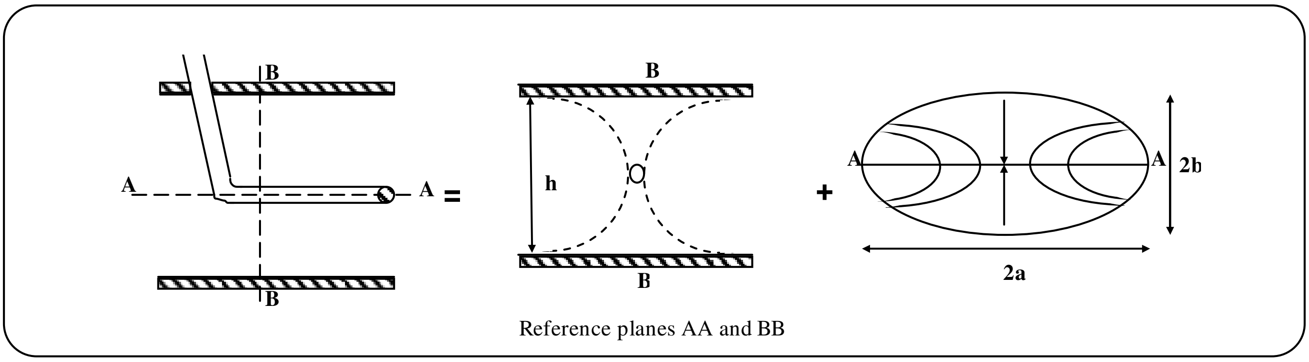

Fig. 2: Division of 3D horizontal-well problem into two 2D problems.

distribution is known, oil production rates can be calculated by Darcy's law. To simplify the mathematical solution, the 3D problem is subdivided into two 2D problems.

Fig. 2 shows the following sub-division of an ellipsoid drainage problem: (1) oil flow into a horizontal well in a horizontal plane and (2) oil flow into a horizontal well in a vertical plane. The solution of these two problems is added to calculate oil production from horizontal well.

Joshi presented the following equation for calculating oil production from horizontal well:

$$q_H = \frac{2\pi k_0 h \Delta P / (\mu B_0)}{\ln\left[\frac{a + \sqrt{a^2 - (L/2)^2}}{L/2}\right] + \frac{h}{L}\ln\left(\frac{h}{2r_w}\right)} \quad (1)$$

For L>h and (L/2) <0.9 r_{eH}

Where r_{eH} is drainage radius and a, is half the major axis of a drainage ellipse in a horizontal plane in which the well is located, is obtained by following formula:

$$(L/2)\left[1/2 + \sqrt{1/4 + \frac{1}{(0.5L/r_{eH})}}\right]^{0.5} \quad (2)$$

Table 1 lists the correspondence between L/ (2a) and $L/(2r_{eH})$ values.

RESULTS

Mathematical Model Development of a Multilateral Well Performance

To derive a new correlation for the performance of a system, the relevant previous works were studied and the existing correlations rationally combined to derive the new one. One may consider the multilateral well as an extension of a horizontal well, so it was basically assumed that the performance of a multilateral well is also an extension of that of the horizontal well. One can also realize a multilateral well as a number of horizontal wells flow in a common casing (well string). This means that these "horizontal wells" interfere each other; therefore the multilateral well performance is the resultant of those horizontal well performances affecting by interference.

Model Approach

A multilateral well as shown in Fig. 3, may be considered as a circular ring of horizontal wells commingled in a common well string. Therefore, each branch of a horizontal may be treated as horizontal well that is affected by other branches (or horizontal wells).

Just for simplicity, it assumed that all branches drilled in a common plane. For applying the interference effect on a multilateral well, a horizontal well performance should be employed.

In order to calculate oil production from the horizontal wells, 3D mathematical equation divided into two 2D problems: 1) Oil flow into a horizontal well in a horizontal plane and 2) Oil flow into a horizontal well in a vertical plane. To derive a correlation for multilateral well, the same approach has been followed.

At first, multilateral well in a horizontal plane has been analyzed and then, the obtained expression generalized to both vertical and horizontal plane. All assumptions are the same as those assumed by *Joshi* (constant pressure at the drainage boundary and at the well bore).

Table1: Relationship between various geometric factors

L/2 r_{eH}	L/2 a	a/r_{eH}
0.1	0.0998	1.002
0.2	0.198	1.01
0.3	0.293	1.024
0.4	0.384	1.042
0.5	0.47	1.064
0.6	0.549	1.093
0.7	0.62	1.129
0.8	0.683	1.171
0.9	0.739	1.218

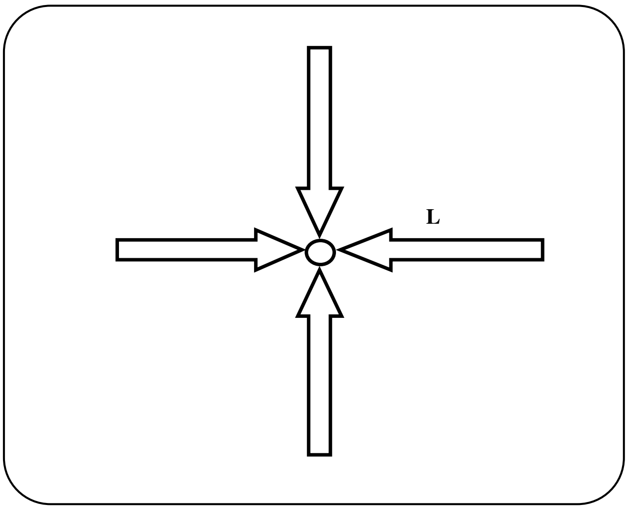

Fig. 3: Schematic drawing of a multilateral well.

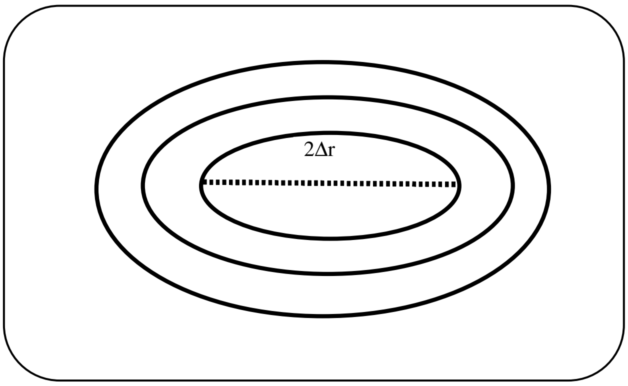

Fig. 4: Schematic of potential flow to a horizontal well horizontal plane.

Multilateral well in a horizontal plane

From an analytical model, *Joshi* derived the following equation:

$$q_1 = \frac{2\pi k_0 \Delta p/\mu}{\ln\left(\frac{a+\sqrt{a^2-\Delta r^2}}{\Delta r}\right)} \tag{2}$$

Where Δr =well half-length =L/2 (Fig. 4) and a = half the major axis of a drainage ellipse in a horizontal plane.

From this equation, *Joshi* presented the Eq. (3) for oil flow to a horizontal well in a horizontal plane:

$$q_1 = \frac{2\pi k_0 \Delta p/\mu B_0}{\ln\left(\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right)} \tag{3}$$

The maximum distance between two branches of a two-branch horizontal well may be obtained from following expression (Fig. 5)

$$d = 2L\sin(\alpha/2) \tag{4}$$

By analogy to *Joshi*'s model and considering the effect of interference phenomena, one can obtain P_d , the pressure at the sand face of second branch when the first branch produces at flow rate of q1, from the following equations:

$$p_e - p_d = \frac{q_1 \mu B_0}{2\pi k_0 h}\ln\left[\frac{a+\sqrt{a^2-d^2}}{d}\right] \tag{5}$$

Where Pe is pressure at the drainage boundary. The term (Pe-Pd) is really the pressure drop in branch 2 induced by production from branch 1.

Substitute the d from Eq. 4 in Eq. 5:

$$p_e - p_d = \frac{q_1 \mu B_0}{2\pi k_0 h}\ln\left[\frac{a+\sqrt{a^2-(2L\sin(\alpha/2))^2}}{2L\sin(\alpha/2)}\right] \tag{6}$$

The static pressure in the second branch is P_d, so that the pressure drop in the second branch when it flows at rate q_2 is:

$$p_d - p_w = \frac{q_2 \mu B_0}{2\pi k_0 h}\ln\left[\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right] \tag{7}$$

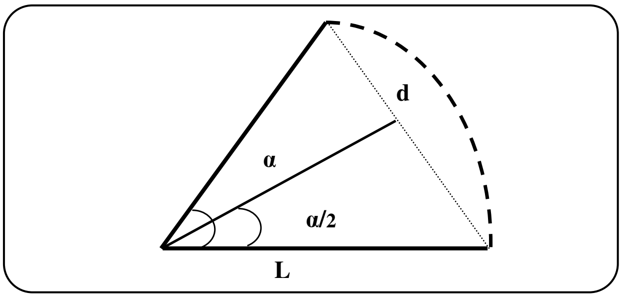

Fig. 5: Schematic of a diagram of a two-branches multilateral well.

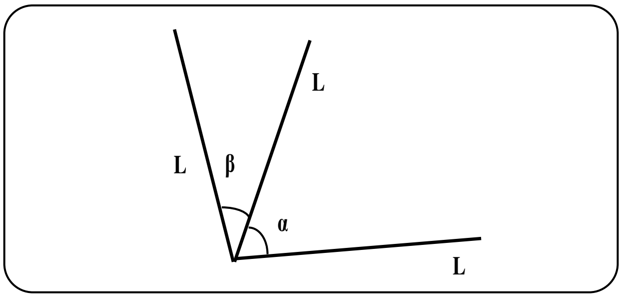

Fig. 6: Schematic diagram of a tree-branches multilateral well.

It is assumed that the length of both branches is the same; L and $q_1=q_2=qw/2$

The total pressure drop is the sum of the Eqs. (6) and (7).

$$(PI)_2=\frac{2\left(2\pi k_0 h/\mu B_0\right)}{\ln\left\{\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\alpha/2\right)\right)^2}}{2L\sin\left(\alpha/2\right)}\right]\left[\frac{a+\sqrt{a^2-\left(L/2\right)^2}}{L/2}\right]\right\}} \qquad (8)$$

From definition, the productivity index of a two-branch multilateral well may be obtained from following expression:

Subscript 2 means PI is for two-branch multilateral well.

$$(PI)_3=\frac{3\left(2\pi k_0 h/\mu B_0\right)}{\ln\left\{\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\alpha/2\right)\right)^2}}{2L\sin\left(\alpha/2\right)}\right]\right.}\times\frac{1}{\left.\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\beta/2\right)\right)^2}}{2L\sin\left(\beta/2\right)}\right]\left[\frac{a+\sqrt{a^2-\left(L/2\right)^2}}{L/2}\right]\right\}} \qquad (9)$$

For a three-branch multilateral well, by using the same approach, the following expression may be derived:

α and β are angles between branches (Fig. 6).

Utilizing the same approach for a larger number of branches for a multilateral well, it can be concluded that for maximizing the Productivity Index (PI) of a multilateral well all branches should be drilled around circular ring at equal angles. It means that in a multilateral well with the number of branches greater than 2, each pair of an branches which are drilled at the same angle from first branch have same effects on the final performance correlation of the well (recalling the assumption of an isotropic reservoir). Therefore, based on above points and investigating the PI expressions for different number branches of multilateral wells, the following general correlations may be derived for a multilateral well that all branches are drilled in a same horizontal plane:

1) In a multilateral well with even number of branches, each of the first and the second branches has its own effect on the final performance equation of the well but each pair of remaining branches have same effect on the final correlation.

$$(PI)=\frac{n\left(2\pi k_0 h/\mu B_0\right)}{\ln\left\{\frac{\left[a+\sqrt{a^2-\left(L/2\right)^2}\right]\left[a+\sqrt{a^2-\left(2L\right)^2}\right]\prod_{i=2}^{n-2}\left[a+\sqrt{a^2-\left(2L\sin\left(\frac{180}{n}\times\frac{i}{2}\right)\right)^2}\right]^2}{4^{n-1}\left(\frac{L}{2}\right)^n\prod_{i=2}^{n-2}\left[\sqrt{\sin\left(\frac{180}{n}\times\frac{i}{2}\right)}\right]^2}\right\}} \qquad n=2,\,4,\,6,\,8, \qquad (10)$$

2) In a multilateral well with odd number of branches, only the first branch has its own effect on the final performance equation of the well and each pair of remaining branches have same effect on the final correlation.

$$(\mathrm{PI})_n = \frac{n\left(2\pi k_0 h/\mu B_0\right)}{\ln\left\{\dfrac{\left[a+\sqrt{a^2-(L/2)^2}\right]\prod_{i=3}^{n}\left[a+\sqrt{a^2-\left(2L\sin\left(\dfrac{180}{n}\times\mathrm{int}\left(\dfrac{i}{2}\right)\right)\right)^2}\right]^2}{4^{n-1}\left(\dfrac{L}{2}\right)^n\prod_{i=3}^{n}\left[\sqrt{\sin\left(\dfrac{180}{n}\times\mathrm{int}\left(\dfrac{i}{2}\right)\right)}\right]^2}\right\}} \quad n=3, 5, 7, 9 \qquad (11)$$

Where ‘int’ means integer part of the parentheses and Π is multiplication symbol.

Therefore, for estimating the performance of a multilateral well drilled in a reservoir with constant pressure at the drainage boundary and at the well bore, the Eqs. (10) and (11) are applicable to even and odd number of branches respectively.

As mentioned, the Eqs. (10) and (11) are derived based on the assumptions made in Joshi's work to generalize the correlations; the equivalent length concept is utilized in following sections.

Equivalent Length of a Multilateral Well

To achieve a general correlation for estimating a multilateral well performance, the concept of equivalent length is utilized. Equivalent length of a multilateral well defined as the length of a horizontal well that has the same performance of that of multilateral wells. This definition is based on the early assumption of an isotropic reservoir; however, this definition is valid for a reservoir that is horizontally isotrope or a reservoir that has an average permeability of the isotropic reservoir of under consideration. To derive a correlation for equivalent length, a fictitious numerical example has been employed. The following numerical values have been used.

L=1000 ft; a=10000 ft; Bo=1.355;
μ=1.7 cp; ko=15 md; h=80 ft.

Based on the Eqs. 10 and 11, above numerical values the plot of productivity indices ratio versus number of branches of multilateral well is shown in Fig. 7.

of a multilateral well by increasing number of branches with respect to horizontal well is computed and plotted (Fig. 8).

$$\text{IPI=Increase in PI}=100\times\frac{PI_M-PI_H}{PI_H}$$

Incremental Increase in PI=IPIi-IPI (i-1)

Where IPIi and IPI (i-1) are increase in PI of a multilateral well with respect to a horizontal well with i and i-1 branches respectively.

As depicted in Figs. 7 & 8, beyond the fourth branch, additional branches have no effect on the well performance. On the other hand, for the above-mentioned numerical values, the productivity indices ratio approaches a constant value of about 1.31. This means that even for an infinite-branch multilateral well, there is a horizontal well with longer length than the length of each individual branch of the multilateral well that give the same performance. This longer length is named equivalent length. The equivalent length of a single-branch horizontal well (Le) that gives the same performance of a multilateral well is calculated by using the *Joshi* horizontal well performance equation (Eq. (12)) and by utilizing a trial and error procedure and plotted against the number of branches of a multilateral well (Fig. 9).

$$q_1^*/\Delta p = \frac{2\pi k_0 h/\mu B_0}{\ln\left[\dfrac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]} \qquad (12)$$

To derive a formula to fit the data that shown in ‘interference data’ curve of Fig. 9, by referring to the figure and from previous results, one can deduce the following points: 1) The equivalent length of whole branches is equal to the sum of the equivalent length of individual branches. 2) By increasing the number of branches, the equivalent length of individual branches decreases, and 3) The equivalent length of individual branches decreases exponentially.

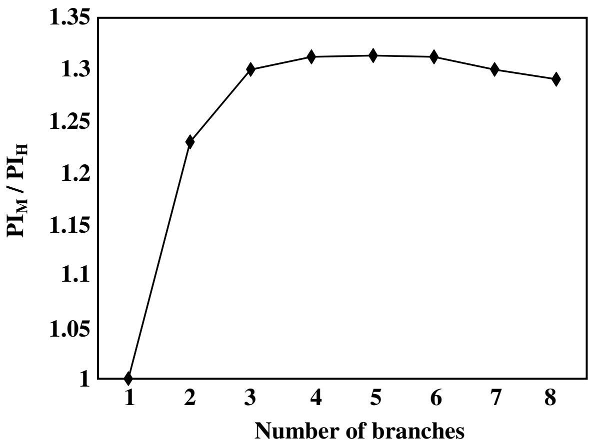

Fig. 7: Comparison of multilateral well performance vs. number of branches.

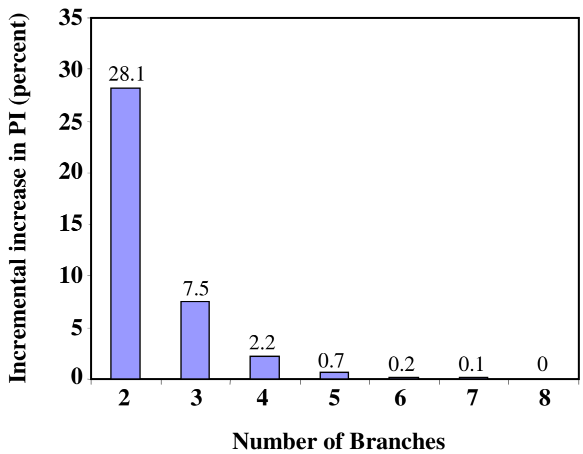

Fig. 8: Incremental increases in PI or flow of a multilateral well with respect to a horizontal well.

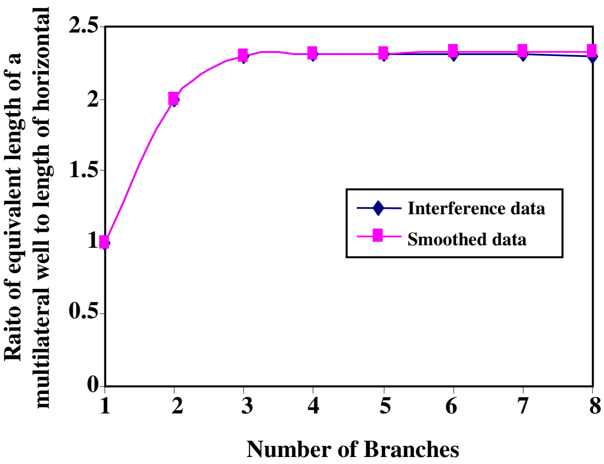

Fig. 9: Equivalent length of a multi lateral well in comparison with horizontal well.

Because in the general multilateral well performance correlations (Eqs. (10) and (11)), the length of branches is multiplied by a Sin function, it is acceptable that one assumes the equivalent length formula has a term containing a Sin function.

Keeping in mind the above points and by utilizing a trial and error procedure the following correlation has been obtained for estimating the equivalent length of a multilateral well:

$$L_e = L\left\{1+\sum_{i=2}^{n}\left[e^{-(i-2)}\sin(180/i)\right]\right\} \tag{13}$$

The equivalent length of a horizontal well obtained from Eq. (13) is shown in Fig. 9 in conjunction with that of calculated from Eqs. (10) or (11) which are in good agreement.

In above equation, it is assumed that the length of all branches is the same but in general, it is not true. The general form of the Eq. (13) may be written as:

$$L_e = L_1 + \sum_{i=2}^{n} L_i\left[e^{-(i-2)}\sin(180/i)\right] \tag{14}$$

Where Li is the length of the ith branch of well.

Performance of a Multilateral Well

Up to now, it has been worked with multilateral well in a horizontal plane. To extend the approach to a multilateral well in both horizontal and vertical planes, one may utilize the concept of equivalent length and the Joshi equation for horizontal well. *Joshi* presented the Eq. (1) for estimating the performance of a horizontal well by substituting equivalent length in above equation; the general multilateral well performance correlation will be obtained:

$$q_M = \frac{2\pi k_0 h\Delta p/(\mu B_0)}{\ln\left[\dfrac{a+\sqrt{a^2-(L_e/2)^2}}{(L_e/2)}\right]+\dfrac{h}{L_e}\ln\left(\dfrac{h}{2r_w}\right)} \tag{15}$$

$$\text{for } L_e > h, \text{ and } (L_e/2) < 0.9 r_e$$

Where q_M is flow rate of multilateral well, Le should be computed from Eq. (14), and all other variables are same as Joshi equation.

If one considers a multilateral well with infinite equal branches, it may deduce that its performance (or flow rate) will be the same as a vertical well with rw equal to length of each branch L. But with following data, PI of an infinite-branch multilateral well would be only 78 percent of that of a vertical well with radius equal to each branch of the multilateral well (calculations are followed). This inequality is due to the effect of incomplete penetration of multilateral in the vertical plane.

Viscosity = 1.7 cp, Bo = 1.355 res. bbl/STB, h = 80 ft, k = 15 md, L = 1000 ft, re ≈ a = 10000 ft, rw = 0.25 ft.

CALCULATIONS

Le: as number of branches approaches infinity, from Eq. (14) or (15), Le=2457ft;

From Eq. (15), PI of a multi-lateral well is computed from following equation:

$$PI_M = \frac{0.000708k_0h/(\mu B_0)}{\ln\left[\frac{a+\sqrt{a^2-(L_e/2)^2}}{(L_e/2)}\right]+\frac{h}{L_e}\ln\left(\frac{h}{2r_w}\right)} \quad (16)$$

By using Eq. (16) and above numerical values, PI_M =1.25 bbl/day/psi

From productivity index definition and PI_v =1.6 bbl/day/psi

$$\frac{PI_M}{PI_V} = \frac{1.25}{1.6} = 0.78$$

ECONOMIC ANALYSIS OF A MULTILATERAL WELL

In engineering economic studies, rate of return on investment is ordinary expressed on an annual percentage basis. The yearly profit divided by the total initial investment necessary represents the fractional return, and this fraction times 100 is the standard percent return on investment. Another rate of return that is helpful in this project is rate of return based on discounted cash flow. The method of approach for profitability evaluation by discounted cash flow takes into account the time value of money and is based on the amount of the investment that is unreturned at the end of each year during the estimated life of the project. A trail-and-error procedure is used to establish a rate of return which can be applied to yearly cash flow so that the original investment is reduced to zero during the project life. Thus, the rate of return by this method (which is named interchangeably interest rate of return or internal rate of return both are abbreviated by IRR) is equivalent to the maximum interest rate at which money could be borrowed to finance the project under conditions where the net cash flow to the project over its life would be just sufficient to pay all principal and interest accumulated on the outstanding principal [14]. The correlations for this method may summarize as:

$$FC = \sum_{n=1}^{N} I_n \frac{1}{(1+i)^n} \quad (17)$$

Where FC = initial investment or capital cost,

I_n = yearly net income,

i = interest rate of return (IRR); the amount of IRR should be obtained by trial-and-error,

n = year of project life to which cash flow applies,

N = whole project life.

If during the project life the yearly net income is constant, then the Eq.17 reduces to the following:

$$FC = I\frac{(1+i)^N - 1}{i(1+i)^N} \quad (18)$$

Where I = constant yearly net income, other terms are the same as before.

Application of IRR to Multilateral Well

It has been shown that in a multilateral well, by increasing the number of horizontal branch, a limited incremental increase in PI or in its flow rate will be obtained (Fig. 7). The net income of each branch may be estimated from the following expression:

$$I_b = 365q_h q_{in} p_o \quad (19)$$

Where I_b = yearly net income of each branch; $US

q_h = horizontal well flow rate; (STB/day),

q_{inc}= incremental increase of a multi-lateral well corresponding to each branch with respect to horizontal well; fraction,

P_o = oil price; $US/STB.

By ignoring the common expenditures, such as drilling up to point of deviation from vertical drilling, and assuming the cost of drilling of deviated section of a horizontal well equal to Ch, the cost of drilling of other branches of a multilateral well may be considered a coefficient greater than 1, fs, multiplied by Ch. The magnitude of this coefficient depends on the several factors; however, it is expected that the cost of subsequent branches increase non-linearly. Based on the above points the following correlation may be proposed

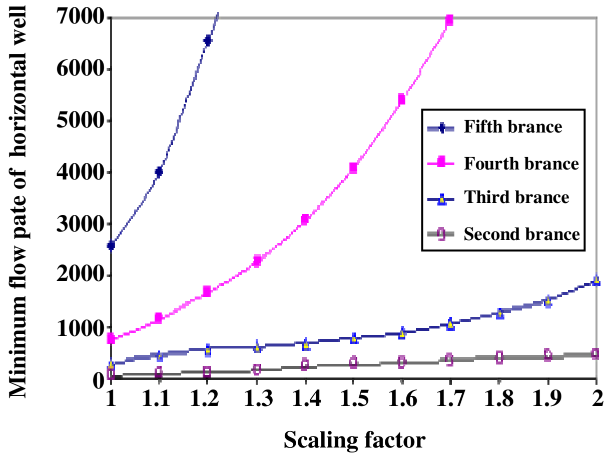

Fig. 10: Minimum flow rate of Horizontal well that make drilling a branch of multilateral well feasible.

for estimating cost of a drilling of each subsequent branch of a multi-lateral well:

$$C_n = (f_s)^n C_h \tag{20}$$

Where f_s = scaling coefficient, greater than 1,

Cn = cost of drilling the nth branch of a multi-lateral well.

Substitute the Eqs. (20) and (19) in Eq. (18),

$$(q_h)_{min} = \frac{(f_s)^n C_h}{365 q_{inc} P_o \dfrac{(1+i)^N - 1}{i(1+i)^N}} \tag{21}$$

By knowing the other variables, one can compute the minimum horizontal well flow rate which is feasible to invest for drilling the corresponding branch of a multilateral well,

$$(q_h)_{min} = \frac{(f_s)^n C_h}{365 q_{inc} P_o \dfrac{(1+i)^N - 1}{i(1+i)^N}} \tag{22}$$

To calculate the minimum horizontal well flow rate to make feasible the drilling of other branches, a numerical example is presented in Fig. 10. Because of lack of information about the scaling coefficient, a sensibility analysis is made.

In Fig. 10, $(q_h)_{min}$ is drawn versus scaling coefficient for branch numbers 2 to 5, with following assumptions:

C_h = cost of drilling the deviated section of horizontal well = $US 500,000,

q_{inc} = incremental increase of a multi-lateral well corresponding to each branch with respect to horizontal well; fraction (from Fig. 8),

P_o = oil price = $US 15,

i = interest rate of return (IRR) = 0.15

N = whole project life = 10 years.

The Fig.10 shows that with scaling factor equal to 1.3 the minimum flow rate of a horizontal well should be 109 STB/D, 533 STB/D, and 2362 STB/D for branch number 2, 3, & 4 respectively; otherwise drilling the corresponding branch would not be feasible.

DISCUSSION

The model developed in this paper is initially based on the two restricted assumptions: 1) Constant pressure at the reservoir boundary, and 2) Constant permeability throughout reservoir (isotropic reservoir). However, the concept of equivalent length was applied in this work; therefore its applicability would be broadened to any type of reservoirs and any boundary conditions. As pointed early, it means a multi-lateral well may be considered as several horizontal wells flowed into a common well string; therefore, the concepts and limitations of a horizontal well are also applicable to a multi-lateral well; so, knowing these concepts and limitations can be helpful in planning the program of drilling a multi-lateral well.

The variables that greatly affect the performance of a horizontal well are the thickness of pay zone, h ; and the ratio of vertical to horizontal permeability, kv/kh. To show effects of these two variables, the Productivity Indices ratio of horizontal to vertical wells (JH/JV) has been utilized, as Joshi pointed out. Fig. 11 shows the productivity indices versus horizontal well length for different thickness of reservoir. As it is obvious from this Figure, horizontal wells are more effective in thin reservoirs than thick reservoirs. In other words, the productivity gain of a horizontal well over a vertical well is very low in thick reservoirs.

Influence of reservoir anisotropy on horizontal and vertical well Productivity Indices ratio is shown in Figure 12. This Figure shows that the low vertical permeability significantly reduces horizontal well productivity. If one considers that vertical permeability is greatly affected by vertical fracturing and especially by density of vertical fractures, he may deduce that the lower vertical fracturing density results the lesser horizontal well productivity over a vertical well.

The same effects can be predicted for a multi-lateral well productivity. Therefore, before planning for any future multilateral well drilling, an extensive study about reservoir thickness and vertical fracturing should be run.

It has been shown in this paper that the performance of a multilateral well strongly depends on the angles between branches and its performance maximizes when angles between all sequential branches are equal. It results that in a multilateral well each pair of branches have the same effect on the increasing the well performance and this is the reason of squaring the brackets in the numerator of "Ln" terms in the denominator of Eqs. (10) and (11).

By a numerical example, it has been shown that improvement of performance of a multilateral well can be achieved up to fourth to fifth branches and drilling other branches has not any effect on the well performance.

The productivity indices ratio of multilateral to horizontal well may obtain by Eq. (23).

$$J_M/J_H = \frac{\ln\left[\frac{a+\sqrt{a^2-(L/2)}}{(L/2)}\right]+\frac{h}{L}\ln\left(\frac{h}{2r_w}\right)}{\ln\left[\frac{a+\sqrt{a^2-(L_e/2)}}{(L_e/2)}\right]+\frac{h}{L_e}\ln\left(\frac{h}{2r_w}\right)} \quad (23)$$

The plot of Productivity Indices ratio (Eq. (23)) versus branch-length (L) is shown in Fig. 13. This figure obviously shows that the multilateral well performance increases by increasing the branch-length; also it reveals that at constant branch-length, the improvement gain of the well performance is negligible over the fourth-branch of a multi-lateral well.

As shown in this paper, in some cases drilling the second branch may also become non feasible. So it is strongly recommendable that before planning to drill any multi-lateral well, an economic analysis should be run for the case of under consideration.

Mathematical models used for predicting horizontal well productivity can be classified into three categories: simple analytical solutions, sophisticated analytical models, and Numerical models. Simple analytical solutions derived in late 1980's and early 1990's based on the assumption of infinite drain hole conductivity; Sophisticated analytical models developed after 1990's for drain holes of finite conductivity; and finally

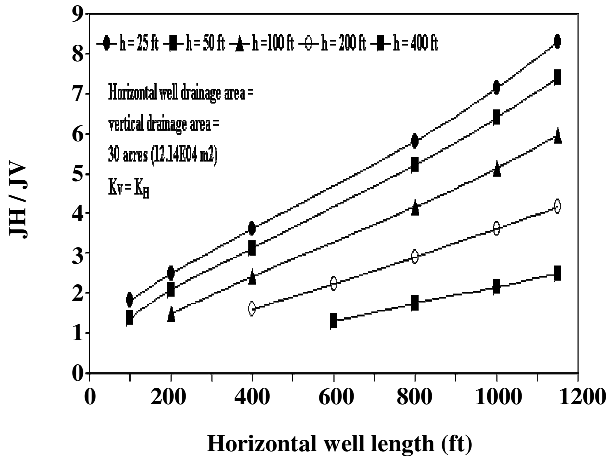

Fig. 11: Influence of reservoir height on horizontal and vertical well productivity index ratio.

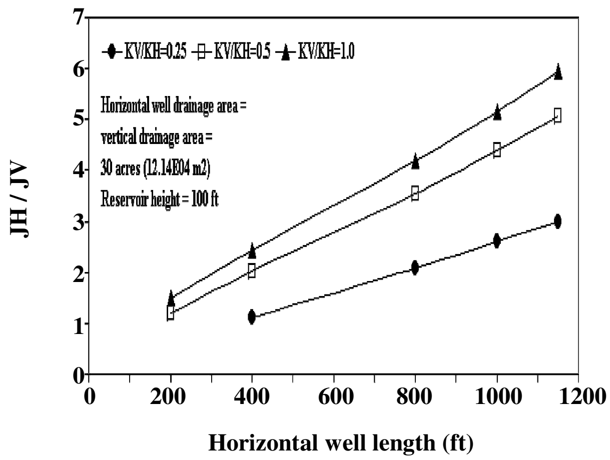

Fig. 12: Influence of reservoir anisotropy on horizontal and vertical well productivity

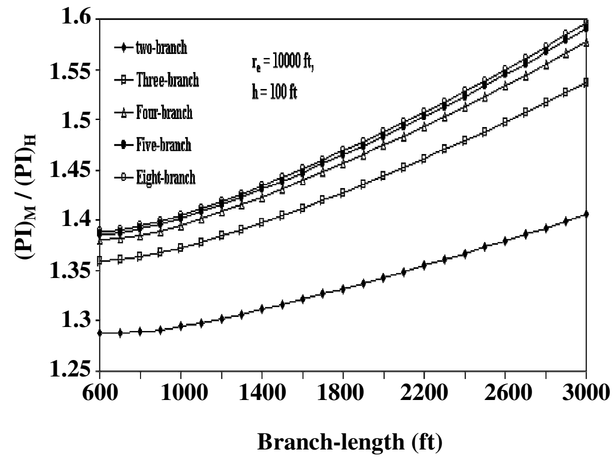

Fig. 13: Effect of branch length on the multilateral performance.

numerical models that considering wellbore hydraulics. This work is based on a simple analytical model and represents inflow performance modeling of multilateral wells by employing the concept of well interference and the joshi's expression. Constant pressure at the drainage boundary and at the wellbore is the assumption of this work. Few works about the multilateral well performance may be found in literature and in these few works no unique expression has been presented for estimating multilateral well performance. Nevertheless, the results obtained in this work are in good agreement with works of *Salas et al.* and *Larsen* that both employed the concept of pseudo-radial skin factor:

- *Larsen*, after presenting an example, deduced that the improvement from 4 to 6 branches is negligible, and even the improvement from 3 to 6 branches marginal. This is in full agreement with results obtained in this work (refer to Figs. 7 and 8).
- *Salas et al.* concluded that in general, drilling fewer multilateral branches, with the tips of the branches at maximum spacing from each other, would give the greatest productivity for the least drilled length. This result is equivalent with the results obtained in this work that state for maximizing the productivity index of a multilateral well, all branches should be drilled around a circular ring at equal angles which in turn maximizes spacing of the tips of branches from each other.

CONCLUSIONS

Based on the results presented here the following conclusions are made:

1. By utilizing the Joshi equation for horizontal well performance and the concept of well-interference, expressions for predicting the multilateral well performance are presented.
2. By utilizing the concept of equivalent length of a multilateral well, all types of horizontal well performance equations may be employed for predicting the performance of a multilateral well in the reservoir model under consideration.
3. Drilling the fourth and higher branch has negligible effect on the improvement of a multilateral well.
4. As in horizontal well, the performance of a multi-lateral well depends on the thickness of pay zone and the extent of vertical fracturing in an anisotropy reservoir.
5. By utilizing the concept of equivalent length, the number of well in a cluster-well can be optimized.
6. In this paper, in order to modeling the inflow performance of multilateral wells, Joshi expression for horizontal well performance was considered. The concept of well interference also used in this work. Finally, two correlations for estimating the multilateral well performance with odd and even branches have been presented. Since this work is extended of *Joshi*'s work, the later method will discuss in this paper.
7. In some cases, drilling of second branch may not economically be feasible. Therefore, economic analysis of the case under consideration, before planning drilling a multilateral well is strongly recommended.

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Appendix A

Productivity Index of a three-branch multi-lateral well in horizontal plane

Schematic diagram of a three-branch multi-lateral well is shown in Fig. A-1. Pressure drop in third branch is the resultant of pressure drops of flow in the first and second branches and flow into itself.

$$\Delta p_3 = \left(p_e - \left(\Delta p_1 + \Delta p_2\right)\right) - p_w \tag{A-1}$$

Where:

Δp_3= total pressure drop in third branch,

Δp_2= pressure drop in third branch due to interference of the second branch flow,

Δp_1= pressure drop in third branch due to interference of the first branch flow,

P_e- $(\Delta p_1+\Delta p_2)$ = static pressure in third branch.

In following expressions, Δp_1 and Δp_2 are computed and substituted in Eq. A-1:

$$\Delta p_1 = p_e - p_{d1} = \frac{q_1 \mu B_o}{2\pi k_o h} \ln\left[\frac{a + \sqrt{a^2 - (d1)^2}}{d1}\right] \tag{A-2}$$

Where $d1=2L\sin(\alpha/2)$ is the maximum distance between third and first branch.

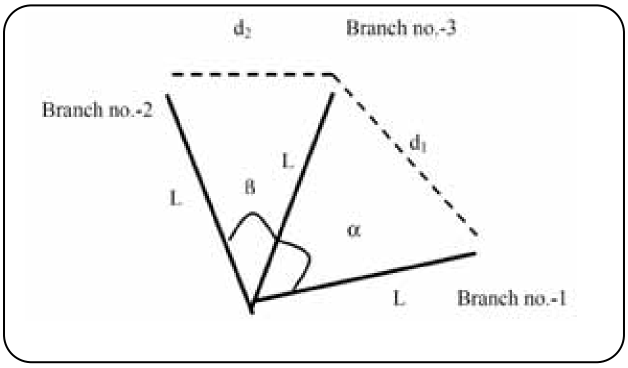

Fig. A-1: Schematic Diagram of a three-branch multi-lateral well.

$$\Delta p_1 = p_e - p_{d1} = \frac{q_1 \mu B_o}{2\pi k_o h} \ln\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\alpha/2\right)\right)^2}}{2L\sin\left(\alpha/2\right)}\right] \tag{A-3}$$

Pressure drop due to interference of second branch is obtained from Eq. (A-4):

$$\Delta p_2 = p_e - p_{d2} = \frac{q_2 \mu B_o}{2\pi k_o h} \ln\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\beta/2\right)\right)^2}}{2L\sin\left(\beta/2\right)}\right] \tag{A-4}$$

Therefore, the total pressure drop in third branch can be obtained by substituting equations (A-3) and (A-4) in Eq. (A-1):

$$\Delta p_3 = (p_e - (\Delta p_1 + \Delta p_2)) - p_w = p_e - \left(\frac{q_1 \mu B_o}{2\pi k_o h} \ln\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\alpha/2\right)\right)^2}}{2L\sin\left(\alpha/2\right)}\right]\right) - \left(\frac{q_2 \mu B_o}{2\pi k_o h} \ln\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\beta/2\right)\right)^2}}{2L\sin\left(\beta/2\right)}\right]\right) - p_w \tag{A-5}$$

But from Joshi work

$$\Delta p_3 = \frac{q_3 \mu B_o}{2\pi k_o h} \ln\left[\frac{a+\sqrt{a^2-\left(L/2\right)^2}}{L/2}\right] \tag{A-6}$$

Where q_1, q_2, and q_3 are individual flow rate of first, second, and third branches respectively.

Combine Eqs. (A-5) and (A-6):

$$\left(\frac{q_3 \mu B_o}{2\pi k_o h} \ln\left[\frac{a+\sqrt{a^2-(L/2)^2}}{(L/2)}\right]\right) = p_e - \left(\frac{q_1 \mu B_o}{2\pi k_o h} \ln\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\alpha/2\right)\right)^2}}{2L\sin\left(\alpha/2\right)}\right]\right) - \left(\frac{q_2 \mu B_o}{2\pi k_o h} \ln\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\beta/2\right)\right)^2}}{2L\sin\left(\beta/2\right)}\right]\right) - p_w \tag{A-7}$$

Rearrange Eq. (A-7):

$$p_e - p_w = \left(\frac{q_3 \mu B_o}{2\pi k_o h} \ln\left[\frac{a+\sqrt{a^2-(L/2)^2}}{(L/2)}\right]\right) + \left(\frac{q_1 \mu B_o}{2\pi k_o h} \ln\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\alpha/2\right)\right)^2}}{2L\sin\left(\alpha/2\right)}\right]\right) + \left(\frac{q_2 \mu B_o}{2\pi k_o h} \ln\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\beta/2\right)\right)^2}}{2L\sin\left(\beta/2\right)}\right]\right) \tag{A-8}$$

Assume flow rate of all branches are equal ($q_w/3=q_1=q_2=q_3$):

$$p_e - p_w = \frac{(q_w/3)\mu B_o}{2\pi k_o h}\left\{\ln\left[\frac{a+\sqrt{a^2-(L/2)^2}}{(L/2)}\right] + \ln\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\alpha/2\right)\right)^2}}{2L\sin\left(\alpha/2\right)}\right] + \ln\left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\beta/2\right)\right)^2}}{2L\sin\left(\beta/2\right)}\right]\right\} \tag{A-9}$$

$$p_e - p_w = \frac{(q_w/3)\mu B_o}{2\pi k_o h}\ln\left\{\left[\frac{a+\sqrt{a^2-(L/2)^2}}{(L/2)}\right] \times \left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\alpha/2\right)\right)^2}}{2L\sin\left(\alpha/2\right)}\right] \times \left[\frac{a+\sqrt{a^2-\left(2L\sin\left(\beta/2\right)\right)^2}}{2L\sin\left(\beta/2\right)}\right]\right\} \tag{A-10}$$

From definition of productivity index $PI = q_w/\Delta p$ and Eq. A-10, productivity index of a three-branch multi-lateral well is obtained from Eq. A-11.

where $M = 2\left(\frac{2\pi k_o h}{\mu B_o}\right)$ and $N = \ln\left[\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]$.

$$(PI)_3 = \frac{3(2\pi k_o h/\mu B_o)}{\ln\left\{\left[\frac{a+\sqrt{a^2-(2L\sin(\alpha/2))^2}}{2L\sin(\alpha/2)}\right]\left[\frac{a+\sqrt{a^2-(2L\sin(\beta/2))^2}}{2L\sin(\beta/2)}\right]\left[\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]\right\}} \quad \text{(A-11)}$$

Appendix B

Maximizing the Productivity Index of a two-branch multi-lateral well

The following expression has been derived for productivity index of a two-branch multi-lateral well in modeling approach section:

$$(PI)_2 = \frac{2(2\pi k_o h/\mu B_o)}{\ln\left\{\left[\frac{a+\sqrt{a^2-(2L\sin(\alpha/2))^2}}{2L\sin(\alpha/2)}\right]\left[\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]\right\}} \quad (8)$$

But there is a question about the optimum angle between two branches for maximizing the PI. To answer this question one can obtain the first derivative of Eq. (8) with respect to sin(α) and by equating it to zero calculate the optimum α.

Change the Eq. (8) to simpler form:

$$(PI)_2 = \frac{2(2\pi k_o h/\mu B_o)}{\left\{\ln\left[\frac{a+\sqrt{a^2-(2L\sin(\alpha/2))^2}}{2L\sin(\alpha/2)}\right]+\ln\left[\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]\right\}} \quad \text{(B-1)}$$

Because the derivative would be with respect to sin(α), the terms that do not contain trigonometric functions are substituted with constants:

$$(PI)_2 = \frac{M}{\left\{\ln\left[\frac{a+\sqrt{a^2-(2L\sin(\alpha/2))^2}}{2L\sin(\alpha/2)}\right]+N\right\}} \quad \text{(B-2)}$$

Take derivative of Eq. (B-2) with respect to sin(α):

$$\frac{d(PI)_2}{d(\sin(\alpha))} = \quad \text{(B-3)}$$

$$\frac{-M*d\left(\ln\left[\frac{a+\sqrt{a^2-(2L\sin(\alpha/2))^2}}{2L\sin(\alpha/2)}\right]\right)\Big/d(\sin(\alpha))}{\left\{\ln\left[\frac{a+\sqrt{a^2-(2L\sin(\alpha/2))^2}}{2L\sin(\alpha/2)}\right]+N\right\}^2}$$

By equating the numerator of Eq. (B-3) to zero, the derivative will become zero; so, the derivative of numerator of Eq. (B-3) is obtained in following paragraph.

From the algebra, the derivative of natural logarithm is obtained from following expression:

$$\frac{d\ln(u)}{dx} = \frac{1}{u}\frac{du}{dx} \quad \text{(B-4)}$$

In Eq.(B-3), $u = \left[a+\sqrt{a^2-(2L\sin(\alpha/2))^2}\Big/2L\sin(\alpha/2)\right]$

and x= sin(α), therefore:

$$\frac{du}{dx} = \frac{2L\sin(\alpha/2)*d\left[a+\sqrt{a^2-(2L\sin(\alpha/2))^2}\right]}{(2L\sin(\alpha/2))^2} - \quad \text{(B-5)}$$

$$\frac{d(2L\sin(\alpha/2))*\left[a+\sqrt{a^2-(2L\sin(\alpha/2))^2}\right]}{(2L\sin(\alpha/2))^2}$$

Compute each derivative terms in Eq. (B-5):

$$\frac{d\left[a+\sqrt{a^2-(2L\sin(\alpha/2))^2}\right]}{d(\sin\alpha)} = \quad \text{(B-6)}$$

$$\frac{-2L^2\sin(\alpha/2)\cos(\alpha/2)}{\left[a+\sqrt{a^2-(2L\sin(\alpha/2))^2}\right]}$$

$$\frac{d\left[2L\sin(\alpha/2)\right]}{d(\sin\alpha)} = L\cos(\alpha/2) \tag{B-7}$$

$$\frac{du}{dx} = \frac{-4L^3\sin^2(\alpha/2)\cos(\alpha/2)}{\left(4L^2\sin^2(\alpha/2)\right)} - \frac{\left(L\cos(\alpha/2)\right)\times\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]}{\left(4L^2\sin^2(\alpha/2)\right)} \tag{B-8}$$

Substitute equations (B-6) and (B-7) in Eq. (B-5):

Or in simpler form:

$$\frac{du}{dx} = \frac{-4L^2\sin^2(\alpha/2)\cos(\alpha/2)}{\left(4L\sin^2(\alpha/2)\right)} - \frac{\left(L\cos(\alpha/2)\right)*\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]}{\left(4L\sin^2(\alpha/2)\right)} \tag{B-9}$$

Compute $\frac{1}{u}\frac{du}{dx}$:

$$\frac{1}{u}\frac{du}{dx} = \frac{-4L^2\sin^2(\alpha/2)\cos(\alpha/2)}{\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]\left(4L\sin^2(\alpha/2)\right)} - \frac{\left(L\cos(\alpha/2)\right)\times\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]}{\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]\left(4L\sin^2(\alpha/2)\right)}\times\left(2L\sin(\alpha/2)\right) \tag{B-10}$$

Simplify the Eq. B-10:

$$\frac{1}{u}\frac{du}{dx} = \frac{-4L^2\sin^2(\alpha/2)\cos(\alpha/2)-}{\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]\left(2\sin(\alpha/2)\right)} - \frac{\left(L\cos(\alpha/2)\right)*\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]}{\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]\left(2\sin(\alpha/2)\right)} \tag{B-11}$$

The Numerator of Eq. B-3 is $-M\times\frac{1}{u}\frac{du}{dx}$, substitute Eq. (B-11) and the magnitude of M and equate to zero; the following expression for condition of maximizing the productivity index of a two-branch multi-lateral well will be obtained:

$$-M\times\frac{1}{u}\frac{du}{dx} = \frac{4L^2\sin^2(\alpha/2)\cos(\alpha/2)}{\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]\left(2\sin(\alpha/2)\right)} + \frac{\left(L\cos(\alpha/2)\right)\times\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]}{\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]\left(2\sin(\alpha/2)\right)}\times M = 0 \tag{B-12}$$

In Eq. (B-12) M is a non-zero constant; therefore, the remaining terms should be equal to zero.

$$4L^2\sin^2(\alpha/2)\cos(\alpha/2) + \left(L\cos(\alpha/2)\right)\times\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right] = 0 \tag{B-13}$$

$$L\cos(\alpha/2)\left(4L\sin^2(\alpha/2)+\left[a+\sqrt{a^2-\left(2L\sin(\alpha/2)\right)^2}\right]\right) = 0 \tag{B-14}$$

But the parenthesis has always a positive non-zero value and also L is always greater than zero; therefore, $\cos(\alpha/2)$ should be equal to zero.

$$\cos(\alpha/2) = 0 = \cos(2n+1)\frac{\pi}{2} \quad , n=0,1,2,3,\ldots \tag{B-15}$$

Therefore,

$$\frac{\alpha}{2} = (2n+1)\frac{\pi}{2} \tag{B-16}$$

or

$$\alpha = (2n+1)\pi \tag{B-17}$$

Thus, the optimum angle between branches of a two-branch multi-lateral well is π radians or 180 degrees.

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