Study of Solute Transfer in the Presence of Williamson Fluid over Exponentially Stretching Surfaces

M., Uma; S., Sushma; B.N., Veena**+*; N. Srikantha

ABSTRACT. *The study finds the mass transfer properties of Williamson fluid through permeable exponential stretched surface with slip boundary conditions. The two-dimensional flow model of Williamson fluid, along with boundary layer equations and mass transfer, are considered in the present paper. Mass transfer characteristics are studied for two instances, i.e., for Prescribed Exponential order Surface Concentration (PESC) and Prescribed Exponential order Mass Flux (PEMF). Similar variables are adapted to simplify the non-linear partial differential equations to ordinary differential equations, and finally, the resultant equations are resolved numerically. The physical characteristics of the flow model are evaluated graphically, and correlation with literature reveals a satisfactory improvement in the current results.*

KEYWORDS*:* *Williamson fluid; Mass transfer; Exponential stretching; PESC and PEM.*

INTRODUCTION

In current years, the investigation of heat and mass transmission behaviour of non-Newtonian fluids has fascinated many investigators owing to their industrial applications. Among the non-Newtonian fluids, one of the repeatedly intervened fluids is pseudoplastic. In 1929 *Williamson* [1] identified a type of fluid which is viscous and elastic and termed it pseudoplastic fluid. After verifying experimentally, he proposed a model and termed it Williamson's model. The flow behaviour of Williamson fluid through the stretching sheet is an exciting area of exploration in the present days because of some of their engineering applications such as in the manufacture of emulsion-treated sheets like photographic films, glass and plastic industry, solutions and melting of polymers with high molecular weight, etc.

Several vital methods include compression, drawing of wires, metal extrusion, rolling, fibre revolving, and polymer sheet extrusion, presenting the crucial uses of stretching surfaces in manufacturing. In 2013 *Nadeem et al*. [2] examined the flow of Williamson fluid for a stretching sheet. *Dapra et al*. [3] conducted an analytical study for a non-Newtonian Williamson fluid and analyzed pulsatile flow in a rock fracture. Due to some of the applications related to the peristaltic motion of Williamson fluid in the fields of Biology and Medicine, *Vajravelu* [4] considered the peristaltic motion of fluid over unequal channels along the porous wall. *Vasudev et al*. [5] investigated the peristaltic pumping of Williamson fluid with a lesser Reynolds number and long wavelength approximation for a planar channel. *Vittal et al*. [6] have done work on heat

**To whom correspondence should be addressed.*
+ *E-mail: mvymath@gmail.com*
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transfer and magnetohydrodynamic stagnation point movement of Williamson liquid by incorporating radiation effects for an exponentially stretching surface.

Because of the applications related to polymer masses, paper fabrication, and the manufacture of glass fibre, *Nagaraja* and *Reddy* [7] inspected the properties of heat transmission of non-Newtonian Williamson fluid by allowing suction and injection through a circular cylinder. *Hayat et al*. [8] explored the collective effects of electric & magnetic fields for a 2D flow of Williamson fluid with thermal radiation through a porous extending surface. By considering a linearly horizontal stretching sheet for a Williamson fluid with nanoparticles, incorporating the effects such as chemical reaction, and heat transmission was studied by *Krishnamurthy et al*. [9]. Some authors [10,11] have done extensive work on heat and mass transfer of fluid through the exponentially stretching sheet. *Elbashbeshy* [12] studied the heat transfer effects for constant, exponentially extending surfaces by including suction. *Nadeem et al*. [13] extended their work on Williamson fluid for a peristaltic flow through the uneven channel. *Lyubimov et al*. [14] considered the behaviour of pseudoplastic liquid movement as a thin oblique layer. *Swati* [15] has extensively worked on the slip effects of magnetohydrodynamic boundary layer fluid flow for an exponential stretching sheet by considering the thermal radiation effects and suction/blowing. *Khan* [16] investigated the impact of non-linear thermal radiation on a two-dimensional unsteady flow of Williamson fluid with the occurrence of heat source & sink. *Partha et al*. [17] considered the heat transfer influence of viscous dissipation on mixed convection through the exponential stretching surface. The impact of thermal radiation on an exponential stretching sheet for a boundary layer flow was studied by *Sajid* [18]. *Bidin* [19] has done numerical work on boundary flow through an exponential stretching sheet with the effect of thermal radiation. *Ishak* [20] analyzed the effects of magnetohydrodynamic boundary layer flow for exponential stretching sheets in the occurrence of radiation.

Hayat et al. [22] have performed work to identify mutual properties of Newtonian and Joule heating in the presence of Williamson fluid over the stretching surface analytically and numerically using a Homotopy Analytic Solution. Williamson fluid flow over an exponential stretching surface in the presence of a magnetic field along with porous medium and convective boundary conditions was discussed numerically by *Lie et al*. [23] and *Srinivasulu* [24], respectively.

Nadeem and Hussain [25] have made numerical and analytical analyses of the effect of heat flow on the Williamson fluid by considering similar and non-similar variables with the aid of the homotopy analysis technique. *Zainal et al*. [26] have used hybrid nanoparticles to perform mathematical scrutiny to study the heat generation/absorption influence on magnetohydrodynamics flow using a bi-directional exponential extending/shrinking sheet. They have shown that the usage of hybrid nanoparticles has improved skin friction coefficient. *Ahmed et al*. [27] have investigated heat transmission variations in the magnetohydrodynamic diverse convective flow of Williamson fluid by considering exponentially extending porous curled surface. He has mainly discussed the effects of various non-dimensional parameters using a graphical approach.

Nagaraja and *Gireesha* [28] studied heat and mass flux and the impact of heat source on magnetohydrodynamic Casson fluid flow above curved exponential extending sheet with chemically reactive fluid. Recently *Ali* and *Acharya* [29, 30] showed the thermal radiation effects of the heat transmission flow phenomenon.

Rashidi et al. [31] have used the homotopy analysis method and found approximate analytical solutions and validated them using the Runge-Kutta and shooting method for the steady flow over a rotating disk in the porous medium.

Majid et al. [32] have used the Chebyshev spectral Newton iterative scheme to study the effects of partial slip and prescribed surface heat flux in moving fluid in a stretching cylinder and compared the results by applying a finite difference scheme.

Bilal and *Mabood* [33] have performed a numerical investigation of exponential stretching sheets with convective boundary conditions in viscoelastic nanofluids using the Runge-Kutta-Fehlberg shooting technique.

Bilal [34] in his paper, has studied mixed convection flow in cylindrical geometry of viscoelastic liquid. He has analytically found the solution using the Homotopy Investigation Strategy (HAM) and found convergent series solutions.

In many industrial applications, rather than linear stretching, there is a need for the study of non-linear stretching sheets, so in the present work, we have attempted to stretch exponentially for a permeable surface. Therefore, the current work aims to analyze the mass transfer effects of Williamson fluid over an exponential stretching permeable surface with suction and injection velocity. Here solute transfer characteristics are studied for two instances, i.e., for PESC and PEMF.

THEORETICAL SECTION

Constitutive equations for Williamson fluid model, the Cauchy stress tensor S (see *Dapra* and *Scarpi* [3], *Nadeem* and *Hussain* [25]) is given by

$$S = -P_1 I + \tau, \tag{1}$$

$$\tau = \left(\mu_\infty + \frac{\mu_0 - \mu_\infty}{1 - \Gamma\gamma}\right) A_1, \tag{2}$$

$\Gamma > 0$ is time constant and γ is given by

$$\gamma = \sqrt{\frac{1}{2}\pi}, \pi = trace(A_1)^2. \tag{3}$$

Considering the case in which $\mu_\infty = 0, \Gamma\gamma > 1$, the value of τ is obtained as

$$\tau = \left(\frac{\mu_0}{1 - \Gamma\gamma}\right) A_1 \tag{4}$$

Mathematical formulation

In the current analysis, we have mathematically formulated the study by considering the 2-D steady flow of Williamson fluid above an exponential stretching surface. The surface of the plate is stretched exponentially in the direction of x the axis with a velocity $U_w = U_0\, exp\left(\frac{x}{l}\right) y = 0$at.

The boundary layer equations for the flow and mass transfer without body force and viscous dissipation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \sqrt{2\nu}\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}, \tag{6}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \quad . \tag{7}$$

Assuming slip boundary condition, the relevant boundary settings for velocity and concentration are as follows

$$u = U_w = U_0\, exp\left(\frac{x}{l}\right), v = -V_w(x), C = C_w \quad \text{When } y = 0, \tag{8}$$

$$u \to 0, C = C_\infty \quad \text{When } y \to \infty.$$

Where $V_w(x) < 0$ is the injection velocity and $V_w(x) > 0$ is the suction velocity, respectively.

We considered two different types of mass transfer problems that is PESC and PEMF case.

The boundary constraint for the PESC and PEMF instances are

$$C = C_w = C_\infty + C_0 exp\left(\frac{x}{2l}\right), \text{ at } y = 0 \text{ (PESC)} \tag{9}$$

$$-D_0\left(\frac{\partial C}{\partial y}\right)_w = C_1 exp\left(\frac{x}{l}\right), \text{ at } y = 0 \text{ (PEMF)} \tag{10}$$

$$C \to C_\infty \text{ When } y \to \infty \text{ for PESC and PEMF case} \tag{11}$$

The similarity transformations (see Reference [21]) are chosen as follows:

$$\eta = \sqrt{\frac{U_0}{2\nu l}}\, y\, exp\left(\frac{x}{2l}\right), \psi(x,y) = \sqrt{2\nu l U_0} f(\eta)\, exp\left(\frac{x}{2l}\right) \tag{12}$$

For the PESC case,

$$C = C_\infty + C_0 exp\left(\frac{x}{2l}\right)\varphi_1(\eta), \tag{13}$$

For the PEMF case,

$$C = C_\infty + \frac{C_1}{D_0}\exp\left(\frac{x}{2l}\right)\sqrt{\frac{2\nu l}{U_0}}\phi_2(\eta), \tag{14}$$

The above transformations are used in Eqs. (5) – (7) therefore Eq. (5) is identically fulfilled, and the resultant velocity components *u* and *v* are obtained as

$$u = U_0\, exp\left(\frac{x}{l}\right) f'(\eta), v = -\sqrt{\frac{\nu U_0}{2l}}\, exp\left(\frac{x}{2l}\right)[\eta f'(\eta) + f(\eta)], \tag{15}$$

Where primes signify derivatives with respect to η.

So that similarity results of Equations. (5) – (11) occurs, we consider it as follows:

$$V_w(x) = V_0\, exp\left(\frac{x}{2l}\right), \tag{16}$$

Where V_w is the special type of wall velocity, with $V_w(x) > 0$ (suction), $V_w < 0$ (injection) and V_0 remains constant. On substituting new variables in Eq. (12), Eqs. (6) and (7) are reduced to the successive system of ODE's:

$$f''' - 2f'^2 + ff'' + \lambda f'' f''' = 0, \tag{17}$$

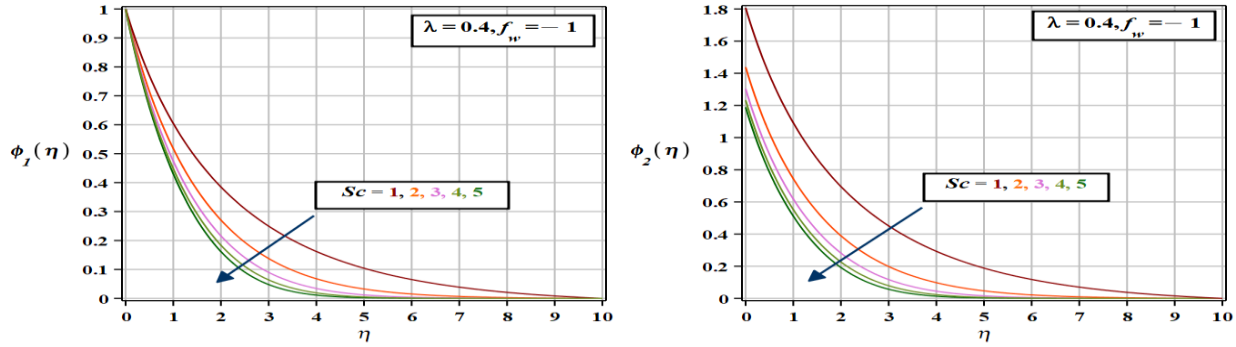

Fig 1: Impact of concentration on the variation of Schmidt number

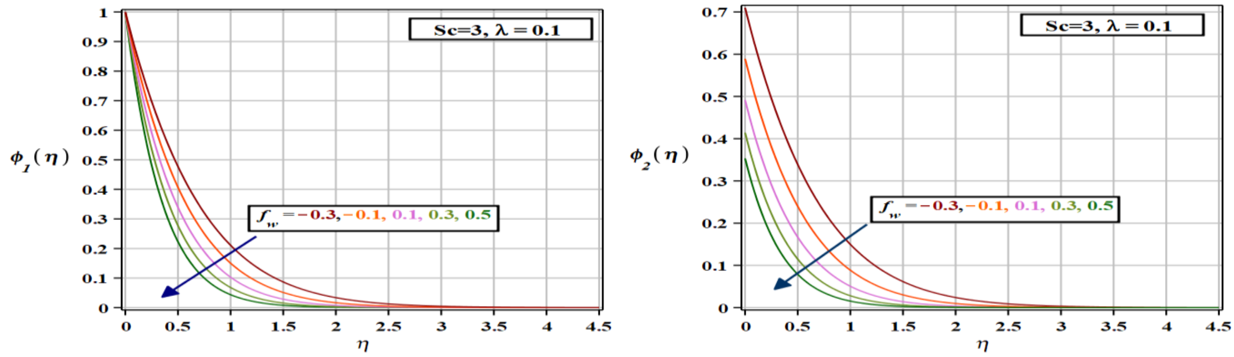

Fig 2: Impact of concentration on suction/injection

$$\varphi_1'' + Sc(\varphi_1' f - f'\varphi_1) = 0, \quad (18)$$

$$\varphi_2'' + Sc(\varphi_2' f - f'\varphi_2) = 0. \quad (19)$$

The boundary conditions for applying similarity transformations are given by

$$f(\eta) = f_w, f'(\eta) = 1, \varphi_1(\eta) = 1, \varphi_2'(\eta) = -1 \text{at} \eta =$$
$$f'(\eta_\infty) = 0, \varphi_1(\eta_\infty) = 0, \varphi_2(\eta_\infty) = 0, \text{ at} \eta = \eta_\infty, \quad (20)$$

where $f_w = -V_0\sqrt{\frac{2l}{\nu U_0}}$is the suction/injection parameter$f_w < 0$, mass injection, $f_w > 0$height suction,η_∞ edge of the boundary layer, $\lambda = \Gamma\sqrt{\frac{U_0\, exp(3x/l)}{\nu l}}$dimensionless Williamson parameter$Sc = \frac{\nu}{D}$, and Schmidt number to the PESC and PEMF case, respectively.

The quantities of practical interest are the local skin friction coefficient, and local Sherwood for PESC cases which are defined as

$$C_{fx} = \frac{\nu(\partial u/\partial y)_{y=0}}{U^2{}_w\, exp(2x/l)}, Sh_x = \frac{x}{(C_w - C_\infty)}(\partial C/\partial y)_{y=0}, \quad (21)$$

$$f''(0) = \sqrt{Re_x}\, C_{fx},$$

$$-\sqrt{\frac{x}{2l}}\varphi_1'(0) = \frac{Sh_x}{\sqrt{Re_x}}, \quad (22)$$

$$Re_x = \frac{U_w x}{\nu}$$

RESULTS AND DISCUSSIONS

Fig. (1) depicts the impact of the Schmidt number on solutal boundary layer thickness in the case of PESC,$\varphi_1(x)$ and PEMF, $\varphi_2(x)$ respectively. It can be observed that as the Schmidt number grows, the solutal boundary layer width drops in PESC & PEMF cases. Physically as the Schmidt number increases, it leads to shrinkage in the molecular diffusivity; correspondingly, the boundary layer width also reduces. It can also be noted that in the case of PESC concentration, boundary layer thickness decreases from $\eta \approx 10$to $\eta \approx 4.5$when Schmidt number increases from $Sc = 1$ to $Sc = 5$where as in the case of PEMF concentration, boundary layer thickness decreases from $\eta \approx 10$to $\eta \approx 4$ when Schmidt number increases from $Sc = 1Sc = 5$to.

Fig. (2), shows the influence of suction/injection in the case of PESC, $\varphi_1(x)$ and PEMF,$\varphi_2(x)$ respectively. It is observed that in the case of PESC and PEMF, solutal

Table 1: Linear regression model for skin friction

Model : $S_f = c_1\lambda + c_2 f_w + c_3$				
Coefficients	Estimate	Standard Error	t-value	P
c_1	-0.09379	0.04930	-1.90229	0.06082
c_2	1.44674	0.00822	176.05700	0
c_3	1.83246	0.00727	252.19300	0

Residuals			
Residual Sum of Squares	Residual Mean Square	Residual Standard Error	Degrees of Freedom
0.01264	0.00016	0.01273	78

R-Squared: 0.99749

Adjusted R-squared: 0.99743

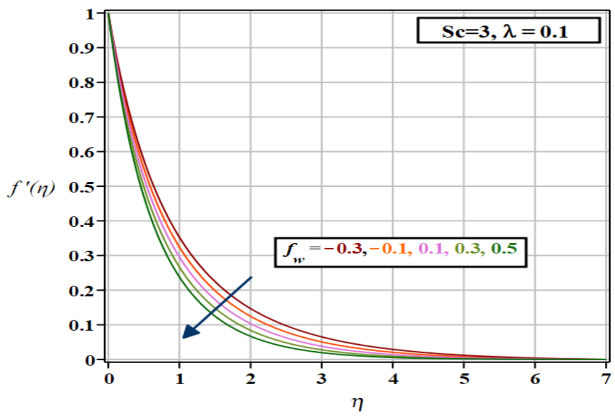

Fig 3: Impact of velocity on Suction/injection

boundary layer thickness increases during injection and decreases during suction. When$f_w < 0$ (injection), it is observed that the fluid is transported from the surface resulting in the decrease of the concentration gradient and hence the role is to maintain the same concentration near the surface, whereas converse behaviour is observed in the case of suction $f_w > 0$in both PESC case & PEMF cases. Also, it can be seen from the same Fig. that in the case of PEMF, the wall's concentration decreases due to the mass flux.

Fig. (3) depicts the influence of Suction/injection due to the Velocity profile, reiterating a similar effect as discussed in Fig. (2).

Fig. (4) depicts the influence of the Williamson factor on the concentration boundary layer thickness for PESC and PEMF. In both the cases with the enhancement in value of the Williamson parameter, the solutal boundary layer thickness increases.

Fig. (5) depicts the influence of the Williamson factor due to Velocity. It can be witnessed that with the rise in the values of the Williamson factor, momentum boundary layer thickness decreases; this is since viscoelastic fluid resists fluid motion.

Fig. (6) presents the deviation of skin friction parameters for various values of λ &f_w. It is detected that in case of suction/injection boundary layer, width drops and surge in Williamson parameter decreases local skin friction parameter.

Regression analysis

A linear regression model for skin friction is obtained for 81 sets of data of f_w and λ. The statistical parameters are tabulated in table 1. P value for the coefficient of λ is slightly larger than the usual significance value, i.e., 0.05. On the other hand, the coefficient of the parameter f_w is statistically significant, and *the* R-squared value and adjusted R-squared value shows more than 99%, for the Linear model of skin friction. Hence linear model justifies the numerically obtained results of the paper. (Ref. Fig. 6).

Validity of the result

Table 2 is comprised of suction/injection values of various Williamson parameters. The so obtained results are compared with the previously existing one of Nadeem and Hussain [25]) and establish a good agreement with the results of the current paper.

CONCLUSIONS

The following conclusion can be drawn from the present paper

- As there is an enhancement in the Williamson parameter, there is fall in momentum boundary layer width is observed.
- As the Schmidt number is raised, there will be a decline in solutal boundary layer width in both PESC & PEMF.
- Enhancement in the Williamson parameter boosts the width of the solutal boundary layer in both PESC and PEMF cases, respectively.
- A rise in suction parameter drops momentum concentration boundary layer thickness, whereas converse behaviour is observed in injection respectively.
- The linear model justifies the numerically obtained results of the paper.

Table 2: Variation of the values in suction/injection for various values of λ

λ	$f_w = -0.2$		$f_w = 0.0$		$f_w = 0.2$	
	Nadeem[25]	Present	Nadeem[25]	Present	Nadeem[25]	Present
0.0	1.37889	1.37898	1.28180	1.28193	1.19298	1.19313
0.1	1.34659	1.34667	1.25153	1.25164	1.16468	1.16483
0.2	1.31031	1.31039	1.21794	1.21806	1.13365	1.13378

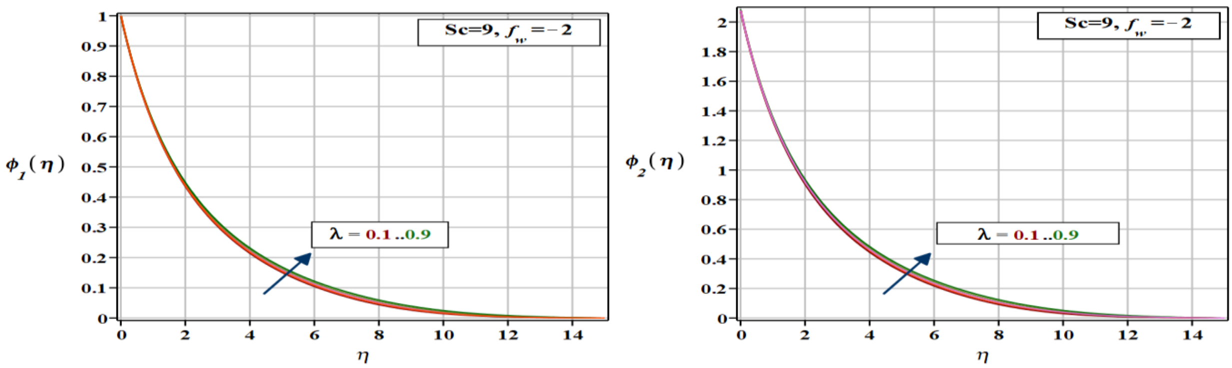

Fig4: Impact of concentration on Williamson parameter

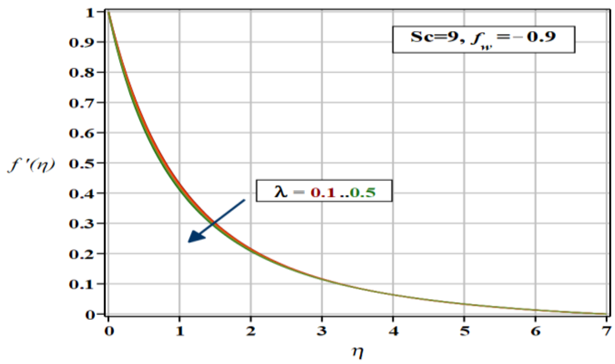

Fig 5: Impact of velocity on Williamson parameter

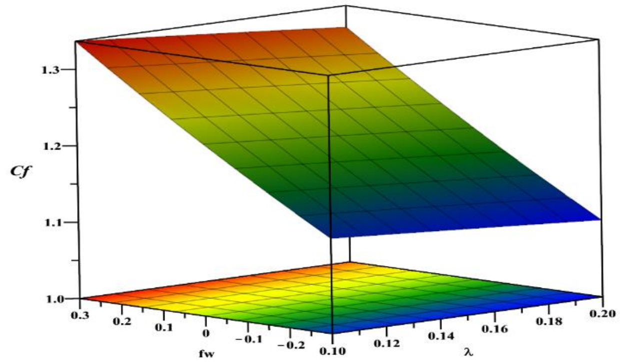

Fig 6: Plot showing the behavior of Local Skin friction due to variation in λ & f_w

Nomenclature

A_1	first Rivilin-Erickson tensor
C	fluid concentration(kg)
D	solutal diffusivity(m^2/s)
I	identity vector
P_1	pressure(kg/ms^2)
U_0,V_0	constant
U_w, T_w	velocity (m/s) and temperature of fluid near the wall (C)
u,v	velocity component in x, y direction (m/s)
U_∞	ambient velocity and concentration
C_∞	ambient concentration
η	similarity variable
μ_0	viscosity at zero shear rate(kg/ms)
μ_∞	viscosity at the infinite shear rate (kg/ms)
ν	kinematic viscosity(m^2/s)
φ_1,φ_2	dimensionless concentration
ψ	stream function
θ	dimensionless temperature
τ	extra stress tensor
PESC	prescribed exponential order surface concentration
PEMF	prescribed exponential order mass flux

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