

# Hydrodynamic Study of a Rising Bubble in the Presence of Cetyltrimethylammonium Bromide Surfactant

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**ABSTRACT:** *The performance of surfactants especially in two-phase systems, depends on their type, Hydrophile-Lipophile Balance (HLB) number, concentration, and whether the surfactant is ionic or non-ionic. The current work was conducted to study the effect of the presence of Cetyltrimethylammonium Bromide (CTAB), a cationic surfactant, in air-water systems. Thus, the behavior of a single air bubble rising in aqueous CTAB solutions was studied experimentally. The independent test variables are solution concentration (0.4-1.6mM) and bubble diameter (3.5-4.6mm). The effect of these variables on rising velocity, bubble shape, and drag coefficient has been studied. Due to the importance of the drag coefficient in two-phase fluid, the effect of Reynolds number, Eötvös number, and aspect ratio on it has also been evaluated. Experiments have been performed at high Reynolds numbers ( $850 < Re < 1000$ ), which are obtained by increasing the surfactant concentration. The results showed that the selected ionic surfactant had a more tangible effect on bubble behavior than nonionic surfactants. Moreover, there is no noticeable difference in the behavior of the bubble rising at concentrations above and below of Critical Micelle Concentration (CMC) of CTAB, which can be attributed to its high aggregation number compared to other surfactants.*

**KEYWORDS:** *Drag coefficient; Bubble rising; Cetyltrimethylammonium bromide; Equivalent diameter; Surfactant Concentration; Terminal velocity.*

## INTRODUCTION

The behavior of bubbles within a fluid is one of the fundamental issues in a two-phase phenomenon which has received a lot of attention so far. In fact, understanding the behavior is essential to control the rising bubble within the fluid and it is of significant importance to know it, especially in industrial applications. Based on studies, different parameters will affect the bubble behavior. For example, the type of fluid of continuous phase has been shown to greatly influence the behavior [1, 2]. Meanwhile,

hydrodynamics of rising bubbles through contaminated fluid by a surfactant has attracted the most attention of researchers [3, 4]. A specific property of surfactants is the aggregate content and form of micelles in different media [5]. Also, the results of studies [6, 7] indicate that the presence of surfactant has a significant effect on two-phase behavior and it changes some fluid properties such as viscosity, density, and surface tension. Moreover, due to the accumulation of surfactant molecules on the liquid-gas

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surface, some parameters such as the bubble surface mobility [8], drag coefficient [9] and bubble velocity [10] will be affected, so that the amount of the effectiveness varies according to the type of surfactant [11]. Generally, adding surfactant causes the decreasing velocity, the prevention of coalescence, and the reduction of mass transfer and internal circulation happens by adsorption layer at the interface. A little amount of surfactant can change the behavior of a rising bubble dramatically. One of the known effects of the presence of surfactant on the movement of the bubble is to reduce the bubble velocity. On the other hand a bubble in aqueous surfactant solution rises more slowly than a bubble in pure water. This phenomenon is explained by "Marangoni's effect", which is due to the non-uniform distribution of surfactant on the bubble surface.

Flotation is a popular industrial technique that leads to increased material production, especially in mineral separation processes. Surfactant is frequently utilized to improve the efficiency of this process. In this method, some mineral particles are brought to the surface with the help of the bubbles formed, while additional particles that are not allowed to attach to the bubble surface are sent to the bottom of the column. Due to the fact that the absorption rate increases as the bubble size grows, so the bubble diameter is an essential parameter in the flotation process [12]. Wang *et al.* study's [13] summarizes the effect of bubble size on the flotation process. According to the research, the bubble size is an essential factor in flotation. Furthermore, if surfactants are utilized, surface tension has a substantial impact on this process [12].

A factor that will be affected by surfactant is drag coefficient. Wenyuan *et al.* [14] demonstrated that the more surfactant concentration, the more drag coefficient. Furthermore, terminal velocity changes inversely with the drag coefficient. Tzounakos *et al.* [15] evaluated the effect of the presence of surfactants on the rising of the bubble in the fluid. Their results showed that increasing concentration of surfactants not only decreases the terminal velocity but also leads to drag coefficient increment. Vecer *et al.* [16] also investigated the behavior of bubble in different types of fluids containing surfactant. They showed that at the high Reynolds number, there is a direct relationship between the Reynolds number and drag coefficient. For  $600 < Re < 800$ , the drag coefficient varies between 1 and 2 which depends on the type of fluid. In some studies,

surfactant has changed the Reynolds number [14, 15]. Their results show that with increasing Reynolds number, the drag coefficient first decreases and then increases. In other words, at high Reynolds number, which corresponds to high concentrations of surfactant, the drag coefficient is directly related to the Reynolds number.

Therefore, present study was done aimed to recognize the effect of CTAB on behavior bubble rising. Thus, the rising behavior of a single bubble of different diameters in CTAB-contaminated fluid with different concentrations was investigated. The parameters that have been studied for this purpose include terminal velocity, drag coefficient and aspect ratio of bubble versus operating conditions and dimensionless numbers. To our knowledge, this is the first time millimeter-sized bubbles have been studied in the presence of a cationic surfactant in the high Reynolds range.

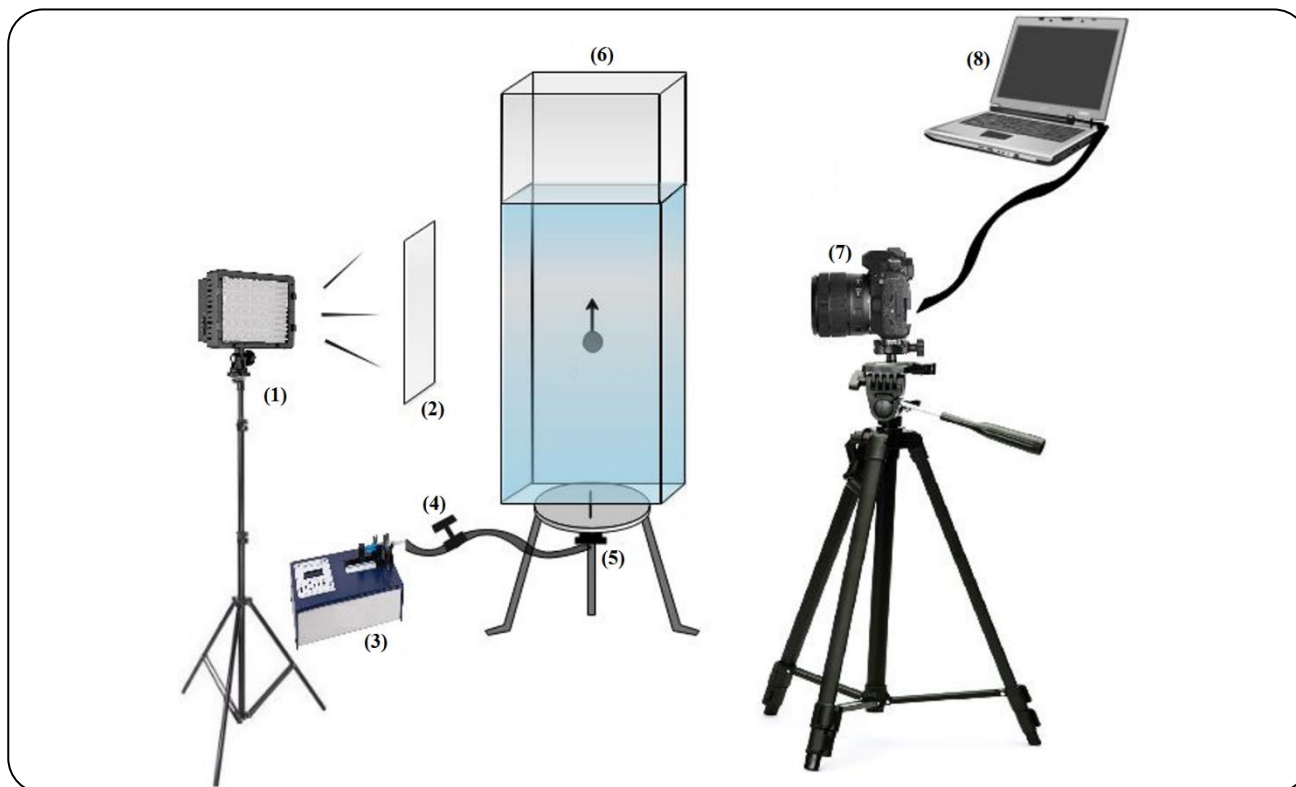
## EXPERIMENTAL SECTION

### *Experimental Set-up*

The used apparatus for the experimental was shown in Fig. 1 where different constitutive parts of the set-up were introduced. The main part of the setup is a Plexiglas rectangular container with dimensions of  $450\text{ mm} \times 105\text{ mm} \times 40\text{ mm}$ . The dimensions of the column are selected so that the effect of the wall on the bubble rising was negligible, called free bubble. The column was in contact with the atmosphere from above and experiments were performed at an ambient temperature ( $20^\circ\text{C}$ ). The container was filled to a depth of 20 cm with different concentration of CTAB solution.

A single bubble was released at various needles of 0.6, 0.7, 0.8, 1.6 mm inner diameter at the bottom of the column. The rate of gas flow was set at 0.4 mL/min. Consequently, this was selected so that the rising bubbles do not interact with each other. In other words, the distance between the two bubbles is so high that there is not practically interaction between them. For each test, experiments were repeated at least three times and their average was presented to minimize error.

The trajectory of single bubble was recorded by a camera that connected to a computer. To achieve high-resolution images of the bubbles, a 100 mm macro lens (AT-X M100 Pro, Tokina, Japan) was used. Illumination system is a LED lamp that was located at adequate distance from the column for lighting balance. The details of image



**Fig. 1:** Schematic of experimental apparatus, 1: LED lamp, 2: Diffuser, 3: Syringe pump (gas flow rate= 0.4 mL/min.), 4: One-way valve, 5: Needle (needle diameters =0.6, 0.7, 0.8, 1.6 mm), 6: liquid column, 7: Camera and macro lens, 8: Image processing system

processing and calculations are provided by Karimi *et al.* [4, 10, 17].

### Materials

In the present work, the selected surfactant to study the bubble rising in the presence of it was Cetyltrimethylammonium bromide (*CTAB*). Aqueous solutions of *CTAB* (different concentrations of 0.4 to 1.4 mmole/L) were used as working fluid.

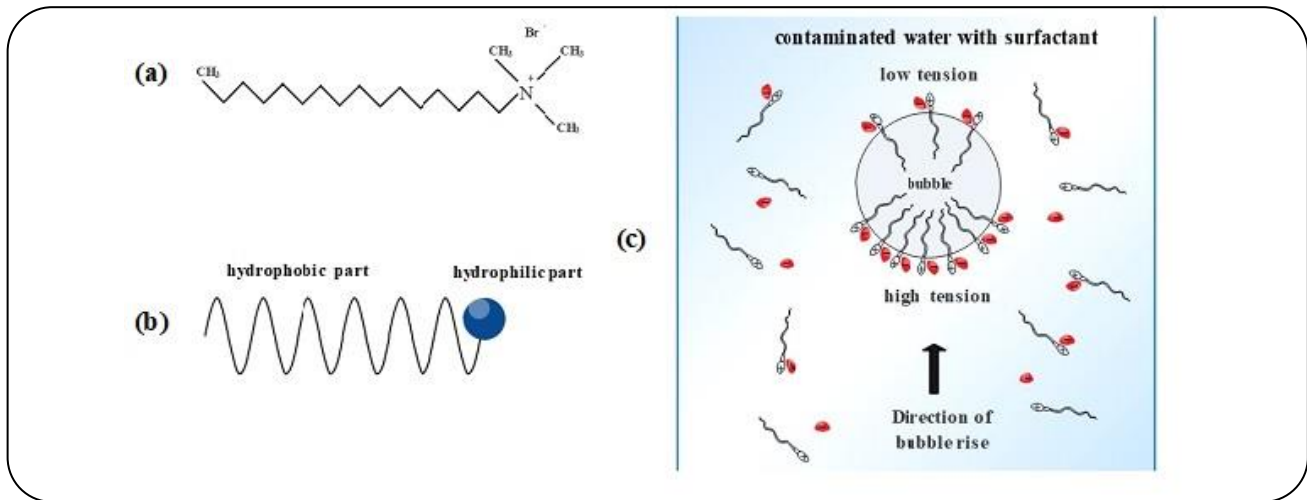
Here, the viscosity was measured using an Ostwald viscometer. The pendant drop method was also used to determine the surface tension of the solutions. Physical properties of the solutions are shown in Table 1. The critical micelle concentration (*CMC*) of *CTAB* at ambient temperature is about  $9.7 \times 10^{-4} \text{ mol/dm}^3$  [16]. Here, the selected concentrations are such that three of them are below *CMC* and two are above it. *CMC* is a concentration of surfactant in which the surface concentration of surfactant molecules reaches such a level that they begin to accumulate to form micellar structures [18]. As shown in Table 1, the viscosity and surface tension decrease with

increasing concentration of the solution, while the density does not depend on the concentration.

*CTAB* is an amphiphilic surfactant contains a hydrophilic ammonium part and a strong hydrophobic alkyl chain in their structure (refer to Fig. 2a &b). This cationic surfactant has a positive polar charge [10]. *CTAB* is an antiseptic agent against bacteria and fungi that is often utilized for disinfection and sanitation in a variety of fields, such as medicines, anticancer, morphology of the gold nanoparticles and food manufacturing [19]. It is the most common surfactant in nanoparticle production. Moreover, due to its positive charge as well as its high capability and selectivity, it is used as a coating agent in heavy metal removal from wastewater [20]. Gurses *et al.* [21] investigated the adsorption mechanism of *CTAB* as a cationic surfactant and showed that it reaches to its adsorption equilibrium rapidly and the dominant mechanism at low concentration of *CTAB* is ion exchange and ion pairing. The study on the effect of different surfactants on boiling heat transfer also showed that *CTAB* works well in this regard [22]. So that by reducing the surface tension, it causes the bubbles to separate more

**Table 1: Physical parameters for different solution concentration.**

Concentration (mmole/lit)	$\mu$ (cp)	$\sigma$ (N/m)	$\rho$ (kg/m <sup>3</sup> )
0.4	0.970	0.068535	1000
0.6	0.967	0.06059	1000
0.8	0.937	0.05590	1000
1	0.923	0.052405	1000
1.2	0.882	0.051113	1000
1.4	0.857	0.048037	1000



**Fig. 2: Schematic of the presence of surfactant CTAB within water: a) Chemical CTAB formula [24], b) hydrophilic and hydrophobic parts in surfactant structure, c) Dispersion of surfactant into water and adsorption of hydrophobic part to the bubble surface.**

easily from the surface and thus the heat transfer increases. *Anchkov et al.* [23] demonstrated that *CTAB* had the highest aggregation number compared to other surfactants. Nevertheless the micelles that they formed were still spherical. In general, it is specified that ionic surfactants have high foamability due to their high adsorption rate, interfacial activity and high number adsorbed molecules at air-water interface [21]. On the other hand, the long tail of a surfactant, the less rising velocity of bubble [21]. The summary of these studies and their results encouraged the authors to study the behavior of a bubble rising in the presence of *CTAB* surfactant, which has not been completely done so far.

Fig. 2 shows the molecular structure of *CTAB* and general schematic of its hydrophilic and hydrophobic parts. The hydrophobic part is adsorbed on the surface of the bubble and the hydrophilic part is drawn towards the water that this is clear in Fig. 2(c). Here the dispersion of the surfactant within the liquid phase and the transfer of the hydrophobic part to the bubble surface are shown.

As a bubble rises through a contaminated continuous fluid with active agents, the accumulation of surfactant behind the bubble increases. It leads to an increase in surface tension behind the bubble. In other words, the bubble movement causes the adsorbed surfactant at the front edge of bubble to migrate into the end edge of it, which results in variation of surface tension at the bubble surface (refer to Fig. 2c). This phenomenon is called the “Marangoni effect” and is attracting a great deal of attention [23, 25]. According to this effect, in most cases, the velocity of rising bubble decreases with increasing surfactant concentration [26]. Most researchers believe that the difference between the results of pure and contaminated fluid is related to the accumulation of surfactant on the bubble surface [27]. Recently, *Karimi et al.* [10] examined the effect of the presence of biosurfactant on free bubble and wall-bounded flow. They showed the decrease in velocity due to the increase in concentration, applies only to free flow and not to wall-bounded flow. This trend was also achieved by

some other studies [10, 28]. Hence, CTAB is selected in the present work.

### Drag coefficient calculation

The transfer equation for a rising bubble ascent, when the forces of gravity, buoyancy, and drag are in equilibrium, is expressed as follows.

$$(\rho_L - \rho_G)g \frac{\pi d_H^3}{6} = C_D \frac{1}{2} \rho_L V^2 \frac{\pi d_H^2}{4} \quad (1)$$

Where  $\rho_L$  and  $\rho_G$  are the densities of liquid and gas, respectively,  $g$  is the gravitational acceleration,  $C_D$  is the drag coefficient,  $V$  is velocity and  $d_H$  is the horizontal diameter of the bubble rising. It should be noted that the relative motion of the bubble and the fluid is in the vertical direction. Therefore, the drag coefficient is calculated using the bubble projection on the horizon. Due to the system's symmetry, the projection is supposed to be a circle with diameter of  $d_H$ . Because the gas density is very small in comparison to the density of the liquid, the drag coefficient may be calculated using the following equation:

$$C_D = \frac{4d_H g}{3V^2} \quad (2)$$

Here, the drag coefficient is calculated using Eq. 2.

### Estimation of measurement precision

It is evident that there is a measurement accuracy error, in image processing and video recording. The following formula presented by Ziqi et al. [29] is used to calculate the uncertainty in determining the bubble velocity:

$$\Delta V_t = \sqrt{\left(\frac{\partial V_t}{\partial P_1} dP_1\right)^2 + \left(\frac{\partial V_t}{\partial P_2} dP_2\right)^2 + \left(\frac{\partial V_t}{\partial \Delta t} d\Delta t\right)^2 + \left(\frac{\partial V_t}{\partial S} dS\right)^2} \quad (3)$$

Where  $V_t$  is bubble terminal velocity,  $P_1$  and  $P_2$  are the vertical placements of bubble centers in two consecutive photos that were taken,  $S$  (mm per pixel) is a scale that determines how accurate the bubble image is,  $\Delta t$  is the difference in time between two imaged (Reverse shutter speed)  $\Delta t = (60fps)^{-1}$ ,  $d\Delta t = (60fps)^{-2}$ .  $dP$  is denoted the positional inaccuracy of the image, which is

related to bubble velocity. The other parameters in Eq. (3) are as follows that the computations are obtained in two successive frames:

$$V_t = \frac{(P_2 - P_1)S}{\Delta t} \quad (4)$$

$$-\frac{\partial V_t}{\partial P_1} = \frac{\partial V_t}{\partial P_2} = \frac{S}{\Delta t} \quad (5)$$

$$\frac{\partial V_t}{\partial \Delta t} = \frac{P_2 - P_1}{\Delta t^2} S \quad (6)$$

$$\frac{\partial V_t}{\partial S} = \frac{P_2 - P_1}{\Delta t} \quad (7)$$

It is assumed that  $dP$  is a function of  $V_t$ , in this case. When  $V_t$  is 400 mm/s,  $dP$  is considered  $\pm 1$ , while the  $V_t$  is zero,  $dP$  is also equal to zero. Consequently, the relationship between  $dP$  and  $V_t$  is expressed as follows:

$$dP = KV_t^2 \quad (8)$$

Where  $K = 6.25 \times 10^{-6} \text{ mm}^2 (\text{s}^2 \cdot \text{pixel})^{-1}$  and  $dS$  is the calibration error:

$$dS = \frac{105.44}{478} - \frac{105}{478} = 9.2 \times 10^{-4} \text{ mm/pixel} \quad (9)$$

By calculating  $\Delta V_t$ , it is determined that the relative error defined by the following relation:

$$E_r = \frac{\Delta V_t}{V_t} \times 100\% \quad (10)$$

Fig. 3 shows the relationship between bubble velocity and the relative error obtained from the equations. According to Fig. 3, for  $0 < V_t < 400 (\text{mm/s})$  the relative error increases as the terminal velocity increases. As shown in Eq. (3), the main cause of the error in the  $dP$  calculation is related to the image processing around the bubble edge.

## RESULTS AND DISCUSSIONS

In this work, the effect of adding CTAB surfactant in water fluid has been investigated experimentally. The experiments were performed by injecting air into water and various concentrations of CTAB solution with four separate needle internal diameters (0.6, 0.7, 0.8 and

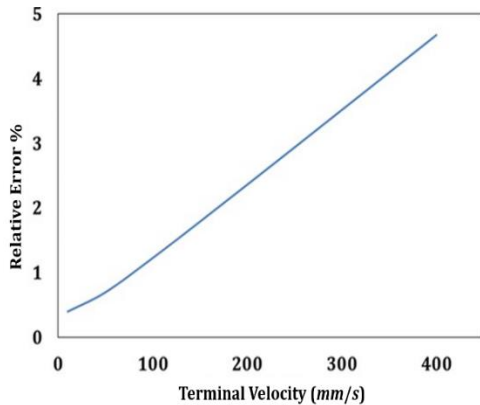


Fig. 3: Dependence of terminal velocity with relative error

1.6 mm). The equivalent diameter varies between 3.5 mm and 4.6 mm, which is controlled by needle diameter.

Table 2 also shows the Peclet number. The reported Peclet number shows a comparison between the penetration characteristic time ( $d_e^2/D$ ) and the surface advection characteristic time ( $d_e/V_r$ ) [30]. Here, the diffusion coefficients of Smith [31], which have been obtained experimentally for CTAB solution at different concentrations, have been utilized to calculate the Peclet number. As can be seen, the Peclet number is relatively high for the experimental conditions of the present study. This means that although the fluid flow is very slow, the mass transfer is affected by the advection [32]. Generally, when  $Pe > 10^3$ , there will be the highest concentration gradient in the bubble surface near, so the boundary layer created by the flow transfer also participates in the mass transfer. As a result, it can be stated that when a spherical bubble rises in high Peclet number and the amount of surfactant is so dilute that it slightly affects the bubble movement, the penetration boundary layer around the surface is very thin [33].

According to the results, which will be reported below, bubble motion, drag coefficient and terminal velocity of bubble are markedly affected by the presence of CTAB surfactant.

### Bubble trajectory

The path of bubbles within a fluid is called trajectory, which is considered relative to X and Y axes [34]. The type and concentration of additives affect the direction of bubble motion within the fluid. Various parameters such

as bubble form, its size, fluid type (Newtonian or non-Newtonian), and the presence of surfactant were influenced the amplitude frequency of bubble oscillation, according to Rafiei Mehrabadi [35]. According to several research, regardless of bubble size, bubbles rise in a direct path in non-Newtonian fluids [1, 4], but bubbles rise in a zigzag course in Newtonian fluids [10, 36, 37]. Moreover, the bubble path is zigzagged in the presence of surfactant, according to Karimi et al. [10]. The researchers discovered that the higher the surfactant content, the lower the amplitude of path changes. Fig. 4 shows the bubble rising path in the present experiments. The time interval between taking each image is 1 second. The amplitude of the motion in the bubble rising path is inversely related to the surfactant concentration in the needle diameter constant, as shown in Fig. 4. The finding is consistent with the findings of other researchers [36, 37].

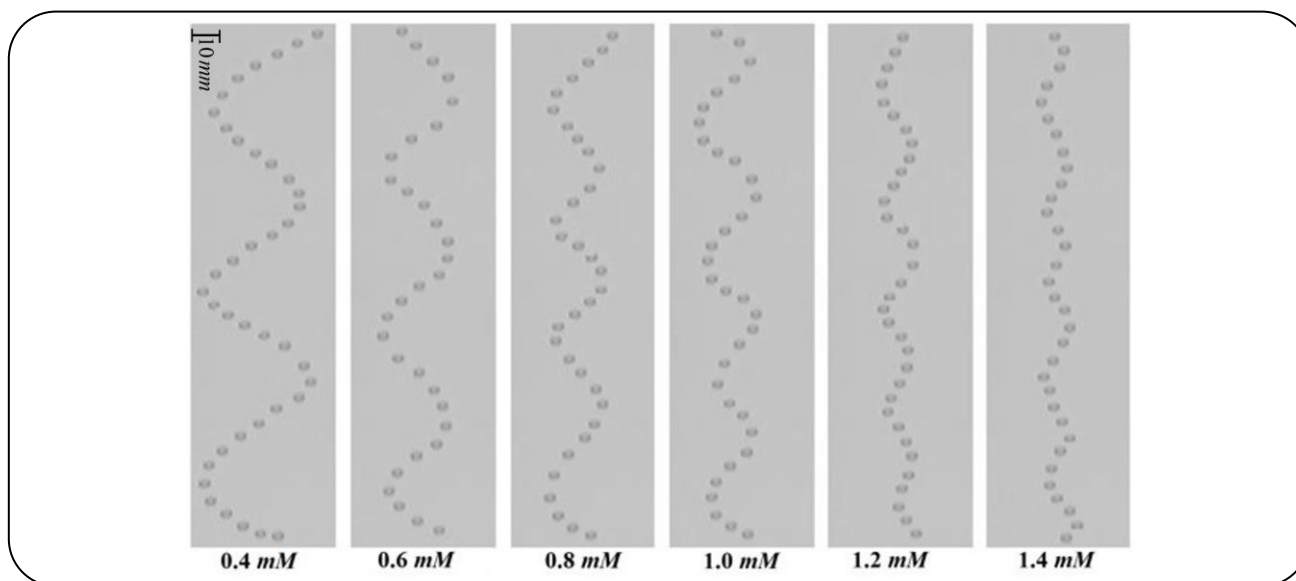
### Shape and size of the rising bubble

As reported by Jamialahmadi and Muller-Steinhagen [19], equivalent diameter of a rising bubble is independent of solution concentration of contaminated water. They obtained the results during the study on the effect of reagent concentration (potassium chloride and ethanol) on bubble equivalent diameter. The same results were observed in the present work. As illustrated in Fig. 5 the bubble size is not affected by CTAB concentration. The differences observed in a concentration in Fig. 5 are only related to the fact that experiments were performed at a concentration with different needle diameters. Therefore, it can be concluded that regardless of the fluid, concentration changes do not cause a significant change in bubble equivalent diameter, whether the concentration was above CMC or not.

Another parameter considered here to evaluate the shape of a bubble is its aspect ratio. The ratio between the largest horizontal diameter ( $0^\circ$  angle with the horizontal axis) and the largest vertical diameter ( $90^\circ$  angle with the horizontal axis) is called aspect ratio. The values of the aspect ratio in the present work were about 0.86, which indicates an elliptical towards spherical shape. This is while in the previous literatures the aspect ratio of bubble air in pure water reported to be about 0.7 [15], which is different from the amount obtained in contaminated water with CTAB. Moreover, as shown in Fig. 5, the aspect ratio remains unchanged versus concentration. Generally,

**Table 2: Peclet number for different equivalent diameter and terminal velocity in various concentration.**

Needle diameter (mm)	Concentration (mM)	$d_e$ (mm)	$V_t$ (mm/s)	Peclet Number
0.6	0.4	3.53	222.86	$1.02 \times 10^6$
	0.6	4.12	221.20	$1.46 \times 10^6$
	0.8	4	218.37	$1.65 \times 10^6$
	1	4.37	213.62	$1.51 \times 10^6$
	1.2	4.15	209.49	$1.22 \times 10^6$
	1.4	3.71	196.89	$0.92 \times 10^6$
0.7	0.4	3.98	214.93	$1.11 \times 10^6$
	0.6	4.31	213.71	$1.48 \times 10^6$
	0.8	4.04	209.22	$1.60 \times 10^6$
	1	4.22	207.69	$1.42 \times 10^6$
	1.2	4.30	204.97	$1.24 \times 10^6$
	1.4	4.07	204.06	$1.04 \times 10^6$
0.8	0.4	4.36	214.37	$1.21 \times 10^6$
	0.6	4.21	210.00	$1.42 \times 10^6$
	0.8	4.03	208.94	$1.59 \times 10^6$
	1	3.81	208.79	$1.29 \times 10^6$
	1.2	4.34	207.66	$1.27 \times 10^6$
	1.4	4.08	207.23	$1.06 \times 10^6$
1.6	0.4	4.41	216.51	$1.24 \times 10^6$
	0.6	4.25	214.46	$1.47 \times 10^6$
	0.8	4.52	210.45	$1.79 \times 10^6$
	1	4.28	203.99	$1.41 \times 10^6$
	1.2	4.47	201.31	$1.27 \times 10^6$
	1.4	4.36	200.11	$1.10 \times 10^6$



**Fig. 4: Bubble trajectory at different concentrations surfactant at a constant needle diameter of 0.6mm.**

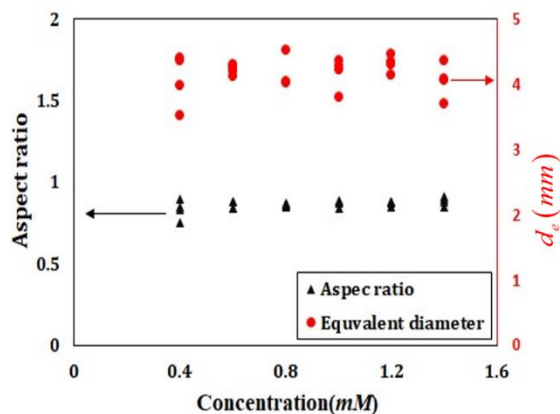


Fig. 5: Equivalent diameter and aspect ratio of rising bubble versus CTAB concentration, needle diameter 0.6-1.6 mm.

bubble deformation in a contaminated solution depends on the type of contaminant. For example, in Tea saponin and Rhamnolipid solutions, there is no significant changes compared to pure water [19, 38]. While in some chemosynthetic surfactant such as Triton X-100, the change is noticeable and bubble deforms into elliptical [15]. Therefore, it can be stated that the effect of the presence of surfactant in water has a stronger effect than the solution contaminated with biosurfactant [37]. Furthermore, it is demonstrated that after the bubble reaches to its terminal velocity aspect ratio remains unchanged, regardless of the type of contaminant [36]. The same result has also been observed in the present work. This can be explained by the fact that at high concentrations of the surfactant or biosurfactant, it is completely absorbed on the bubble surface and the surface is saturated [37] and low concentrations this is justified by reduced interface mobility [36].

#### Terminal velocity

Fig. 6 shows the obtained terminal velocity of single bubble injected from various needle diameters versus CTAB concentration. As illustrated in Fig. 6, the more solution concentration, the less terminal velocity. In the other words, the present of CTAB decreases bubble rising velocity, regardless of whether the concentration is higher or lower than CMC. It is related to absorption of surfactant molecules on the bubble surface and formation of a layer on it. For the first time, Yao *et al.* evaluated the effect of the presence of surfactant in the controlled turbulence flow. Their study points to the inverse relationship between bubble velocity and surfactant concentration and

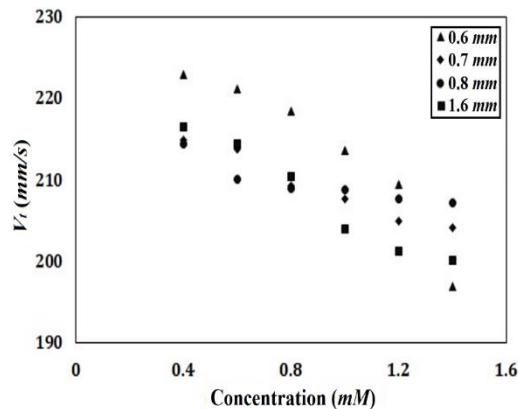


Fig. 6: Bubble terminal velocity versus CTAB concentration; needle diameter 0.6-1.6 mm.

the turbulence severity [39]. Similar results were observed by other researchers in their experiments [15, 36, 37]. They attributed the behavior of the bubble to the accumulation of surfactant behind the bubble, named stagnant cap, that reduced surface mobility [15]. This is called Marangoni effect described earlier. Consequently, the more dynamic structure of adsorption layer, the more bubble rising velocity [40].

To study the effect of concentration, Krzan *et al.* [41] also reported a decrease of about 59% in terminal velocity of bubbles with an equivalent diameter of 1.42 to 1.48 mm in CTAB solution. Wang *et al.* and Li *et al.* [36, 37] showed that at different concentrations between 0 to 0.33 mmole/l for their working fluid, the terminal velocity decreases from 300 to 190 mm/s. Moreover, comparing their results indicates that both type and concentration of surfactant would be affected the hydrodynamics of a rising bubble. The same results were also reported by Tzounakos *et al.* [15]. Furthermore, they stated that the decrease is observed only in bubbles with an equivalent diameter less than 0.8 cm and in larger diameters, the effect of concentration is negligible.

#### Range of Reynolds number

Fig. 7 shows the range of resulted Reynolds number in the experiments ( $850 < Re < 1000$ ) and its variation with CTAB concentration. As mentioned above, the viscosity decreases with increasing concentration, resulted in Reynolds number enhancement. On the other hand, according to Fig. 7, the increment is not very significant, which is due to the simultaneous decrease in rising velocity with increasing the concentration. In other words,



the reduction in velocity and viscosity are such that Reynolds changes slowly. According to the Reynolds number of experiments (relatively high), such a result was to be expected. *Tzounakos et al.* [15] showed that the increase was more severe in the lower Reynolds. Although their system was also contaminated water SDS. Therefore, it can be concluded that in low Reynolds numbers dependence of Reynolds to surfactant concentration is high and in medium and high Reynolds numbers the dependence will decrease. [42]

### Bubble rising drag coefficient

According to previous studies [10, 15, 36], for a rising bubble, the drag coefficient can be a function of different dimensionless groups such as Reynolds ( $Re$ ), Eötvös ( $Eo$ ) and Morton ( $Mo$ ). In the present study, the calculated drag coefficient was investigated in terms of  $Re$  (Fig. 8a),  $Eo$  (Fig. 8b), equivalent diameter (Fig. 9a), terminal velocity (Fig. 9b), concentration of CTAB (Fig. 10a), and aspect ratio (Fig. 10b). In the following, the obtained results are presented and compared with other finding of researchers. The remarkable thing about all these figures is that there is no noticeable difference in the behavior of the bubble rising at concentrations above and below CMC. It should be noted that in the present experiments the concentrations are selected in such a way that three of them are below CMC and two are above it.

As mentioned before, in the present work, the experiments were performed at relatively high Reynolds numbers ( $850 < Re < 1100$ ), which have been achieved thanks to the presence of surfactant. Fig.8(a) shows the drag coefficient as a function of Reynolds number in CTAB solution and has been compared with obtained value of pure water in *Tzounakos et al.*[15] work. The results indicate that the drag coefficient in the contaminated water not only is higher than the one in pure water, but also it slightly increases by Reynolds number. Whereas, it is reported a decrease and then an increase in drag coefficient with Reynolds number in studies that have examined a wide range of Reynolds numbers [11]. *Ziqi et al.* [29] had also examined the drag coefficient in the Reynolds number ranges from 130 to 1200 and reported a value between 0.2 and 1.4 for it. Moreover, they showed that the more Reynolds number, the more drag coefficient. According to the range of Reynolds number in the present study, the obtained results were in good agreement with *Ziqi et al.* [29] and other mentioned studies.

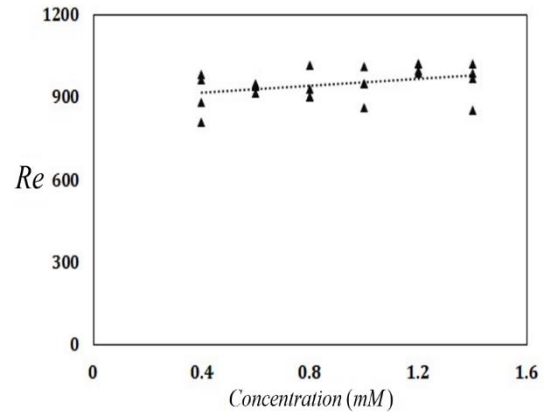


Fig. 7: Calculated Reynolds number at different concentration of CTAB; needle diameter 0.6-1.6 mm.

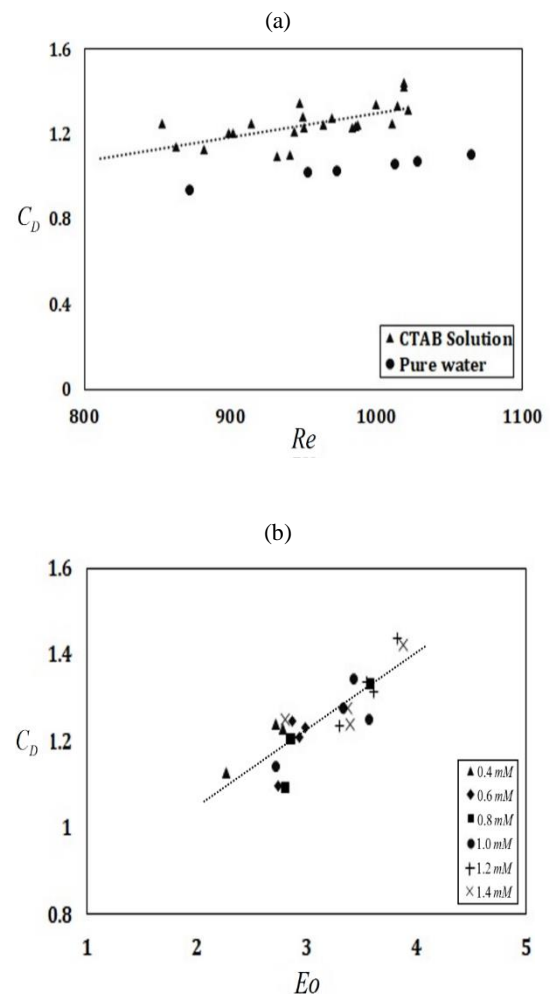


Fig. 8: bubble drag coefficient versus (a) Reynolds number, (b) Eötvös number; needle diameter 0.6-1.6 mm; CTAB concentration 0.4-1.4 mM.

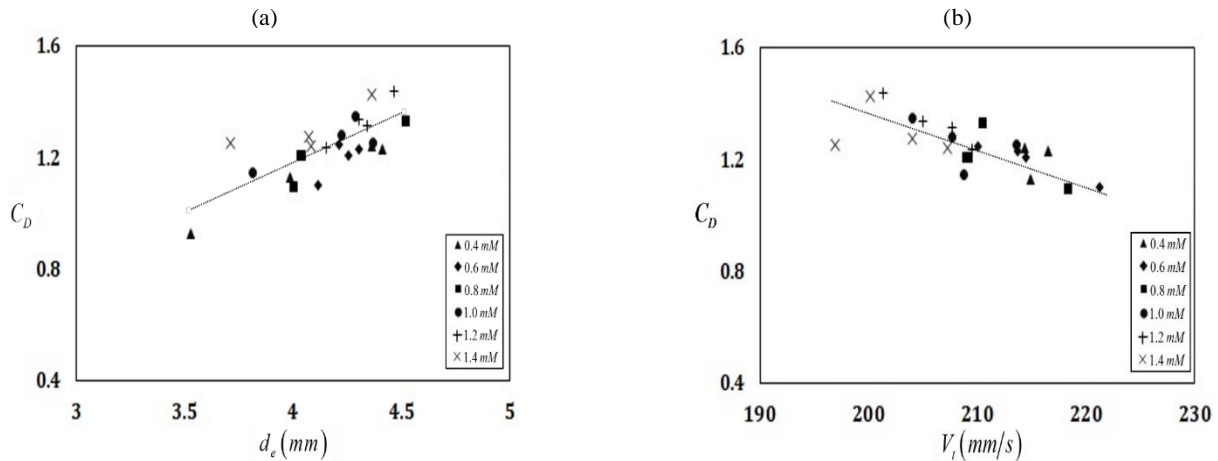


Fig. 9: Dependence of drag coefficient to (a) equivalent diameter, (b) terminal velocity; needle diameter 0.6-1.6 mm; CTAB concentration 0.4-1.4 mM.

In contrast to Reynolds, another dimensionless number that researchers have found to have a greater effect on bubble behavior, especially in contaminated water, is Eötvös number. Fig.8 (b) shows the relatively linear dependence of the drag coefficient on Eötvös number. Similar trend was also reported by Zhang *et al.*[43] when they examined the rising of bubbles in the presence of different aqueous solutions and by Kurimoto *et al.*[44] in studying the behavior of Taylor bubbles in the presence of surfactant. In a study conducted by Karimi *et al.*[10], the same relationship between drag coefficient and Eötvös number was observed when the behavior of a bubble in rhamnolipid solution in the presence of walls was investigated. They observed that the drag coefficient varies between 0.9 and 1.2 in the range of  $2.8 < Eo < 4.4$ . The reported results are in good agreement with the results of the present experiments.

Equivalent diameter is a fundamental parameter on which the drag coefficient is strongly dependent, as shown in Fig. 9 (a). Sun *et al.* [45] used different fluids in their study to investigate the effect of surface tension on the behavior of a bubble rising. For the range of  $3.5 \text{ mm} < d_e < 4.5 \text{ mm}$ , they estimated that drag coefficient varies from 0.2 to 1.2, depends on the type of the stagnant fluid. The results of Deng *et al.* [46] experiments, which were done on a rising droplet ranged from 0.1 to 10 mm in diameter, indicate that the drag coefficient first decreases by increasing diameter until it reaches a minimum value ( $d_e=2 \text{ mm}$ ), then increases. The uptrend is consistent with the results of the present experiment. However, the amounts of the reported drag coefficient in the two studies are different,

mainly due to the differences in the working fluids.

The balance of forces acting on a rising bubble indicates the dependence of drag coefficient on bubble velocity. The dependency is shown in Fig. 9(b). Accordingly, the more bubble velocity, the less drag coefficient. The same results were also reported by McHale *et al.* [28] for superhydrophobic droplet. They stated that the drag coefficient decreases with increasing terminal velocity which is attributed to the high Reynolds number.

As mentioned above, the terminal velocity decreases with increasing concentration and the drag coefficient decreases by enhancing velocity. Therefore, it can be resulted that drag coefficient and fluid concentration are directly related to each other. Fig. 10(a) shows similar behavior for a bubble rising in CTAB solutions. The result is in good agreement with the result of Tzounakos *et al.* [15]. They investigated the influence of surfactant on the drag coefficient and showed that the concentration of surfactant affects the drag coefficient. They stated that the main cause of this is the surface mobility of the bubble. According to their results, the drag coefficient tends to increase by increasing concentration.

Surfactant contaminated fluid affects bubble shape and rising bubble due to the accumulation of surfactant molecules on the bubble-liquid interface. The concentration changes the bubble's shape, which affects the drag coefficient. Fig.10 (b) shown the variation of the drag coefficient with aspect ratio. As it is clear, the obtained drag coefficient is directly related to aspect ratio. According to Fig.10 (b), as the bubble becomes more spherical, its drag coefficient increases.

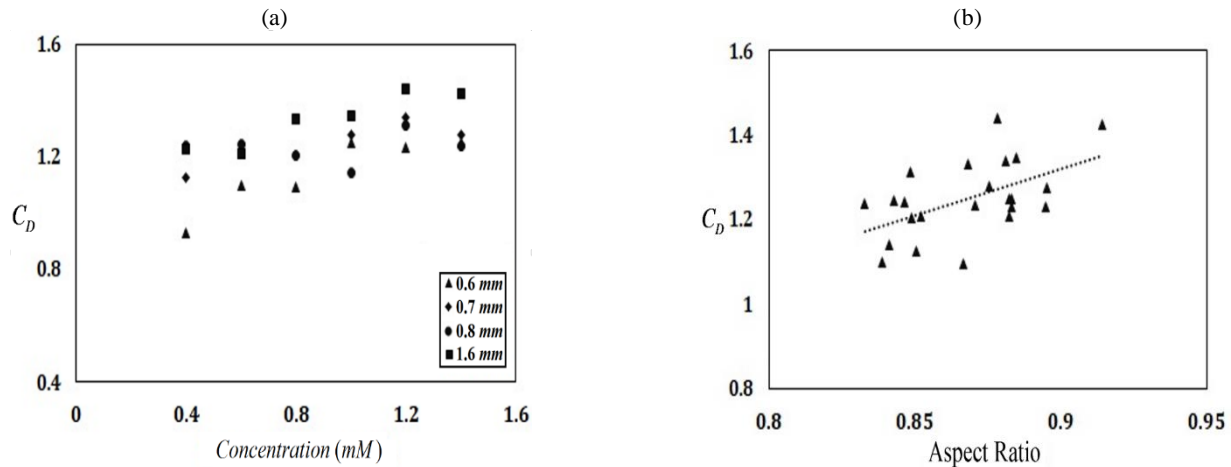


Fig. 10: Variations of bubble drag coefficient with (a) concentration in different diameter of needle, (b) aspect ratio; needle diameter 0.6-1.6 mm; CTAB concentration 0.4-1.4 mM.

## CONCLUSIONS

In the present study, the motion of a single bubble in a fluid contaminated with CTAB surfactant was studied. The operation variables are the liquid concentration and the diameter of the bubble controlled by needle diameter. Different important parameters such as terminal velocity, drag coefficient and aspect ratio were evaluated. Surfactant concentration is an important parameter that has a great effect on dimensionless numbers. It has been shown that by increasing the concentration of surfactant in the fluid, the drag coefficient increases relative to pure water. Increasing the fluid concentration reduces the bubble terminal velocity, due to the "Marangoni effect", the accumulation of surfactant molecules behind the bubble and the mobility of the bubble surface. According to the fact that the type of used fluid affects the shape of the bubble, in the present work, due to using CTAB surfactant, the aspect ratio is almost constant and equal to 0.86.

## Nomenclature

$C_D$	Drag coefficient
CMC	Critical micelle concentration
CTAB	Cetyltrimethylammonium Bromide
D	Diffusion coefficient
$d_H$	Horizontal diameter, mm
$d_e$	Equivalent diameter, mm
dp	Positional inaccuracy of the image
ds	Calibration error
$E_o$	Eötvös number
$E_r$	Relative Error

HLB	Hydrophile-lipophile balance
K	$6.25 \times 10^{-6} \text{mm}^2 (\text{s}^2 \cdot \text{pixel})^{-1}$
Mo	Morton number
$P_e$	Peclet number
$P_1$	Vertical placements of bubble centers in two consecutive photos that where taken
$P_2$	Vertical placements of bubble centers in two consecutive photos that where taken
Re	Reynolds number
S	Scale for accurate the bubble image determine
$V_t$	Terminal velocity, mm/s

## Greek

$\Delta V_t$	Uncertainty in determining the bubble velocity, mm/s
$d\Delta t$	$(60 \text{ fps})^{-2}$
$\Delta t$	Is the difference in time between two imaged (Reverse shutter speed), $(60 \text{ fps})^{-1}$
$\mu$	Liquid viscosity, cp
$\rho$	Densities of the liquid, $\text{kg/m}^3$
$\sigma$	Surface tension between fluid and bubble, N/m

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