Numerical Solution of MHD Flow over a Nonlinear Porous Stretching Sheet

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ABSTRACT: *In this paper, the MagnetoHydroDynamic (MHD) boundary layer flow over a nonlinear porous stretching sheet is investigated by employing the Homotopy Perturbation Transform Method (HPTM) and the Pade´ approximation. The numerical solution of the governing non-linear problem is developed. Comparison of the present solution is made with the existing solution and excellent agreement is noted. Graphical results have been presented and discussed for the pertinent parameters. The results attained in this paper confirm the idea that HPTM is powerful mathematical tool and it can be applied to a large class of linear and nonlinear problems arising in different fields of science and engineering.*

KEY WORDS: *Homotopy perturbation transform method, MHD boundary layer equation, Nonlinear porous stretching sheet, Pade´ approximants.*

INTRODUCTION

Recently, many scientist and engineers have paid more attention on new methods for solving boundary layer equations, arising from mathematical modeling of fluid mechanics problems [1-7]. The study of laminar boundary layer flow of an incompressible fluid has several important engineering applications such as the aerodynamic extrusion of plastic sheets, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along liquid film condensation process, glass and polymer industries.

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In 1973, *McCormack & Crane* [8] introduced the stretching sheet problem. The stretching problems for steady flow have been extensively in various aspects, for example to non-Newtonian fluids, MHD flows, porous plate, porous medium and heat transfer analysis. The literature on the topic is quite extensive and hence can not be described here in detail. However, some most recent works of eminent researchers regarding the flow over a stretching sheet may be mentioned in the Refs [9-12].

Although the numerical approach allows studying more complex boundary conditions, the importance of analytical solutions is undeniable and it is witnessed by the large amount of work performed, particularly in recent years, on this subject. Various kinds of solutions methods [13-29] were used to handle the boundary layer problem. Recently *Khan & Wu* [30] developed the Homotopy Perturbation Transformation Method (HPTM) by combining the standard homotopy perturbation and Laplace transformation method for solving nonlinear problems. The Laplace transformation method was also combined with the well-known the variational iteration method [31] and the Adomian decomposition method [32-34] to produce a highly effective technique for handling many nonlinear problems.

This technique basically illustrates how the Laplace transform can be used to approximate the solutions of the nonlinear differential equations by manipulating the homotopy perturbation method which was introduced first by *He* [35] and was further developed by *Hesameddini & Latifizadeh* [36]. The method is very well suited to physical problems since it does not require unnecessary linearization, discretization and other restrictive methods and assumptions which may change the problem being solved, sometimes seriously. The basic motivation of the present study is to extend our previous approach proposed in [30] to solve MHD boundary layer problem over a nonlinear porous stretching sheet on semi-infinite domain. The HPTM is much easier to implement as compared with the adomian decomposition method where huge complexities are involved. To the best of authors knowledge no attempt has been made to exploit this method to solve nonlinear equation on semi-infinite domain. Also our aim in this article is to compare the results with solutions to the existing ones [3].

THEORITICAL SECTION

Homotopy perturbation transform method

In order to elucidate the solution procedure of the homotopy perturbation transform method, we consider the following general form of 3rd order non-homogeneous nonlinear ordinary differential equation with initial conditions is given by

$$f''' + b_1(x)f'' + b_2(x)f' + b_3(x)f = g(y), \tag{1}$$

$$f(0) = \alpha,\ f'(0) = \beta,\ f''(0) = \gamma, \tag{2}$$

According to Homotopy perturbation transform method [30], we apply Laplace transform (denoted throughout this paper by L) on both sides of Eq. (1):

$$s^3 L[\,f\,] - s^2\alpha - s\beta - \gamma + L[b_1(x)f''] + L[b_2(x)f'] + L[b_3(x)f] = L[g(f)] \tag{3}$$

Using the differentiation property of Laplace transform, we have

$$L[f] = \frac{\alpha}{s} + \frac{\beta}{s^2} + \frac{\gamma}{s^3} + \frac{1}{s^3}L[g(f)] - \frac{1}{s^3}L[b_1(x)f'' + b_2(x)f' + b_3(x)f] \tag{4}$$

Operating with Laplace inverse on both sides of Eq. (4) gives

$$f = G(x) + \frac{1}{s^3}L[g(f)] - L^{-1}\left[\frac{1}{s^3}L[b_1(x)f'' + b_2(x)f' + b_3(x)f]\right] \tag{5}$$

Now we apply the homotopy perturbation method

$$f = \sum_{n=0}^{\infty} p^n f_n \tag{6}$$

the nonlinear term can be decomposed as

$$g(y) = Nf = \sum_{n=0}^{\infty} p^n H_n \tag{7}$$

for some He's polynomials H_n (see [37]) that are given by

$$H_n = \frac{1}{n!}\frac{d^n}{dp^n}\left[N\left(\sum_{i=0}^{\infty} p^i f_i\right)\right]_{p=0}, \quad n = 0,1,2,3...$$

Substituting Eq. (7) and Eq. (6) in Eq. (5) we get

$$\sum_{n=0}^{\infty} p^n f_n = G(x) + \tag{8}$$
$$p\left(\begin{array}{l} L^{-1}\left[\frac{1}{s^3} L\left[\sum_{n=0}^{\infty} p^n H_n\right]\right] \\ -L^{-1}\left[\frac{1}{s^3} L\left[b_1(x)\sum_{n=0}^{\infty} p^n f_n'' + b_2(x)\sum_{n=0}^{\infty} p^n f_n' + b_3(x)\sum_{n=0}^{\infty} p^n f_n\right]\right] \end{array}\right)$$

Where G(x) represents the term arising from prescribe initial condition.

Comparing co-efficient of like powers of p, following approximations are obtained

$$p^0 : f_0 = G(x) \tag{9}$$
$$p^1 : f_1 = L^{-1}\left[\frac{1}{s^3} L\left[H_0\right]\right] -$$
$$L^{-1}\left[\frac{1}{s^3} L\left[b_1(x) f_0'' + b_2(x) f_0' + b_3(x) f_0\right]\right]$$
$$p^2 : f_2 = L^{-1}\left[\frac{1}{s^3} L\left[H_1\right]\right] -$$
$$L^{-1}\left[\frac{1}{s^3} L\left[b_1(x) f_1'' + b_2(x) f_1' + b_3(x) f_1\right]\right]$$
$$\vdots$$

Problem formulation

Let us consider the MHD flow of an incompressible viscous fluid over a non-linear porous stretching sheet at y=0. The fluid is electrically conducting under the influence of an applied magnetic field B(x) normal to the stretching sheet. The induced magnetic field is neglected. The resulting boundary-layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B_0^2(x)}{\rho} u \tag{11}$$

Here u and v are the velocity components in the x - and y -directions respectively, ν is the kinematic viscosity, ρ is the density and σ is the electrical conductivity of the fluid. In Eq. (11), the external electric field and polarization effects are negligible, therefore

$$B(x) = B_0 x^{\frac{n-1}{2}} \tag{12}$$

The boundary conditions corresponding to the non-linear porous stretching sheet are given below

$$u(x,0) = cx^n,\ v(x,0) = -V_0 \tag{13}$$
$$u(x,y) \to 0 \text{ as } y \to \infty$$

Where c is the stretching parameter V_0 is the porosity of the plate (where $V_0>0$ corresponds to suction and $V_0<0$ corresponds to injection) . Upon making use of the following substitutions

$$\eta = \sqrt{\frac{c(n+1)}{2\nu}} x^{\frac{n-1}{2}} y,\ u = cx^n f'(\eta) \tag{14}$$
$$dv = -\sqrt{\frac{c\nu(n+1)}{2}} x^{\frac{n-1}{2}} [f(\eta) + \frac{n-1}{n+1}\eta f'(\eta)]$$

The resulting non-linear differential equation and boundary conditions are of the following form

$$f''' + ff'' - \beta f'^2 - Mf' = 0 \tag{15}$$
$$f(0) = K,\ f'(0) = 1,\ f'(\infty) = 0$$

where

$$\beta = \frac{2n}{n+1}, M = \frac{2\sigma B_0^2}{\rho c(1+n)},\ K = \frac{V_0}{\sqrt{\frac{c\nu(n+1)}{2}} x^{\frac{n-1}{2}}} \tag{16}$$

β is the non-dimensional parameter, M is the magnetic parameter, and K is the wall mass transfer parameter. In order to seek the solution of Eq. (15) through the homotopy perturbation transform method, we assume $f''(0) = \alpha < 0$, in this work. By applying the aforesaid method subject to the initial conditions, we have

$$L[\ f] = K + \frac{s+\alpha}{s^3} + \frac{1}{s^3} L\left[\beta(f')^2 + Mf' - ff''\right] \tag{17}$$

The inverse of Laplace transform implies that

$$f(\eta) = K + \eta + \frac{\alpha\eta^2}{2} + \frac{1}{s^3} L\left[\beta(f')^2 + Mf' - ff''\right] \tag{18}$$

Applying the homotopy perturbation method, we have

$$\sum_{m=0}^{\infty} p^m f_m(\eta) = K + \eta + \frac{\alpha\eta^2}{2} + \tag{19}$$
$$p\left(L^{-1}\left[\frac{1}{s^3} L\left[\beta\left(\sum_{m=0}^{\infty} p^m H_{1m}(\eta)\right) - \left(\sum_{m=0}^{\infty} p^m H_{2m}(\eta)\right) + M\sum_{m=0}^{\infty} p^m f_m'(\eta)\right]\right]\right)$$

In the above equation $H_{1m}(\eta)$ and $H_{2m}(\eta)$are the *He*'s polynomials [37] that represent the nonlinear terms. The few components of the He's polynomials are given as follows

$$H_{10}(\eta)=(f_0')^2(\eta) \tag{20}$$
$$H_{11}(\eta)=2f_0'(\eta)f_1'(\eta)$$
$$H_{12}(\eta)=(f_1')^2(\eta)+2f_0'(\eta)f_2'(\eta)$$
$$H_{13}(\eta)=2f_1'(\eta)f_2'(\eta)+2f_0'(\eta)f_3'(\eta)$$
$$H_{14}(\eta)=(f_2')^2(\eta)+2f_1'(\eta)f_3'(\eta)+2f_0'(\eta)f_4'(\eta)$$
$$H_{15}(\eta)=2f_0'(\eta)f_5'(\eta)+2f_1'(\eta)f_4'(\eta)+2f_2'(\eta)f_3'(\eta),$$
$$\vdots$$
$$\vdots$$
$$H_{1m}(\eta)=\sum_{i=0}^{m}f_i'(\eta)f_{m-i}'(\eta)$$

And for $H_{2m}(\eta)$ we find

$$H_{20}(\eta)=f_0(\eta)f_0''(\eta) \tag{21}$$
$$H_{21}(\eta)=f_0(\eta)f_1''(\eta)+f_1(\eta)f_0''(\eta)$$
$$H_{22}(\eta)=f_0(\eta)f_2''(\eta)+f_1(\eta)f_1''(\eta)+f_2(\eta)f_0''(\eta)$$
$$H_{23}(\eta)=f_0(\eta)f_3''(\eta)+f_1(\eta)f_2''(\eta)+f_2(\eta)f_1''(\eta)+f_3(\eta)f_0''(\eta)$$
$$H_{24}(\eta)=f_0(\eta)f_4''(\eta)+f_1(\eta)f_3''(\eta)+f_2(\eta)f_2''(\eta)+ f_3(\eta)f_1''(\eta)+f_4(\eta)f_0''(\eta)$$
$$H_{25}(\eta)=f_0(\eta)f_5''(\eta)+f_1(\eta)f_4''(\eta)+f_2(\eta)f_3''(\eta)+ f_3(\eta)f_2''(\eta)+f_4(\eta)f_1''(\eta)+f_4(\eta)f_1''(\eta)$$
$$\vdots$$
$$\vdots$$
$$H_{2m}(\eta)=\sum_{i=0}^{m}f_i(\eta)f_{m-i}''(\eta)$$

Comparing the coefficient of like powers of p, we have

$$p^0: f_0(\eta)=K+\eta+\frac{\alpha\eta^2}{2} \tag{22}$$

$$p^1: f_1(\eta)=L^{-1}\left[\frac{1}{s^3}L\left[\beta\left(H_{10}\right)-H_{20}+Mf_0'(\eta)\right]\right] \tag{23}$$

$$p^2: f_2(\eta)=L^{-1}\left[\frac{1}{s^3}L\left[\beta H_{11}(\eta)-H_{21}+Mf_1'(\eta)\right]\right],$$
$$\vdots$$

Writing $f_0(\eta)=K+\eta+\frac{\alpha\eta^2}{2}$ in Eq. (23), the other components are

$$f_1(\eta)=\left(\frac{M}{6}+\frac{\beta}{6}-\frac{K\alpha}{6}\right)\eta^3+\left(-\frac{\alpha}{24}+\frac{\alpha M}{24}+\frac{\alpha\beta}{12}\right)\eta^4+ \left(-\frac{\alpha^2}{120}+\frac{\alpha^2\beta}{60}\right)\eta^5 \tag{24}$$

$$f_2(\eta)=\left(\frac{K^2\alpha}{24}-\frac{KM}{24}-\frac{K\beta}{24}\right)\eta^4+ \left(-\frac{\beta}{60}+\frac{M^2}{120}-\frac{M}{60}+\frac{M\beta}{40}+\frac{\beta^2}{60}+\frac{K\alpha}{40}-\frac{K\alpha M}{60}-\frac{K\alpha\beta}{30}\right)\eta^5+ \left(\frac{\alpha}{240}-\frac{\alpha M}{90}+\frac{\alpha M^2}{720}-\frac{\alpha\beta}{60}+\frac{\alpha M\beta}{72}+\frac{\alpha\beta^2}{72}+\frac{\alpha^2K}{144}-\frac{\alpha^2K\beta}{90}\right)\eta^6+ \left(\frac{11\alpha^2}{5040}-\frac{\alpha^2M}{630}-\frac{2\alpha^2\beta}{315}+\frac{\alpha^2M\beta}{504}+\frac{\alpha^2\beta^2}{252}\right)\eta^7+ \left(\frac{11\alpha^3}{40320}-\frac{\alpha^3\beta}{1260}+\frac{\alpha^3\beta^2}{2016}\right)\eta^8$$

The series solution is given as

$$f(\eta)=K+\eta+\frac{\alpha\eta^2}{2}+\left(\frac{M}{6}+\frac{\beta}{6}-\frac{K\alpha}{6}\right)\eta^3+ \left(\frac{K^2\alpha}{24}-\frac{KM}{24}-\frac{K\beta}{24}-\frac{\alpha}{24}+\frac{\alpha M}{24}+\frac{\alpha\beta}{12}\right)\eta^4+ \left(-\frac{\beta}{60}+\frac{M^2}{120}-\frac{M}{60}+\frac{M\beta}{40}+\frac{\beta^2}{60}+\frac{K\alpha}{40}-\frac{K\alpha M}{60}-\frac{K\alpha\beta}{30}-\frac{\alpha^2}{120}+\frac{\alpha^2\beta}{60}\right)\eta^5+ \left(\frac{\alpha}{240}-\frac{\alpha M}{90}+\frac{\alpha M^2}{720}-\frac{\alpha\beta}{60}+\frac{\alpha M\beta}{72}+\frac{\alpha\beta^2}{72}+\frac{\alpha^2K}{144}-\frac{\alpha^2K\beta}{90}\right)\eta^6+ \left(\frac{11\alpha^2}{5040}-\frac{\alpha^2M}{630}-\frac{2\alpha^2\beta}{315}+\frac{\alpha^2M\beta}{504}+\frac{\alpha^2\beta^2}{252}\right)\eta^7+ \left(\frac{11\alpha^3}{40320}-\frac{\alpha^3\beta}{1260}+\frac{\alpha^3\beta^2}{2016}\right)\eta^8+.... \tag{25}$$

The padé approximants

Our aim in this section is mainly concerned with the mathematical behaviour of the solution $f(\eta)$ in order to determine the value of free parameter $\alpha=f(0)$. It was formally shown by *Wazwaz & Boyd* [38, 39] this goal can easily be achieved by forming the Padé approximants [40] which have the advantage of manipulating the polynomial approximation into a rational function to obtain the more information about $f(\eta)$. It is well known fact that Padé approximants will converges on the entire real axis if $f(\eta)$ is free of singularities on the entire real axis. More importantly, the diagonal approximants are most accurate approximants, therefore we will construct on diagonal approximants. Using the boundary condition $f'(\infty)=0$, the diagonal approximants [M/M] vanish if the coefficients of numerator vanish with the highest power in the η. Choosing the coefficients of the highest power of η equal to zero, we get a polynomial equations in α which can be solved very easily by using the built in utilities in the most manipulation languages such as Maple and Mathematica.

RESULTS AND DISCUSSION

Tables 1 and 2 clearly reveal that present solution method namely HPTM shows excellent agreement with the existing solutions in the literature [3]. This analysis shows that HPTM suits for MHD boundary layer flow problems.

The effects of the mass suction parameter K, the magnetic parameter M, and the non-dimensional parameter β are shown in Figs. 1-3. It is evident from Fig. 1 that when K increases, the velocity profiles are far away from the wall for mass injection, and the boundary layer thickness is more and more thicker. Fig. 2 and 3 are prepared for f' against η for M, β respectively. It is observed from Fig. 2 that when M, increases, the velocity profile is more and more far away from the wall, and the boundary layer thickness is more and more thicker. Fig. 3 indicates that when β increases; the velocity profile is more and more far away from the wall, whereas the boundary layer thickness increases. Therefore, it is concluded from Fig.1 that f' decreases when K increases. One can see from the Fig. 2 and Fig. 3 shows similar effect with respect to M and β for f'.

CONCLUSIONS

The homotopy perturbation transform method is applied for numerical treatment of nonlinear differential equations that appear on boundary layers in fluid mechanics. The HPTM accelerates the rapid convergence of the series solution, dramatically reduces the size of work. The obtained series solution is combined with the diagonal Pade´ approximants to handle the boundary condition at infinity. The convergence of HPTM is also shown in Table.1 and Table 2. Comparison of the present solution is made with the existing solution [3]. An excellent agreement between the present and existing solutions is achieved. The analysis given here further shows confidence on HPTM.

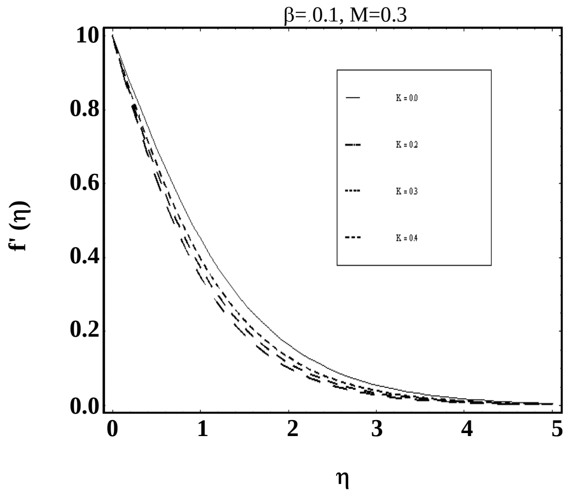

Fig. 1: Variation of f' for different values of K.

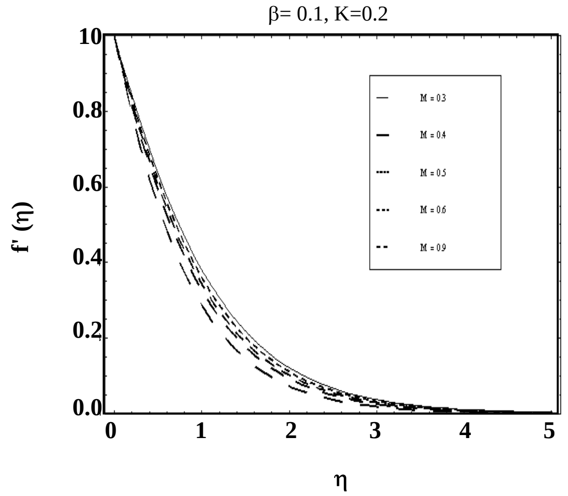

Fig. 2: Variation of f' for different values of M.

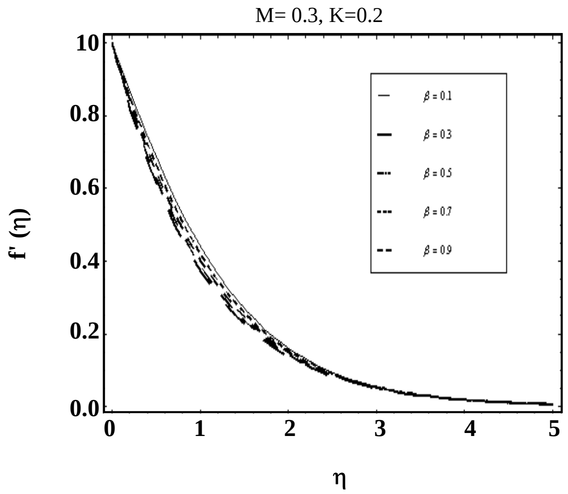

Fig. 3: Variation of f' for different values of β.

Table 1: Comparison of the numerical value of $f''(0) = \alpha$, obtained by HPTM.

K	β	M	Croco Transformation [3]	Shooting Method [3]	HPTM
0	1.5	0	-1.4902	-1.4860	-1.4902
		1	-1.5253	-1.52527	-1.5253
		5	-2.51616	-2.5162	-2.5161
		10	-3.36632	-3.36631	-3.3658
		50	-7.16471	-7.16471	-7.16354
		100	-10.0664	-10.0664	-10.0648

Table 2: Comparison of the numerical value of $f''(0) = \alpha$, obtained by HPTM.

K	β	M	Croco Transformation [3]	Shooting Method [3]	HPTM
0	5	0	-1.90433	-1.9025	-1.9031
		1	-2.15344	-2.1529	-2.1529
		5	-2.94150	-2.94144	-2.94142
		10	-3.69567	-3.6956	-3.6956
		50	-7.32561	-7.3256	-7.3256
		100	-10.1816	-10.1816	-10.1816

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