Optimal Control of Nonlinear Multivariable Systems

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ABSTRACT: *This paper concerns a study on the optimal control for nonlinear systems. An appropriate alternative in order to alleviate the nonlinearity of a system is the exact linearization approach. In this fashion, the nonlinear system has been linearized using input-output feedback linearization (IOFL). Then, by utilizing the well developed optimal control theory of linear systems, the compensated nonlinear system has been controlled. Thus, the structure of the objective function will be converted into a quadratic form which is qualitativly comparable with usual cost functions, and from operating viewpoint is more favorable. To qualify such a procedure, it has been applied to two minimum and nonminimum-phase chemical processes, and its performance is verified through computer simulations.*

KEY WORDS: *Optimal control, Multivariable control, Input-qutput feedback linearization.*

INTRODUCTION

Over the years, the optimal control of dynamic systems has received a growing attention from researchers [1, 2]. During these researches, various approches have been developed to deal with practical processes. These approches mainly can be devided into three categories; decentralized architectures, interacting MPC based methodologies, and hybrid approaches. The decetralized architectures are the most straightforward approaches which can be designed using RGA or Neiderlinski index criteria. However, for highly integrated systems, the optimal pairing may become a challenging problem. On the other hand, MPC based methods are capable to predict the behavior of the system, but solving the resulting nonlinear program to find the global optimum is their main drawback. This urges us to simplify the structure of the problem. Transforming the original system into a structure which

can be analyzed through a more flexible, yet powerful farmework is an appropriate choice to compensate the nonlinearity of the system. Based on this idea, hybrid approches emerge. Linearization seems to be an appropriate alternative which transforms the original nonlinear system into a linear one. This way, the linear optimal control tools could be used to control the system.

The hope to eliminate nonlinear behavior of such systems led to solutions for exact input-state linearization [3], state feedback decoupling [4], and input-output feedback linearization [5, 6] based on a differential geometric framework. By developing theoretical basis of input-output feedback linearization (IOFL) in 1980s, because of the limitations which restricted the linearizable systems to square, minimum phase and non-singular ones, the approximate linearization methods seem to be more attractive to reaserchers (a survey on

various approximate approaches is performed by *Guardabassi* and *Savaressi* [7]). Parallel to developing approximate approaches, the exact methods progressed to the point that now it is possible to return to exact methods with a more comprehensive knowledge about systems. To date, the previous tight conditions are alleviated and the new theorems and results for nonminimum-phase [8-10], non-square [11-13] and singular [14] systems have been derived. These attempts paved the way to redefine linearizability conditions to cover a wide range of systems. Hence, coupling a nonlinear system with a compensator to make the overall system linear is an alternative in order to achieve control objectives. The linearization provides new coordinates in which the behavior of multivariable system is simpler and more predictable. Furthermore, we can utilize well-developed linear control theory tools [15, 16] to control new system.

In this work, we will study the optimal control of nonlinear systems via input-output feedback linearization; so, the rest of this paper is organized as follows:

In section 2, we introduce the concepts of relative order and characteristic matrix as the principal differential geometric definitions for nonlinear systems. Then, these definitions are used in section 3 to present input-output feedback linearization for minimum-phase and nonminimum-phase systems. This is because of the fact that the major restriction for linearization of nonlinear systems is the minimum-phaseness condition . It will be followed in section 4, with a topic on optimal control theorem for linear systems. In section 5 the control methodology has been described. Simulation results for both a minimum-phase and a non-minimum-phase systems along with the concluding remarks are covered in section 6.

MATHEMATICAL PRELIMINARIES

Consider the continuous-time state space representation of a nonlinear multivariable system as:

$$\dot{x} = f(x) + \sum_{i=1}^{m} u_i g_i(x) \; ; \; y = (h(x) \tag{1}$$

Where

$$u = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} \quad , \quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad , \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}$$

Denote inputs ($u \in U \subset R^m$), states ($x \in X \subset R^n$) and outputs, respectively. It is assumed that the vector fields $f, g_i : X \rightarrow R^n$ and $h : X \rightarrow R^m$ are real analytic and smooth functions whose number of inputs and outputs are equal (square system). In the literature, there are various cases in which the system is assumed to be square. It is due to the fact that the major part of the linearization theorems is developed upon this foundation. Some other analogous definitions and equations have been derived that take into account non-square systems [13]. Since the assumption of squareness makes no restriction to our work, for the sake of simplicity and clarity we proceed with square MIMO systems. We start by reviewing definitions of relative order and characteristic matrix for nonlinear systems.

Definition 1: A nonlinear MIMO system of form (1) is said to have relative order of r_i with respect to output y_i if

$$L_g L_f^k h_i(x) \left[L_{g1} L_f^k h_i(x) \; \cdots \; L_{g_m} L_f^k h_i(x) \right] = 0 \tag{2}$$

$$K = 0, 1, \dots, r_i - 2$$

$$L_g L_f^k h_i(x) \left[L_{g1} L_f^k h_i(x) \; \cdots \; L_{g_m} L_f^k h_i(x) \right] \neq 0$$

$$K = r_i - 1$$

where r_i is the smallest integer k for which the vector $L_g L_f^k h_i(x)$ has at least one nonzero component. This definition shows that r_i is the smallest order of derivative of the output y_i which is explicitly affected by manipulated variables vector, u [17].

Definition 2: The characteristic matrix for a system of the form (1) with finite relative orders r_i is defined as:

$$C(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \dots & L_{g_m} L_f^{r_1-1} h_1(x) \\ \vdots & & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix} \tag{3}$$

This matrix plays an important role in multivariable control. It can be used for linearization as well as input-output decoupling. Due to the structure of this matrix, it depends on input and output properties of the system.

Based on the definitions for relative order and characteristic matrix, the systems that do not abide these conditions are classified as singular systems. For some systems whose output y_i does not have a well-defined

relative order, there is no dependence of y_i and its derivatives, and manipulated variables. This means that probably the problem formulation is not correct, and output y_i is not controllable. Another source of singularity which is of more concern is the singularity of the characteristic matrix. In this situation, the characteristic matrix is not invertible. As a result, the system is not feedback linearizable.

INPUT-OUTPUT EXACT LINEARIZATION

Kravaris and *Soroush* extended the GLC methodology to MIMO systems [5]. They derived necessary and sufficient conditions and studied the stability of the input-output linearized systems. Consequently, they developed a static-state feedback law as follows:

$$u\left\{\left[\beta_{1r_1}\cdots\beta_{mr_m}\right]L_g\begin{bmatrix}L_f^{r_1}h_1(x)\\\vdots\\L_f^{r_m-1}h_m(x)\end{bmatrix}\right\}^{-1}\left\{v-\sum_{i=1}^{m}\sum_{k=0}^{r_i}\beta_{ik}L_f^{k}h_i(x)\right\} \quad (4)$$

Whose corresponding closed-loop response is:

$$v=\sum_{i=1}^{m}\sum_{k=0}^{r_i}\beta_{ik}\frac{d^k y_i}{dt^k} \quad (5)$$

Based on the necessary and sufficient conditions, as the equation (4) shows, the system is input-output feedback linearizable if it possesses definite relative orders and non-singular characteristic matrix. Additionally, an input-output linearized system is internally stable if its zero dynamics is stable. In other words the system must be minimum-phase.

Linearization of non-minimum phase systems

As it has just been stated, the key restriction for input-output feedback linearization (IOFL) is the minimum-phaseness condition. For linear systems, it is possible to separate the minimum-phase and nonminimum-phase components from each other and use the factorized stable part to control the system. However, it cannot be a general methodology to be used for all systems. In the case of nonlinear systems, modifying the input and output variables of the system is a more suitable choice. Since, usually the number of manipulated variables are limited, the major effort is to select outputs as appropriate as possible to fit the system into the linearizability conditions.

Kravaris and *Soroush* showed it is possible to fit the system into the necessary conditions by modifying the outputs [5]. To do so, the original outputs should be substituted with statically equivalent ones to make the system minimum-phase. This way, *Niemiec* and *Kravaris* proposed an algorithm to synthesize new outputs [9]. They constructed vanishing manifolds on the equilibrium manifold of a nonlinear system to generate new outputs. In general, this set of vanishing manifolds is not unique, and there are infinite possible synthetic outputs for a certain system. The generated outputs are statically equivalent to the original ones, while during transitions they have distinct behavior. This causes an opportunity to assign desired transmission zeros to the system and make the overall system nonminimum-phase. The algorithm for producing vanishing manifolds as well as transmission zeros is outlined in [9]. The new outputs are in the form of equation (6).

$$h^*(x)=h(x)+\Lambda\eta(x) \quad (6)$$

Where, h and $\dot{h}$ represent the original and synthetic outputs, respectively. η is the vector of vanishing manifolds and Λ is the matrix of weighting parameters to place transmission zeros of linearized system to pre-assigned locations. The weighting parameters can be constant values or functions of state variables. In the former case, the corresponding Sylvester equation must be solved whose main drawback is being ill-condition or low-rank for high dimensional systems. For latter, the problem will be converted into a singular PDE system and can be solved only for moderate nonlinear systems, in a reasonable time.

OPTIMAL CONTROL OF LINEAR SYTSTEMS

Due to the simplicity of the linear time-invariant systems, in this direction the control theorem has been extended widely. Moreover, many numerical packages have been developed and optimized for such systems. Multi-model and IOFL are two examples of control structures which make use of linear systems theorem facilities. A linear system without direct affection of inputs on outputs can be showed as follows:

$$\dot{x}=Ax+Bu \quad ; \quad y=Cx \quad (7)$$

The major goal of converting systems into the linear format is to take advantage of the low computational cost.

Mathematically, an optimal control problem for linear systems can be solved efficiently if the objective function is in the following quadratic form:

$$J = \frac{1}{2} \int_{t_0}^{t_f} (\overline{x}^T Q \overline{x} + \overline{u}^T R \overline{u}) dt \tag{8}$$

where R, the weight of input vectors, is a positive definite matrix. Also, Q is the matrix of weight factors of the state variables. Furthermore, if other constraint and bounds are taken into account they must be of linear type.

INCENTIVES OF PROPOSED METHOD

Nowadays, the integrity is the essential property of chemical processes whose main drawback is difficulty in designing the control systems. This makes the use of multivariable control configurations a necessity to fulfill the control objectives. The proposed method is supposed to take advantage of both exact input-output linearization and linear optimal control theory to control nonlinear systems. This goal can be achieved in a two-step approach. First, the nonlinear system is linearized via IOFL. In the next step, this linear time-invariant system is controlled via the linear multivariable optimal control theory. Based on this idea, the structure of such a control configuration is illustrated in Fig. 1. The inner loop consists of a nonlinear compensator in which the nonlinear system is linearized using IOFL, and the outer loop is utilized to control this linear system.

Optimal setpoint tracking which is a kind of a fixed-endpoint problem [16] is an appropriate case-study to verify this configuration. In mathematical formulation, minimizing the functional subject to the dynamics of the linear system and end-point values is the main objective. This problem can be showed as follows:

$$\min \quad J = \frac{1}{2} \int_{t_0}^{t_f} (z^T Q z + v^T R v) dt \tag{9}$$

Subjet to $\quad \dot{z} = Az + Bv$; $z_0 = z(t_0)$; $z_f = z(t_f)$

where z is the vector of state variables, and v is the vector of manipulated variables of linear system in new coordinates.

Since the optimization problem in this form is a quadratic functional of virtual variables (v, z) which may have no physical meaning, the contemporary cost functions cannot be interpreted as the objective function. In fact, such a kind of function is a qualitative criterion
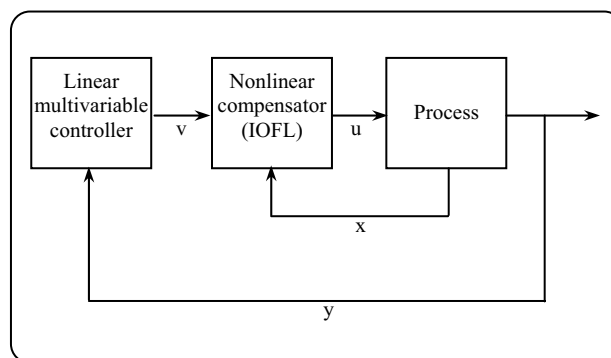

Fig. 1: The hybrid optimal control structure for nonlinear systems system.

which is more favorable from the operating viewpoint. With such a rationale, there is not much difference between using original and virtual variables. In this fashion, the optimal results for virtual variables can be considered as near-optimal solutions for the original system.

SIMULATION EXAMPLES

In this section, the proposed method is used to optimally control the transitions of nonlinear systems. We consider two nonlinear systems, one of which is a minimum-phase and the other one is a non-minimum-phase real chemical process. For both simulation examples the objective function has the formulation of equation (9). Specially, there are no other constraints on the systems. Both simulation studies were run on a Pentium IV with 1.73 GHz Dual Core CPU and 1 GB RAM.

Optimal Control for a minimum phase system

One of the most common and severely nonlinear polymerization reactions is free radical polymerization of methyl methacrylate (MMA) which takes place in a CSTR where Azo-bis-isobutyronitrile (AIBN) is the initiator and toluene is the solvent. The reaction is exothermic and its heat is removed through a jacket cooling system. As a major simplification it is assumed that the process is standard free-radical polymerization, no gel effect has been considered and the flow of initiator is negligible. The process consists of 2 inputs as well as 2 outputs. The manipulated variables, flow of initiator (F_I) and flow of cooling water (F_{cw}), have been used to control the number average molecular weight (D_I/D_0) and reactor temperature (T) at desired points.

More information about the dynamic model of the system and its parameters are given in appendix A. This system is minimum-phase, non-singular and consequently input-output feedback linearizable. Both outputs have relative order of 2 which means the overall linear system has 4 state variables. The decoupled closed loop response for this process is selected as follows [6]:
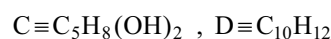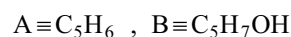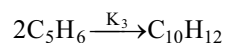
$$\frac{y(s)}{u(s)} = \begin{bmatrix} \dfrac{1}{0.016s^2 + 0.44s + 1} & 0 \\ 0 & \dfrac{1}{0.016s^2 + 0.44s + 1} \end{bmatrix} \quad (10)$$

The selected normal operating condition for the process is $T^e = 329.6C$ and $D_I/D_0 = 67348.1$. Increasing reactor temperature by 6 C and decreasing average molecular weight by 10000 units during 1 hour is the major goal. In this case, we have only considered the effect of input variables in the objective function (R=I, Q=0). In Figs. 2 through 7 the variations of linearized system inputs (v), process inputs (u) and process outputs (y) have been plotted. It should be noted that the inputs of linearized system have no physical meaning, and rapid changes in their values make no instability in the system. To prove the claim, it has been illustrated in Figs. 4 and 5, that the variations of process inputs (u) are completely mild and valid.

As it is expected this procedure is very fast, and calculations of the optimal trajectory for this case took 0.012 s which is incomparable with nonlinear optimal control algorithms [2].

Optimal Control for a non-minimum phase system

This example is an optimal control problem for a non-isothermal continuous stirred tank reactor which consists of parallel and serial reactions [18]. The details of the model and kinetic data are given in appendix B.

$$C_5H_6 \xrightarrow{k_1/+(H_2O)} C_5H_7OH \xrightarrow{k_2/+(H_2O)} C_5H_8(OH)_2$$

$$2C_5H_6 \xrightarrow{K_3} C_{10}H_{12}$$

$$A \equiv C_5H_6 \;,\; B \equiv C_5H_7OH$$

$$C \equiv C_5H_8(OH)_2 \;,\; D \equiv C_{10}H_{12}$$

The objective is to control the temperature and the molar concentration of component B at desired points by dilution rate (F/V) and heat duty (Q_H).
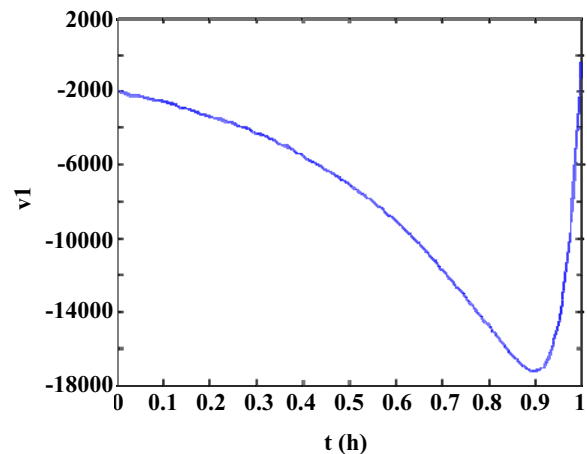

Fig. 2: Optimal trajectory for first input of linearized MMA process.
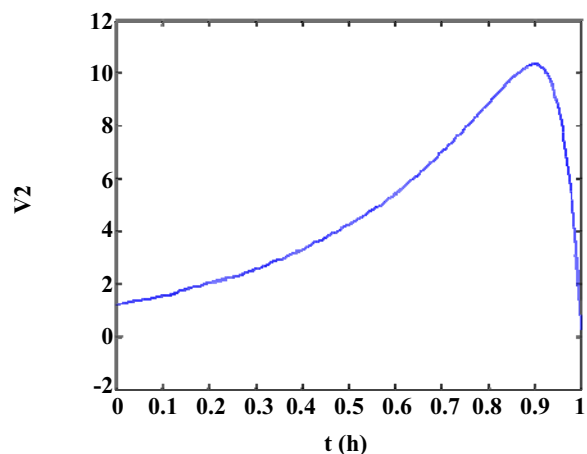

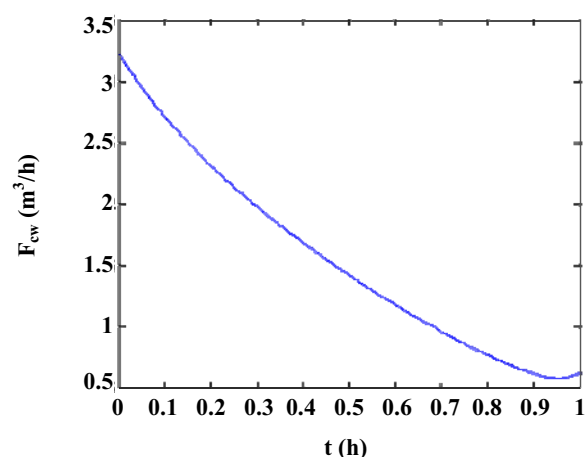*Fig. 3: Optimal trajectory for second input of linearized MMA process.*

Fig. 4: Optimal trajectory for cooling water flow in MMA process (First input).
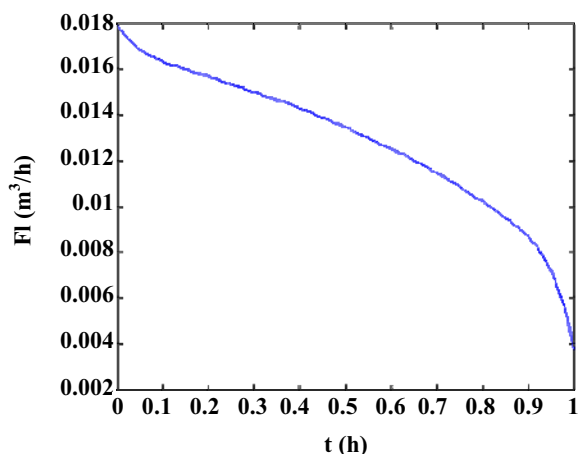
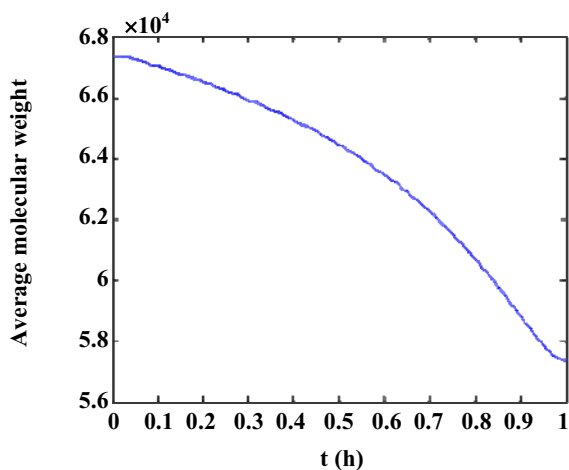**Fig. 5: Optimal trajectory for initiator flowrate in MMA process (Second input).**

Fig. 6: Optimal trajectory for average molecular weight in MMA process (First output).
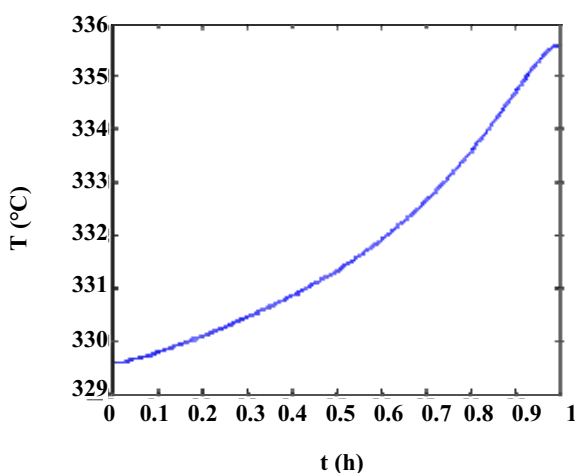

Fig. 7: Optimal trajectory for temperature in MMA process (Second output).

The initial operating conditions of the reactor are at $C_A^e = 1.25$ mol/L, $C_B^e = 0.9$ mol/L and $T^e = 407.15$ K while their equivalent manipulated variables are $u_1^e = 19.52 h^{-1}$ and $u_2^e = -451.51$ kjh^{-1}. The process is stable while it has one unstable zero at 122.71, [9]. By finding synthetic outputs for the system and placing transmission zeros at desired locations through the algorithm proposed by *Niemiec* and *Kravaris* [9], the new system is input-output feedback linearizable. Since the relative orders of both outputs of the system are 2, we select the following closed-loop response:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{s^2+2s+1} & 0 \\ 0 & \dfrac{1}{s^2+2s+1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \quad (11)$$

In this way the outputs are linearized and decoupled at the same time. Our objective is to take the system to the desired conditions which are $C_B^e = 0.8$ mol/L and $T^e = 390$ K during 10 hours. To do so, we have selected the identity matrix as the inputs and states weighting matrices (R=Q=I). Figs. 7 through 11 show the variations in virtual inputs (v), process inputs (u) and process outputs (y) during transitions.

The variations near the end of the time span have their greatest values. This is due to the fact that the objective function is concerned with the weighted values of input and state variables, while there is no constraint on their rate of changes. The size of the linearized system is the same as it was at the former simulation example. Therefore, for this case the calculations took a time of 0.012 seconds too.

CONCLUSIONS

A framework for the optimal control of nonlinear systems based on input-output feedback linearization has been presented. This framework consists of two steps, which are design of a nonlinear compenstaor based on input-output linearization theory used as the inner loop, and an external controller designed based on the optimal control theory. The formulations and algorithms required to linearize minimum-phase and non-minimum-phase system are reviewed. This leads to the definition of the linear-quadratic optimal control problem.
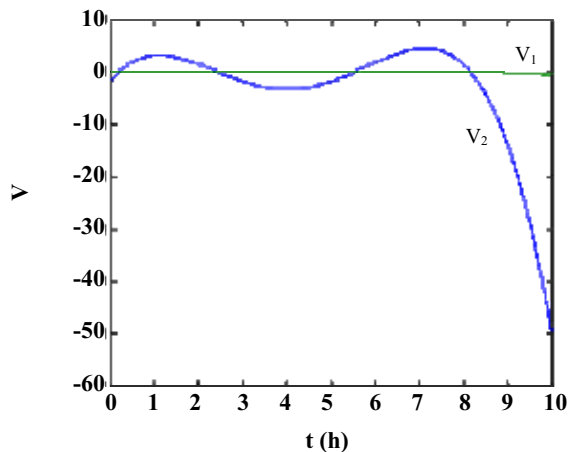
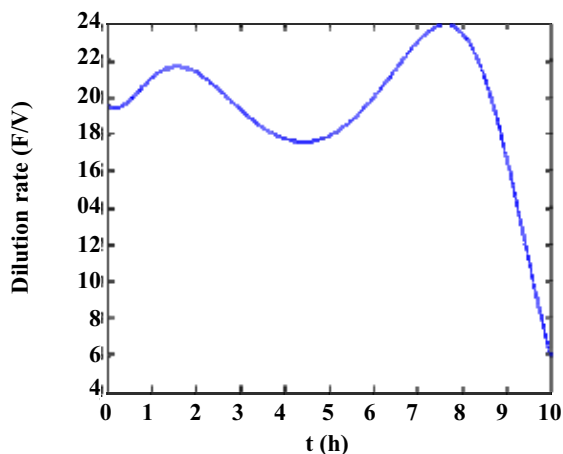**Fig. 8: Optimal trajectory for inputs of linearized non-isothermal CSTR.**

Fig. 9: Optimal trajectory for dilution rate in non-isothermal CSTR (First input).
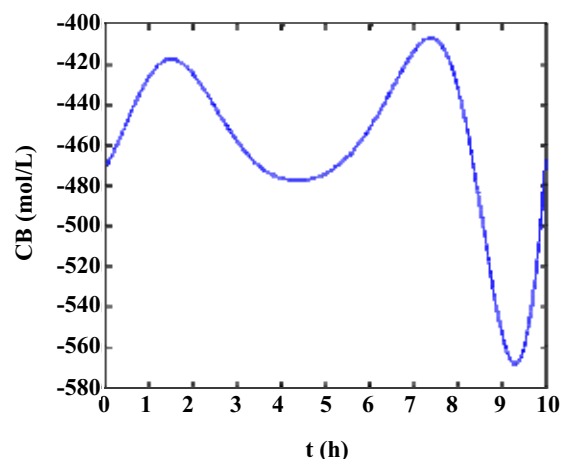

Fig. 10: Optimal trajectory for heat duty in non-isothermal CSTR (Second input).

The rationale and benefits of the proposed method have been elaborated. Two MIMO case-studies, one minimum-phase and one non-minimum-phase system, were used to evaluate the performance of the proposed method. The results show that the proposed method could be used to optimally control various nonlinear systems.

Appendix A: Kinetic model of MMA free radical polymerization

$$\frac{dC_m}{dt} = -\left[Z_p \exp\left(\frac{-E_p}{RT} \right) + Z_{f_m} \exp\left(\frac{-E_{f_m}}{RT} \right) \right] \times \tag{A1}$$

$$C_m P_0(C_1, T) + \frac{F(C_{m_{in}} - C_m)}{V}$$

$$\frac{dC_1}{dt} = -Z_1 \exp\left(\frac{-E_1}{RT} \right) C_1 + \frac{F_1 C_{1_{in}} - FC_1}{V}$$

$$\frac{dT}{dt} = Z_p \exp\left(-\frac{E_p}{RT} \right) C_m \frac{(-\Delta H_p)}{\rho C_p} P_0(C_1, T) -$$

$$\frac{UA}{\rho C_p V}(T - T_j) + \frac{F(T_{in} - T)}{V}$$

$$\frac{dD_0}{dt} = \left[0.5 Z_{T_c} \exp\left(\frac{-E_{T_c}}{RT} \right) + Z_{T_d} \exp\left(\frac{-E_{T_d}}{RT} \right) \right] P_0(C_1, T)^2 +$$

$$Z_{f_m} \exp\left(\frac{-E_{f_m}}{RT} \right) C_m P_0(C_1, T) - \frac{FD_0}{V}$$

$$\frac{dD_1}{dt} = M_m \left[Z_p \exp\left(\frac{-E_p}{RT} \right) + Z_{f_m} \exp\left(\frac{-E_{f_m}}{RT} \right) \right] \times$$

$$C_m P_0(C_1, T) - \frac{FD_1}{V}$$

$$\frac{dT_j}{dt} = \frac{F_{cw}}{V_o}(T_{w_0} - T_j) + \frac{UA}{\rho_w c_w V_o}(T - T_j)$$

$$P_0(C_1, T) = \left[\frac{2 f^* C_1 Z_1 \exp\left(\frac{-E_1}{RT} \right)}{Z_{T_c} \exp\left(\frac{-E_{T_c}}{RT} \right) + Z_{T_d} \exp\left(\frac{-E_{T_d}}{RT} \right)} \right]^{0.5} \tag{A2}$$

The kinetic coefficients and parameters of the system are presented in table A1 and A2 in order.

Appendix B: Kinetic model of non-isothermal CSTR

$$\frac{dC_A}{dt} = -k_1(T)C_A - k_3(T)C_A^2 + (C_{A0} - C_A)\frac{F}{V} \tag{B1}$$

$$\frac{dC_B}{dt} = k_1(T)C_A - k_2(T)C_B + C_B\frac{F}{V}$$

Table A1: Kinetic coefficients of MMA free radical reactions.

ij	Zij	Eij
Tc	3.8223E+10	2.9442E+03
Td	3.1457E+11	2.9442E+03
ij	3.7920E+18	1.2877E+05
P	1.7700E+09	1.8283E+04
fm	1.0067E+15	7.4478E+04

Table A2: Parameters of MMA free radial reactions model (Normal operating condition).

F=1	F_{cw}=3.26363	V=0.1	T_{W_0} =293.2
R=8.314	F_I=0.01679	M_m=100.12	y_1^{sp} =25000
$C_{I_{in}}$ =6	ρ=866	A=2	y_2^{sp} =335
U=720	$-\Delta H_p$=57800	V_0=0.02	
T_{in}=350	C_w=4.2	f^{*}=0.58	

Table B1: Kinetic parameters of the non-isothermal CSTR.

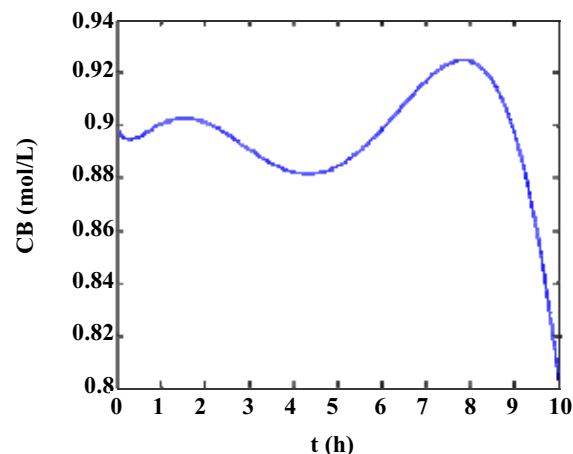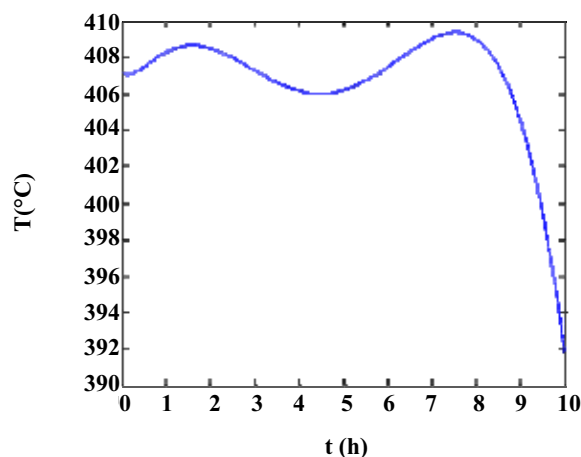| C_{A_0} C_{A0} =5 mol/l	ΔH_2 = -11 kJ/mol	E_1/R = -9758.3 K
T_0=403.15 K	ΔH_3 = -41.85 kJ/mol	E_2/R = -9758.3 K
ρ = 0.9342 kg/l	k_{10} = 1.287×10^{12}/h	E_3/R = -8560.0 K
C_p = 3.01 kJ/(kg K)	K_{20} = 1.287×10^{12}/h	
ΔH_1 = 4.2 KJ/mol	K_{30} = 9.403×10^9 l/(mol h)	

$$\frac{dT}{dt} =$$

$$\frac{(-\Delta H_1)k_1(T)C_A +(-\Delta H_2)k_2(T)C_B +(-\Delta H_3)k_3(T)C_A^2 +Q_H}{\rho C_p}$$

$$+(T_0 - T)\frac{F}{V}$$

where The kinetic parameters of the system are presented in table B1.

Fig. 11: Optimal trajectory for C_B in non-isothermal CSTR (First output).

Fig. 12: Optimal trajectory for temperature in non-isothermal CSTR (Second output).

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