Simulation of Boiling in a Vertical Channel Using Ensemble Average Model

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ABSTRACT: *Simulation of turbulence boiling, generation of vapour and predication of its behaviour are still subject to debate in the two-phase flow area and they receive a high level of worldwide attention. In this study, a new arrangement of the three dimensional governing equations for turbulence two-phase flow with heat and mass transfer are derived by using ensemble averaging two-fluid model and utilizing the latest approved constitutive equations. Then, the governing equations are simplified for bulk boiling in a vertical channel. A computer program with SIMPLE algorithm is written for the simplified equations, and the results are compared with available experimental data and a boiling water reactor in operating condition.*

KEY WORDS: *Boiling, Two-phase flow, Heat and mass transfer, Two-fluid model, Ensemble averaging*

INTRODUCTION

Forced convective heat transfer of boiling flow in channels is used in a wide variety of areas including the petroleum and chemical processing industries, steam generations, refrigeration equipment, and nuclear reactors (specially boiling water reactors). The pressure gradient and void fraction are more importance when predicting parameters in the boiling systems. The prediction of two-phase pressure gradient is an essential step in designing of a variety of power plants and process industries. Researches for developing boiling models are receiving a high level of worldwide consideration since this subject has wide range of practical applications in many fields, including chemical micro reactors, aerospace science, micro electro-mechanical systems (MEMS), process intensification, and compact heat exchangers. Therefore, boiling is one of the most popular two-phase flow including heat and mass transfer.

In today's world, the scientific knowledge is improving all the time by enhanced available experimental devices and also accessing articles data bases electronically via fast methods of communication. Consequently, it needs to modify the mathematical models continuously.

In the recent decades, significant developments in the two-phase flow formulation have been accomplished by introducing and improving the two-fluid model. In the present state-of-the-art, the two-fluid model can be considered the most detailed and accurate macroscopic formulation of the thermo-fluid dynamics of two-phase

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systems [1-3]. In the two-fluid model, the field equations are expressed by the six conservation equations consisting of mass, momentum and energy equations for each phase. There are also jump conditions for all of the conserved equations at the phase interface. The field equations are obtained from an appropriate averaging of the local instantaneous balance equations. The two-fluid models of two-phase flow are formulated based on space, time or ensemble averaging of the local instantaneous phasic balance equations. As a consequence, the two-fluid model can only simulate the average flow behaviour, provided sufficiently accurate empirical correlations are used to describe heat, mass and momentum transfer processes at the phasic interface and at the boundary walls. The two-fluid model of two-phase flow, which is based on a single time- averaging, has been extensively investigated by *Ishii* [4]. *Dalhaye* [5] has developed both spatial and space/time averaging techniques. *Nigmatulin* [6] has derived a volumetric-averaged set of balance equations for multiphase flow. In addition, he developed a cell model as a means of deriving closure relations for his equations. Ensemble averaging has been proposed as the fundamentally correct form of averaging by *Buyevich* [7] and *Batchelor* [8].

In addition, a number of other averaging techniques have been developed. These include a five-stage space/time averaged scheme developed by *Drew* [9], volume averaging using a weighting function by *Iwanaga* and *Ishihara* [10] and a volume average using a variable size averaging by *Gray* [11]. Additionally, *Bataille* [12] have reported how to transform the space and time coordinates to a four dimensional space. He derives a technique whereby flow parameters are averaged over the four dimensions, while being weighted by a smoothing function, which enhances the smoothness of derivatives of averaged parameters.

The ensemble averaged two-phase flow equations using the interfacial forces between two phases were developed by many authors such as *Arnold* [13], *Park* [14], *Arnold et al.* [15], *Antal* [16], *Drew* and *Lahey* [17, 18]. *Drew* and *Passman* [19] give a very broad and detailed overview of averaging methods for multi-component flows. They discuss ensemble-averaging model in detail as well. Moreover, *Drew et al.* and *Lahey et al.* have continued their research in the ensemble averaging model [20-24].

In this study, for investigation of boiling flow inside a heated channel, a new arrangement the three dimensional governing equations for turbulence two-phase flow including heat and mass transfer are derived using ensemble averaging two-fluid model by utilizing the latest approved constitutive equations. The governing equations are simplified by using overall energy balance equation. The simplified equations are solved and the results are compared with available experimental boiling data and also with a real boiling water reactor.

MATHEMATICAL MODEL

The correct formulation of the basic two-fluid conservation equations, and the most appropriate closure laws, are still subject to debate and depend strongly on the particular problem. It means in different problems, which terms should or should not be included [25]. The following approach is based on the phase-weighted ensemble average two-fluid model which is described by *Drew* and *Passman* [19] and it is adopted and improved by many researchers [20-24] for simulation in multiphase flow systems. The phasic conservation equations are:

The phasic mass equation,

$$\frac{\partial}{\partial t}\left(\alpha_k \langle \rho_k \rangle^{\chi}\right) + \nabla \cdot \left(\alpha_k \langle \rho_k \rangle^{\chi} \langle v_k \rangle^{\chi\rho}\right) = \dot{m}_k''' - \cancel{\dot{m}_k'''} \qquad (1)$$

The phasic momentum equation,

$$\frac{\partial}{\partial t}\left(\alpha_k \langle \rho_k \rangle^{\chi} \langle v_k \rangle^{\chi\rho}\right) + \nabla \cdot \left(\alpha_k \langle \rho_k \rangle^{\chi} \langle v_k \rangle^{\chi\rho} \langle v_k \rangle^{\chi\rho}\right) = \qquad (2)$$
$$-\nabla\left(\alpha_k \langle p_k \rangle^{\chi}\right) + \nabla \cdot \left[\alpha_k \left(\langle \tau_k \rangle^{\chi} + \tau_k^t\right)\right] + \sum F_k +$$
$$\alpha_k \langle \rho_k \rangle^{\chi} \langle b_k \rangle^{\chi\rho} + v_{ki}\dot{m}_k''' - v_k \cancel{\dot{m}_k'''}$$

The phasic energy equation,

$$\frac{\partial}{\partial t}\left[\alpha_k \langle \rho_k \rangle^{\chi}\left(\langle u_k \rangle^{\chi\rho} + \frac{1}{2}\langle v_k \rangle^{\chi\rho} . \langle v_k \rangle^{\chi\rho} + u_k^t\right)\right] + \qquad (3)$$
$$\nabla \cdot \left[\alpha_k \langle \rho_k \rangle^{\chi} \langle v_k \rangle^{\chi\rho}\left(\langle u_k \rangle^{\chi\rho} + \frac{1}{2}\langle v_k \rangle^{\chi\rho} . \langle v_k \rangle^{\chi\rho} + u_k^t\right)\right] =$$
$$\nabla \cdot \alpha_k \left[\left(\langle \tau_k \rangle^{\chi} + \tau_k^t\right) . \langle v_k \rangle^{\chi\rho} - \langle q_k'' \rangle^{\chi} - q_k^t\right] + \alpha_k \langle q_k''' \rangle^{\chi} +$$
$$\alpha_k \langle \rho_k \rangle^{\chi} \langle b_k \rangle^{\chi\rho} . \langle v_k \rangle^{\chi\rho} + E_k + W_k +$$
$$\left[u_{ki} + \frac{1}{2}\left(v_{ki} . v_{ki}\right)\right]\dot{m}_k''' - \left[u_k + \frac{1}{2}\left(v_k . v_k\right)\right]\cancel{\dot{m}_k'''}$$

where $\dot{m}'''$ is phasic interfacial mass generation rate, subject to many reasons such as chemical production and/or phase change and/or mass source and/or flashing, $\dot{m}'''$ is phasic interfacial mass reduction rate during the process, and F_k is the interfacial momentum transfer. In the above equations the components are assumed to be not under the influence of electromagnetic fields.

JUMP CONDITIONS

The Jump conditions of mass, momentum and energy are:

$$\sum(\dot{m}_k''' - \dot{m}_k''') = 0 \tag{4}$$

$$\sum(\sum F_k + v_{ki}\dot{m}_k''' - v_k\dot{m}_k''') = M \tag{5}$$

$$\sum\left(E_k + W_k + \left[u_{ki} + \frac{1}{2}(v_{ki}.v_{ki})\right]\dot{m}_k''' - \left[u_k + \frac{1}{2}(v_k.v_k)\right]\dot{m}_k'''\right) = \in \tag{6}$$

INTERFACIAL MOMETUM TRANSTER

The Interfacial momentum transfer is modelled with the interfacial forces. For boiling flow in vertical channels, the following forces must be taken into account: the drag force F_k^{drag}, the lift force F_k^{lift}, the turbulent dispersion force $F_k^{turbulent}$, the virtual mass force $F_k^{virtual}$ and the wall lubrication force F_k^{wall}. The total interfacial force per unit volume is the sum of the forces,

$$\sum F_k = F_k^{drag} + F_k^{lift} + F_k^{turbulent} + F_k^{virtual} + F_k^{wall} \tag{7}$$

The drag force is a vector directed along the relative velocity of the vapour phase which is exerted by the vapour phase on the liquid phase. Thus, it depends strongly on relative velocity of phases and interfacial area. Some studies suggest the drag force relation which implies interfacial area for only spherical bubbles [24, 26-28] (However, prediction for bubbly flow it means the void fraction should be less than 25 % [29]). In this study, it is used an expression which includes interfacial area term in common form as [19-23, 30-32].

$$F_v^{drag} = -F_l^{drag} = -\frac{1}{8}\rho_l A''' C_D (v_v - v_l)|v_v - v_l| \tag{8}$$

Where C_D and A''' are the drag force coefficient and the interfacial area density, respectively. The C_D is a flow-regime dependent, and is usually calculated by using a correlation, but for boiling in the general case, it needs a C_D for wide range of void fraction as [33].

$$C_D = \begin{cases} 24(1+0.1Re_b^{0.75})Re_b^{-1} & \text{when } 0 < \alpha_v \le 0.1 \\ \frac{2D_b}{3}\sqrt{\frac{g(\rho_l-\rho_v)}{\sigma_s}}\left[\frac{1+17.67(1-\alpha_v)^{1.285}}{18.67(1-\alpha_v)^{1.5}}\right]^2 & \text{when } 0.1 < \alpha_v \le 0.25 \\ 9.8(1-\alpha_v) & \text{when } 0.25 < \alpha_v \end{cases} \tag{9}$$

where the bubble Reynolds number Re_b is:

$$Re_b = \frac{|v_v - v_l| D_b \rho_l}{\mu_l} \tag{10}$$

However, some of the valuable works [18, 20, 21, 23] didn't mention any relation for C_D. The interfacial area density also should be determined for large range of vapour volume fraction by using [34]:

$$A''' = \frac{4.5(\alpha_v - \alpha_{sb})}{D(1-\alpha_{sb})} + \frac{6\alpha_{sb}(1-\alpha_v)}{D_b(1-\alpha_{sb})} \tag{11}$$

where α_{sb} is the small bubble void fraction, (then it should be equal to void fraction "α_v" within bubbly flow region) and it can be calculated by [35] where α_{sb} is the small bubble void fraction, (then it should be equal to void fraction "α_v" within bubbly flow region) and it can be calculated by [35]:

$$\alpha_{sb} = \begin{cases} \alpha_v & \text{when } \alpha_v < 0.25 \\ 0.3929 - 0.5714\alpha_v & \text{when } 0.25 \le \alpha_v < 0.6 \\ 0.05 & \text{when } 0.6 \le \alpha_v \end{cases} \tag{12}$$

The lift force arises from a velocity gradient of the continuous phase in the lateral direction. It acts perpendicular to the main flow direction and is proportional to the gradient of the liquid velocity field. Lift force is the important parameter to obtain correct radial distribution of the two phases. The following expression is offered by Drew & Lahey [36], and has been addressed by recent acceptable works [23, 26, 27, 32, 37].

$$F_v^{lift} = -F_l^{lift} = C_L \alpha_v \rho_l (v_v - v_l) \times (\nabla \times v_l) \tag{13}$$

where C_L is the lift force coefficient and depends on Eötvos number (Eö) as [38]:

$$C_L = 0.288 \tanh(0.121\, Re_b) + \begin{cases} 0, & \text{when } Eö < 4 \\ -0.096Eö + 0.384, & \text{when } 4 < Eö < 10 \\ -0.576 & \text{when } 10 < Eö \end{cases} \quad (14)$$

where the Eötvos number (Eö) is:

$$Eö = \frac{gD_b^2(\rho_l - \rho_v)}{\sigma_s} \quad (15)$$

The turbulent dispersion force accounts for the effect of the dispersion of vapour bubbles caused by liquid turbulence and then smoothes void fraction distribution. The following turbulent dispersion force was derived by *Lopez de Bertodano* [39] and it has been widely used [18, 23, 25, 30, 40, 41]

$$F_v^{turbulent} = -F_l^{turbulent} = -C_{TD}\rho_l k_l \nabla\alpha_v \quad (16)$$

In the present model, C_{TD} and k_l are "the turbulent dispersion force coefficient" and "the kinetic energy of the turbulence of the liquid phase per unit mass", respectively. The turbulent dispersion force coefficient (C_{TD}) is 0.1 for bubbles.

The virtual mass force, which comes into calculation whenever one phase is accelerating relative to the other one. In case of a bubble accelerating in a continuous liquid phase, this force can be described by the following expression [23, 24, 30].

$$F_v^{virtual} = -F_l^{virtual} = \alpha_v\rho_l C_{VM}[(\frac{\partial v_l}{\partial t} + v_l.\nabla v_l) - (\frac{\partial v_v}{\partial t} + v_v.\nabla v_v)] \quad (17)$$

where, the virtual mass coefficient C_{VM} is equal to 0.5 for spherical bubbles.

The wall lubrication force is introduced to eliminate the effect of vapour sticking to the wall, thus it is also another parameter which influences obtaining the correct radial distribution of the two phases. The most detail model is [23]

$$F_v^{wall} = \left\{\max\left[\frac{2\alpha_v\rho_l v_{axial}^2}{D_b}\left(C_{W1} + C_{W2}\frac{D_b}{2L}\right), 0\right]\right\}\delta_w n_w - \frac{0.3\rho_l\alpha_v}{D_b}\sqrt{\frac{\tau_w}{\rho_l}}(v_v - v_l)\delta_w \quad (18)$$

and $F_v^{wall} = -F_l^{wall}$ where, n_w is the unit vector normal to the wall. Also, the other parameters are defined as:

$C_{W1} = -0.104 - 0.06|v_r|$, C_{w2}=0.142,

$$L=(x_b-x_w)n_w,\ v_{axial}=v_r-(n_w-v_r)\,n_w, \quad (19)$$

$$\delta_w = \begin{cases} 1.0 & L \le D_b \\ 0.0 & \text{otherwise} \end{cases}$$

The first term on the right hand side of Eq. (18) is a force which is normal to the wall and the second term is a force parallel to the wall. These forces are important in the nodes adjacent to the wall.

TURBULENT MODEL

There is no standard model for two-phase flow like the k-ε for single-phase. However, many authors argue that if the dispersed phase elements are small and/or the void fraction is low, the standard k-ε formulation can be used. In this study, more elaborate model includes extra source terms is used. The model represents the increased generation of turbulence by the presence of the bubbles as [23]:

The turbulent kinetic energy for liquid phase

$$\frac{D(\alpha_l k_l)}{Dt} - \nabla.(\alpha_l\frac{\upsilon_l^T}{\sigma_k}\nabla k_l) = \alpha_l(P_l - \varepsilon_l) + \alpha_l S_k \quad (20)$$

Turbulent dissipation for liquid phase

$$\frac{D(\alpha_l\varepsilon_l)}{Dt} - \nabla.(\alpha_l\frac{\upsilon_l^T}{\sigma_\varepsilon}\nabla\varepsilon_l) = \frac{\alpha_l}{k_l}\left(C_{1\varepsilon}\varepsilon_l P_l - C_{2\varepsilon}\varepsilon_l^2\right) + \alpha_l S_\varepsilon \quad (21)$$

where,

$$P_l = \mu_{eff}\nabla v_l.(\nabla v_l + (\nabla v_l)^T) - \frac{2}{3}\nabla.v_l(\mu_{eff}\nabla.v_l + \rho_l k_l) \quad (22)$$

$C_{1\varepsilon} = 1.44$, $C_{2\varepsilon} = 1.92$

$\sigma_\varepsilon = 1.3$, $k_l = \frac{1}{2}v_l'.v_l'$, $\sigma_k = 1.0$

The turbulence viscosity induced by the bubbles in the liquid phase is modeled as [42]:

$$\upsilon_l^T = 0.09\frac{k_l^2}{\varepsilon_l} + 1.2R_b\alpha_v\left|v_v - v_l\right| \quad (23)$$

The interaction terms are defined as [24]:

$$S_k = \frac{k_l}{C_{\varepsilon 2}\varepsilon_l} S_\varepsilon = 0.25\alpha_v(1+C_D^{4/3})\frac{\left|v_v - v_l\right|^3}{D_b} \quad (24)$$

Similar turbulent transport equations could utilize for the vapour phase.

SIMPLIFICATION FOR BULK BOILING

To validate the present model (which is a new arrangement of governing and constitutive equations) and comparison with the available data, it is considered a saturated boiling problem. According to Fig. 1 the subcooled water enters to a channel with hydraulic diameter of D. Saturated water and then saturated boiling occurs in consequence of convective heat transfer.

One could find a relation for the vapour quality by using the energy balance equation between inlet and a saturated section. In the steady state case, the quality of any two-phase flow section (saturated boiling) could derive as:

$$x_Q(z) = \frac{\dot{Q}(z_s)\big/\dot{m}_{in} + h_{in} - h_l - gz_s + 0.5v_{in}^2 - 0.5v_l^2}{(h_{fg} - 0.5v_l^2 + 0.5v_v^2)} \quad (25)$$

where the subscript "in" indicates the inlet condition. V_l and V_v denote the cross sectional average velocity of the liquid and vapour phase, respectively. So, in order to use this simplification for three-dimensional problems, the average phasic velocities could obtain as:

$$v_k^2 = \left\langle u_k^2 \right\rangle + \left\langle v_k^2 \right\rangle + \left\langle w_k^2 \right\rangle \quad (26)$$

According to equation (25) for a given heat distribution and available inlet conditions, the quality for any cross section in boiling region will be function of average phasic velocities (V_l,V_v), and the pressure (enthalpies are function of pressure in saturated boiling). Moreover, in the saturation boiling the temperatures of phases are function of pressure as well. Thus, it is possible to use equation (25) instead of equation (3) for saturated boiling. This saves significant computer time.

For the first loop iteration process, it can eliminate the velocity effect, thus the Eq. (25) could change to:

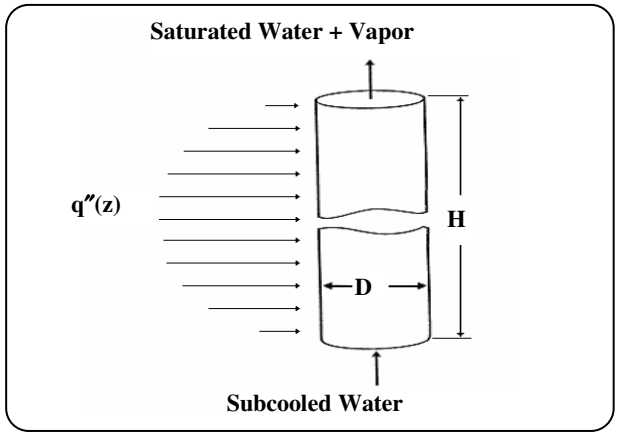

Fig. 1: The Simple Schematic of the problem.

$$x_Q(z) = \frac{\dot{Q}(z_s)\big/\dot{m}_{in} + h_{in} - h_l - gz_s}{h_{fg}} \quad (27)$$

Sharifpur et al. [43] used the Eq. (27) instead of Eq. (3) by applying linear assumption for pressure gradient in laminar bubbly flow regime (void fraction less than 25 %). In present study the Eq. (25) is derived and used for wide range of void fraction in the case of turbulence bulk boiling with no simplification in pressure gradient.

NUMERICAL PROCEDURE

In this work, a finite volume method was used for the differencing of the various conservation one-dimensional equations, and the associated interfacial jump conditions. The equations were discretised on a collocated mesh using standard differencing techniques. The sets of discretised equations were solved iteratively in a sequential manner.

The solution algorithm utilizes a procedure similar to the SIMPLE method in which the pressure-velocity correction technique is extended to two-phase flows [44]. In present approach, it is used energy balance equation (Eq. (25)) instead of energy equation (Eq. (3)), for bulk boiling, iteratively. This saves significant computer time. The numerical accuracy of the solution was verified by carrying out an appropriate grid refinement study for the test cases which yielded almost identical results. Moreover, error analysis investigation are done by comparing the numerical results with available experimental data and a boiling water reactor type "PB2 BWR/4 NPP" in the operating condition.

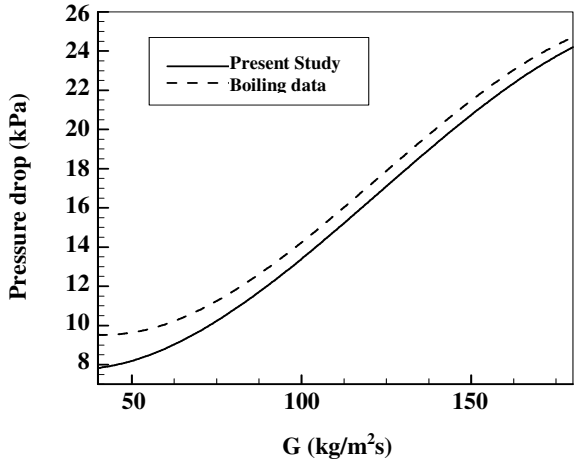

Fig. 2: Comparison of pressure drop between experimental boiling data and current model.

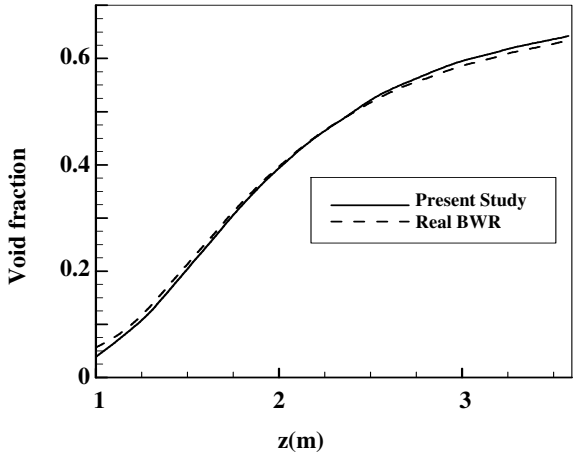

Fig. 4: The void fraction comparison with real BWR.

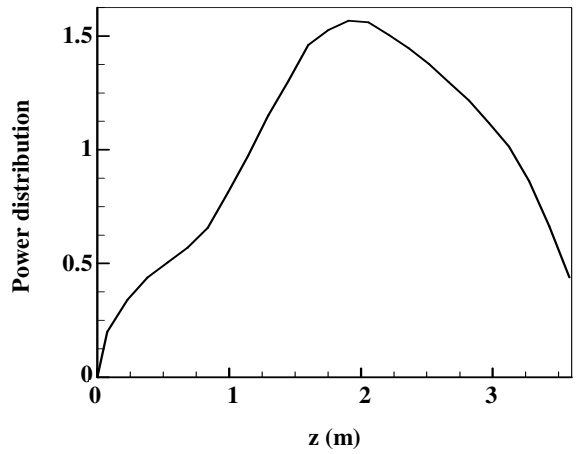

Fig. 3: Normalized power distribution of real BWR [46].

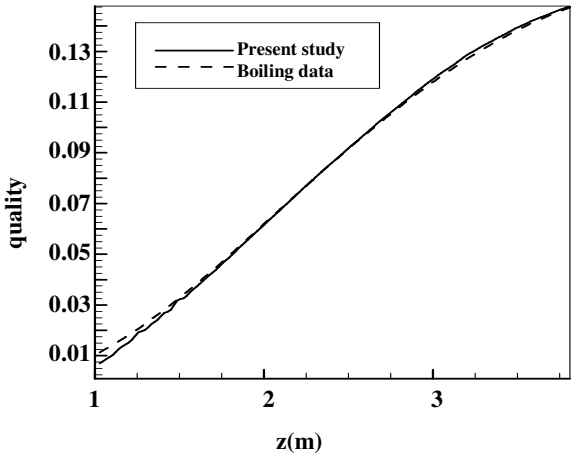

Fig. 5: The quality comparison with boiling data.

RESULTS AND DISCUSSION

In order to validate the result of present model, some comparisons with available experimental data and boiling in a boiling water reactor are done. Since modelling of two-phase flow systems are used to predict the pressure gradient, the void fraction (phase fraction) and velocity of phases, and these objectives are most important. Thus, they need to be investigated to guarantee the validity.

Shunyu et al. [45] have studied boiling heat transfer experiments in vertical narrow channels with pure water. They were conducted their experiments in a low pressure domain within the following range: mass flux 45-180 $kgm^{-2}s$, system pressure 2-3.5 MPa, temperature of inlet water 50-180 ºC, heat flux 40-210 kw m^{-2} and the length of their test section was 1300 mm. The pressure drop comparison of their boiling data and present study is shown in Fig. 2. It is obvious that the pressure drop increases as the fluid velocity increase because of the enhancement of frictional resistance. The maximum relative error of this pressure drop comparison is 14.58 %.

Fig. 2 shows that the agreement between experimental data and predicted pressure drops is reasonable.

The most challenging part of the boiling water reactor (BWR) steady-state analysis is the prediction of the void fraction distribution [46]. To validate the proposed model, comparisons have been made with available real BWR (PB2 BWR/4 NPP) data [46, 47].

The physical and operating conditions for this simulation consist of the active core height 358.14 cm, the normalized core thermal power 3293 MW

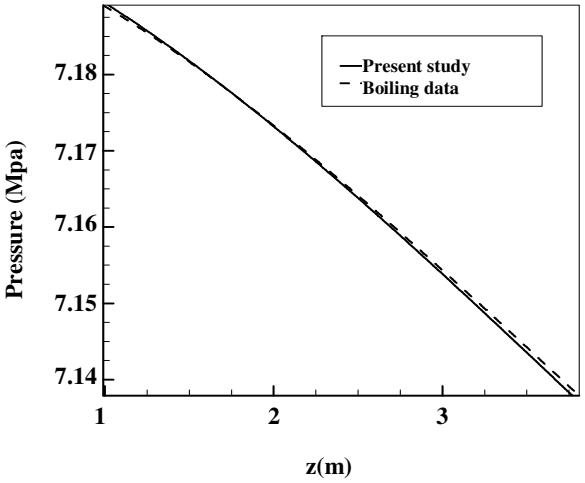

Fig. 6: The pressure changes along the channel.

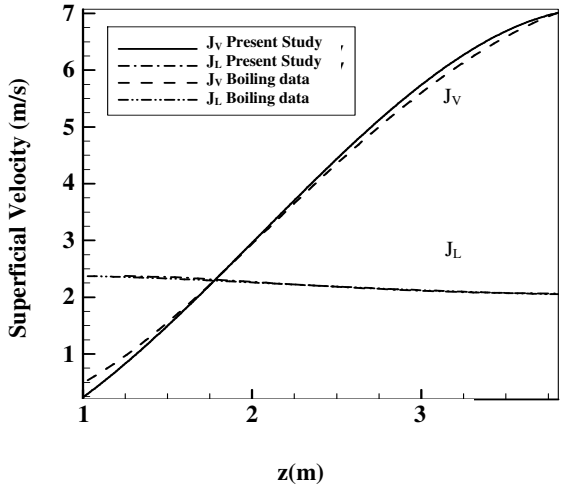

Fig. 7: The variation of the superficial velocities.

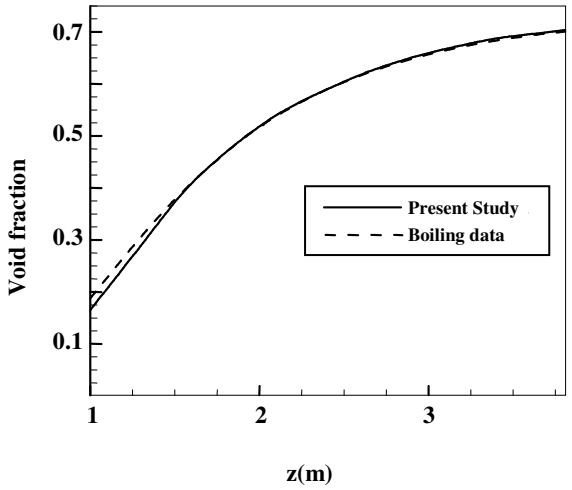

Fig. 8: The void fraction comparison with boiling data.

the proposed theoretical axial void fraction distribution in the saturated boiling region, agrees well with the real (the axial power distribution is approximately cosine shape which is shown in Fig. 3), the water mass flow rate 12915 kg/s, channel inlet pressure 73.084 bar, the inlet enthalpy 1212.5 kJ/kg, in steady state condition. Fig. 4 shows that BWR data. The maximum relative error of this void fraction comparison is 12.68 %.

The results confirm that, present model has good agreement with boiling data of the real boiling water reactor.

The comparison of current study and boiling data of *Yadigaroglu* and *Askari* [48] are shown in Figs. 5 to 8. The following physical and operating conditions have been used for simulation of this boiling data; the channel length 381 cm, the water mass flow rate 17.3797 kg/s, inlet subcooling 10 K, channel cross section flow area 0.0098 m², channel inlet pressure 72 bar, the channel heat generation 4.6791 MW (cosine shape), in steady state condition. Fig. 5 to 8 denote the comparisons of quality, void fraction, pressure drop and superficial velocities, respectively.

Fig. 6 shows the comparison of pressure drop along the channel, which the maximum relative error is less than 1%. The superficial velocity of liquid (Jl) and vapour phase (Jv) along the saturated boiling zone of channel are shown in Fig. 7. Fig. 8 also shows the proposed model void fraction distribution is acceptable as well. The maximum relative error of this void fraction comparison is 11.34 %. Since the temperature and density of phases in the saturation zone depend on local pressure, thus the densities and temperature for any locations could be predictable. According to the comparative study of present model and available data, it can express that in the case of saturated boiling, using Eq. (25) instead of general energy Eq. (3) is acceptable

CONCLUSIONS

A new arrangement of three-dimensional, phase-weighted ensemble averaged two-phase turbulence flow, two-fluid model for simulation wide range of void fraction with heat and mass transfer has been developed. The model is simplified for saturated boiling in a heated channel. The simplification is based on using overall energy balance analysis instead of usual two-phase flow energy equation. This saves significant computer time. The results (pressure drop, void fraction, quality …) are well-

matched with the available experimental boiling data and also with a real BWR (PB2 BWR/4 NPP) data.

The computational results highlight that in the case of saturated boiling it would be better to use overall energy balance analysis, but the accuracy is higher for high pressure case. This could simplify the solution of two phase turbulent flows.

Nomenclatures

A''	Interfacial area density
b_k	Phasic body force, k = l, ν
BWR	Boiling Water Reactor
D	Channel equivalent diameter
D_b	Bubble diameter
E_k	Phasic interfacial heat source, k = l, ν
F_k	Phasic interfacial momentum transfer, k = l, ν
g	Gravitational acceleration
G	Mass flux
h	Enthalpy
H	Channel height
h_{fg}	Latent heat
J	Superficial velocity vector
$\dot{m}_k'''$	Phasic interfacial mass generation, k = l, ν
$\dot{m}_k'''$	Phasic interfacial mass reduction, k = l, ν
$\dot{m}$	Mass flow rate
M	Surface tension source
NPP	Nuclear Power Plant
PB	Peach Bottom
p_k	Phasic pressure, k = l, ν
q''	Heat flux
q'''	Volumetric heat source
$\dot{Q}(z_s)$	Total heat transferred to the channel from inlet to z_s
Re_b	Bubble Reynolds number
u	Velocity component in x axis direction
u_k	Phasic internal energy, k = l, ν
ν	Velocity component in y axis direction
v_k	Phasic velocity vector, k= l, ν
v_r	Relative velocity (v_v-v_l)
w	Velocity component in z axis direction
W_k	Phasic interfacial work, k = l, ν
x_b	Position of a bubble
$x_Q(z)$	Quality at z
x_w	Wall position
z	Vertical direction in Cartesian coordinate
z_s	Any location in the saturation region along the channel

Greek

α_k	Phasic volume fraction, k = l, ν
χ	Phase indicator function
ρ_k	Phasic density, k= l, ν
σ_s	Surface tension
τ_k	Phasic stress tensor, k= l, ν
τ_k^t	Phasic ensemble averaged Reynolds stress tensor, k = l, ν
∈	Interfacial energy source
∇	Del operator

Subscripts

b	Bubble
in	At the inlet of the channel
ki	Phasic Interface, k = l, ν
k	Each phase, k = l, ν
l	Liquid phase
r	Relative
v	Vapour phase

Superscripts

t	Related to Reynolds stress
χ	Weighted average with phase indicator function
χρ	Weighted average with phase indicator function and density

Notation Convention

$\langle\ \rangle$	Ensemble average
$\langle\ \rangle^{\chi}$	χ - Phase weighted ensemble average
$\langle\ \rangle^{\chi\rho}$	χρ - Phase weighted ensem

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